



Holographic entanglement entropy in metal/superconductor phase transition with Born–Infeld electrodynamics

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Abstract

We investigate the holographic entanglement entropy in the metal/superconductor phase transition for the Born–Infeld electrodynamics with full backreaction and note that the entropy is a good probe to study the properties of the phase transition. For the operator $\langle \mathcal{O}_- \rangle$, we find that the entanglement entropy decreases (or increases) with the increase of the Born–Infeld parameter b in the metal (or superconducting) phase. For the operator $\langle \mathcal{O}_+ \rangle$, we observe that, with the increase of the Born–Infeld parameter, the entanglement entropy in the metal phase decreases monotonously but the entropy in the superconducting phase first increases and forms a peak at some threshold b_T , then decreases continuously. Moreover, the value of b_T becomes smaller as the width of the subsystem A decreases.

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1. Introduction

As the holographic principle provides a new insight into the investigation of strongly interacting condensed matter systems [1,2], there are a lot of works applying the anti-de Sitter/conformal

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field theory (AdS/CFT) duality [3–5] to condensed matter physics and in particular to superconductivity [6–22]. It states that the bulk AdS black hole becomes unstable and scalar hair condenses as one tunes the temperature for black hole. As a matter of fact, in order to understand the influences of the $1/N$ or $1/\lambda$ (λ is the 't Hooft coupling) corrections on the holographic superconductors, the higher derivative corrections to the gauge field should be taken into consideration. For the high order correction related to the gauge field, one of the important nonlinear electromagnetic theories is Born–Infeld electrodynamics [23–28]. As is well known, the Born–Infeld electrodynamics, which was proposed in 1934 by Born and Infeld to avoid the infinite self-energies for charged point particles arising in Maxwell theory [23], displays good physical properties including the absence of shock waves and birefringence. It was also found that the Born–Infeld electrodynamics is the only possible nonlinear version of electrodynamics that is invariant under electromagnetic duality transformation [24]. Jing and Chen observed that the Born–Infeld coupling parameter makes it harder for the scalar condensation to form [29]. Then, the analytic study of properties of holographic superconductors in Born–Infeld electrodynamics was presented in Ref. [30]. In this paper, we would like to investigate the phase transition in the Born–Infeld electrodynamics with full backreaction of the matter fields electrodynamics on the AdS black hole geometry.

Recently, according to the AdS/CFT duality, Ryu and Takayanagi [31,32] have presented a proposal to compute the entanglement entropy of conformal field theories (CFTs) from the minimal area surface in gravity side. Since this proposal provides a simple and elegant way to calculate the entanglement entropy of a strongly coupled system which has a gravity dual, the holographic entanglement entropy is widely used to study various properties of holographic superconductors at low temperatures [33–41]. The entanglement entropy in the metal/superconductor system was studied in Ref. [42]. It was shown that the entanglement entropy in superconducting phase is always less than the one in the metal case and the entropy as a function of temperature is found to have a discontinuous slope at the transition temperature T_c in the case of the second order phase transition. Ref. [43] considered the case with higher derivative corrections and studied the holographic entanglement entropy in Gauss–Bonnet gravity. Ref. [44] studied the holographic entanglement entropy for general higher derivative gravity and proposed a general formula for calculating the entanglement entropy in theories dual to higher derivative gravity. Then, the holographic entanglement entropy in the insulator/superconductor model was studied in Refs. [45–48] and it turned out that the entanglement entropy is a good probe to investigate the holographic phase transition. Furthermore, Kuang et al. examined the properties of the entanglement entropy in the four-dimensional AdS black hole and found that the entanglement entropy can be considered as a probe of the proximity effect of a superconducting system by using the gauge/gravity duality in a fully backreacted gravity system [49]. More recently, the entanglement entropy of general Stückelberg models both in AdS soliton and AdS black hole backgrounds with full backreaction was studied in Ref. [50]. However, the models mentioned above are all in the frame of Maxwell electromagnetic theory. It is of interest to investigate holographic entanglement entropy in the nonlinear electromagnetic generalization. We have studied in Ref. [51] the holographic entanglement entropy in the insulator/superconductor phase transition for the Born–Infeld electrodynamics, and found that the entanglement entropy increases with the increase of the Born–Infeld factor in the superconductor phase and the critical width ℓ of confinement/deconfinement phase transition is dependent of the Born–Infeld parameter. As a further study along this line, in this paper we will extend the previous works to investigate the entanglement entropy in metal/superconductor phase transition with Born–Infeld electrodynamics. We find that the entanglement entropy decreases with the increase of the Born–Infeld parameter

b for both operators in the metal phase. However, In the superconducting phase, the entanglement entropy of operator $\langle O_- \rangle$ increases with the increase of Born–Infeld factor. Interestingly, for the operator $\langle O_+ \rangle$, the effect of the Born–Infeld parameter on the entanglement entropy is non-monotonic and the value of the threshold b_T is related to the belt width of subsystem A .

The framework of this paper is as follows. In Section 2, we will derive the equations of motions and introduce the boundary conditions of the holographic model. In Section 3, we study the phase transition with Born–Infeld electrodynamics in AdS black hole spacetime. In Section 4, we calculate the holographic entanglement entropy in AdS black hole gravity with Born–Infeld electrodynamics. In Section 5, we conclude our main results of this paper.

2. Equations of motion and boundary conditions

The action for the gravity and Born–Infeld electromagnetic field coupling with a charged scalar field is described by

$$S = \int d^d x \sqrt{-g} \left[\frac{1}{16\pi G} (R - 2\Lambda) \right] + \int d^d x \sqrt{-g} \left[\frac{1}{b^2} \left(1 - \sqrt{1 + \frac{b^2 F^{\mu\nu} F_{\mu\nu}}{2}} \right) - |\nabla\Psi - iqA\Psi|^2 - m^2|\Psi|^2 \right], \tag{1}$$

where g is the determinant of the metric, ψ represents a scalar field with charge q and mass m , $\Lambda = -(d-1)(d-2)/2L^2$ is the cosmological constant, A is the gauge field, $F^{\mu\nu}$ is the strength of the Born–Infeld electrodynamic field $F = dA$, and b is the Born–Infeld coupling parameter. In the limit $b \rightarrow 0$, the Born–Infeld field will reduce to the Maxwell field.

To study the holographic entanglement entropy in Born–Infeld electrodynamics, we will take the full backreaction into account. The metric for the planar black hole can be taken as

$$ds^2 = -f(r)e^{-\chi(r)} dt^2 + \frac{dr^2}{f(r)} + r^2 h_{ij} dx^i dx^j. \tag{2}$$

The Hawking temperature of this black hole is

$$T_H = \frac{f'(r_+)e^{-\chi(r_+)/2}}{4\pi}, \tag{3}$$

where r_+ is the black hole horizon.

We consider the electromagnetic field and the scalar field in the forms

$$A = \phi(r)dt, \quad \psi = \psi(r). \tag{4}$$

Then, the equations of motion from the variation of the action with respect to the matter and metric can be obtained as

$$\psi'' + \left(\frac{d-2}{r} - \frac{\chi'}{2} + \frac{f'}{f} \right) \psi' + \frac{1}{f} \left(\frac{q^2 e^\chi \phi^2}{f} - m^2 \right) \psi = 0, \tag{5}$$

$$\phi'' + \left(\frac{d-2}{r} + \frac{\chi'}{2} \right) \phi' - \frac{(d-2)b^2 e^\chi}{r} \phi'^3 - \frac{2q^2 \psi^2 (1 - b^2 e^\chi \phi'^2)^{\frac{3}{2}}}{f} \phi = 0, \tag{6}$$

$$\chi' + \frac{2r}{d-2} \left(\psi'^2 + \frac{q^2 e^\chi \phi^2 \psi^2}{f^2} \right) = 0, \tag{7}$$

$$f' - \left(\frac{(d-1)r}{L} - \frac{(d-3)f}{r} \right) + \frac{r}{d-2} \left[m^2 \psi^2 + f \left(\psi'^2 + \frac{q^2 e^\chi \phi^2 \psi^2}{f^2} \right) + \frac{1 - \sqrt{1 - b^2 e^\chi \phi'^2}}{b^2 \sqrt{1 - b^2 e^\chi \phi'^2}} \right] = 0, \tag{8}$$

where a prime denotes the derivative with respect to r , and $16\pi G = 1$ was used. For this system, the useful scaling symmetries are

$$r \rightarrow \alpha r, \quad (x, y, t) \rightarrow (x, y, t)/\alpha, \quad \phi \rightarrow \alpha \phi, \quad f \rightarrow \alpha^2 f, \tag{9}$$

$$L \rightarrow \alpha L, \quad r \rightarrow \alpha r, \quad t \rightarrow \alpha t, \quad q \rightarrow \alpha^{-1} q, \tag{10}$$

$$e^\chi \rightarrow \alpha^2 e^\chi, \quad \phi \rightarrow \alpha^{-1} \phi, \quad t \rightarrow t\alpha. \tag{11}$$

Using the scaling symmetries (9), we can take $r_+ = 1$. Further employing the symmetries (10), we can let $L = 1$.

At the horizon r_+ , the regularity condition gives the boundary conditions

$$\phi(r_+) = 0, \quad f(r_+) = 0. \tag{12}$$

And at the asymptotic AdS boundary ($r \rightarrow \infty$), the asymptotic behaviors of the solutions are

$$\chi \rightarrow 0, \quad f \sim r^2, \quad \phi \sim \mu - \frac{\rho}{r^{d-3}}, \quad \psi \sim \frac{\psi_-}{r^{\Delta_-}} + \frac{\psi_+}{r^{\Delta_+}}, \tag{13}$$

where μ and ρ are interpreted as the chemical potential and charge density in the dual field theory, and the exponent Δ_\pm is defined by $((d-1) \pm \sqrt{(d-1)^2 + 4m^2})/2$ for d -dimensional spacetime. Notice that, provided Δ_- is larger than the unitarity bound, both ψ_- and ψ_+ can be normalizable. According to the AdS/CFT correspondence, they correspond to the vacuum expectation values $\psi_- = \langle \mathcal{O}_- \rangle$, $\psi_+ = \langle \mathcal{O}_+ \rangle$ of an operator \mathcal{O} dual to the scalar field [52,53]. In the following calculation, we impose boundary condition that either ψ_- or ψ_+ vanishes.

3. Phase transition with Born–Infeld electrodynamics

In this section, we will concretely study the phase transition for Born–Infeld electrodynamics with full backreaction in the 4-dimensional AdS black hole spacetime. For the normal phase, the metric becomes the Reissner–Nordström AdS black hole as the Born–Infeld factor approaches to zero. Thus, we have

$$\chi = \psi = 0, \quad \phi = \rho \left(1 - \frac{1}{r} \right), \quad f = r^2 - \frac{1}{r} \left(1 + \frac{\rho^2}{4} \right) + \frac{\rho^2}{4r^2}. \tag{14}$$

However, if the Born–Infeld factor is not equal to zero, the solution is the Born–Infeld AdS black hole.

For purpose of getting the solutions in superconducting phase where $\psi(r) \neq 0$, we can introduce a new variable $z = r_+/r$. Then, the equations of motion can be rewritten as

$$\psi'' - \left(\frac{\chi'}{2} - \frac{f'}{f} \right) \psi' - \frac{1}{z^4 f} \left(m^2 - \frac{e^\chi q^2 \phi^2}{f} \right) \psi = 0, \tag{15}$$

$$\phi'' + \frac{1}{2} \chi' \phi' + 2z^3 b^2 e^\chi \phi'^3 - \frac{2q^2 \psi^2 (1 - b^2 e^\chi z^4 \phi'^2)^{\frac{3}{2}}}{z^4 f} \phi = 0, \tag{16}$$

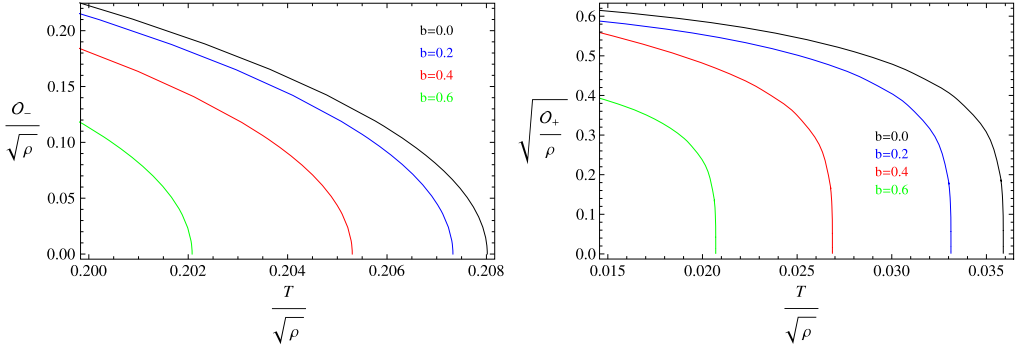


Fig. 1. (Color online.) The operators O_- (left plot) and O_+ (right plot) versus temperature after condensation with different values of Born–Infeld parameter. The four lines from top to bottom correspond to $b = 0$ (black), $b = 0.2$ (blue), $b = 0.4$ (red), and $b = 0.6$ (green), respectively.

$$\chi' - z\psi'^2 - \frac{e^X q^2 \phi^2 \psi^2}{z^3 f^2} = 0, \tag{17}$$

$$f' - \frac{f}{z} + \frac{3r_+^2}{z^3} - \frac{1}{2z^3} \left[m^2 \psi^2 + f \left(z^4 \psi'^2 + \frac{1}{f^2} e^X q^2 \phi^2 \psi^2 \right) + \frac{1 - \sqrt{1 - b^2 z^4 e^X \phi'^2}}{b^2 \sqrt{1 - b^2 z^4 e^X \phi'^2}} \right] = 0, \tag{18}$$

where the prime now denotes the derivative with respect to z . Using the shooting method, we can solve the equations of motion numerically and then discuss the effects of the Born–Infeld parameter b on the condensation of the scalar operators. In this paper, we set $m^2 = -2$ and $q = 1$. Since there are scaling symmetries described by Eq. (9) for the equations of motion, the following quantities can be rescaled as

$$\mu \rightarrow \alpha \mu, \quad \rho \rightarrow \alpha^2 \rho, \quad \langle O_- \rangle \rightarrow \alpha \langle O_- \rangle, \quad \langle O_+ \rangle \rightarrow \alpha^2 \langle O_+ \rangle. \tag{19}$$

In Fig. 1, we plot the behaviors of condensate with the changes of the temperature and the Born–Infeld parameter in the dimensionless quantities $\langle O_- \rangle / \sqrt{\rho}$, $\sqrt{\langle O_+ \rangle} / \sqrt{\rho}$ and $T / \sqrt{\rho}$. From the left plot of Fig. 1, we can see that if the temperature $T > T_c$, there is no condensation and this can be thought of as the metal phase. However, when the temperature decreases to be lower than the critical value T_c , the condensation of the operator $\langle O_- \rangle$ emerges and this corresponds to a superconducting phase. It should be noted that the value of the critical temperature T_c becomes smaller as the Born–Infeld factor b increases, which means that the Born–Infeld correction to the usual Maxwell field makes the scalar hair harder to form in the full-backreaction model. For the operator $\langle O_+ \rangle$ (right plot), we also find that the critical temperature T_c decreases with the increase of the parameter b . Our result is the same as the result in Ref. [53] as the factor b approaches to zero.

4. Holographic entanglement entropy of the holographic model

After obtaining the solutions to the metal phase and superconducting phase for the Born–Infeld electrodynamics in the AdS black hole geometry with full backreaction, we are now ready to study the behavior of holographic entanglement entropy in the Born–Infeld electrodynamics.

The entanglement entropy in conformal field theories can be calculated from the area of minimal surface in AdS spaces [31,32], and its formula is given by the “area law”

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N}, \tag{20}$$

where G_N is the Newton constant in the Einstein gravity on the AdS space, S_A is the entanglement entropy for the subsystem A which can be chosen arbitrarily, γ_A is the minimal area surface in the bulk which ends on the boundary of A .

We consider the entanglement entropy for a straight geometry which is described by $-\frac{l}{2} \leq x \leq \frac{l}{2}$ and $-\frac{R}{2} < y < \frac{R}{2}$ ($R \rightarrow \infty$), where l is defined as the size of region A . The holographic surface γ_A starts from $r = \frac{1}{\epsilon}$ at $x = \frac{\ell}{2}$, extends into the bulk until it reaches $r = r_*$, then returns back to the AdS boundary $r = \frac{1}{\epsilon}$ at $x = -\frac{\ell}{2}$. Thus, the induced metric on γ_A can be obtained as follows

$$ds^2 = \left[\frac{1}{f(r)} + r^2 \left(\frac{dx}{dr} \right)^2 \right] dr^2 + r^2 dy^2. \tag{21}$$

By using the proposal given by Eq. (20), the entanglement entropy in the strip geometry is

$$S_A = \frac{R}{2G_4} \int_{\frac{1}{\epsilon}}^{z_*} dz \frac{z^2}{z^2 \sqrt{(z_*^4 - z^4)z^2 f(z)}} = \frac{R}{2G_4} \left(\frac{1}{\epsilon} + s \right), \tag{22}$$

with

$$\frac{l}{2} = \int_{\frac{1}{\epsilon}}^{z_*} dz \frac{z^2}{\sqrt{(z_*^4 - z^4)z^2 f(z)}}, \tag{23}$$

where z_* satisfies the condition $\frac{dz}{dx}|_{z_*} = 0$ with $z = \frac{1}{r}$. The term $1/\epsilon$ in Eq. (22) is divergent, while the term s is a finite term which is physically important. Under the scaling symmetries of Eq. (9), we can rescale the l and s as

$$l \rightarrow \alpha^{-1}l, \quad s \rightarrow \alpha s. \tag{24}$$

Therefore, in the following calculation we can use these dimensionless quantities

$$\ell\sqrt{\rho}, \quad s/\sqrt{\rho}. \tag{25}$$

We now show the behavior of the holographic entanglement entropy of the operator $\langle O_- \rangle$ or $\langle O_+ \rangle$ with respect to the temperature T , the Born–Infeld factor b and the belt width l , respectively.

4.1. Holographic entanglement entropy of the operator $\langle O_- \rangle$

The behavior of the entanglement entropy of the operator $\langle O_- \rangle$ is shown in Fig. 2 in which the dot-dashed lines describe the normal phases and the solid ones show the superconducting phases. It can be seen from the figure that the entanglement entropy presents a discontinuous change at a critical temperatures T_c denoted by vertical dashed lines for different strengths of the Born–Infeld parameter b . The discontinuous change of the entanglement entropy indicates the phase transition point from the normal state to the superconducting state and the value of the T_c becomes smaller with the increase of the Born–Infeld factor b . Which indicates that the holographic entanglement

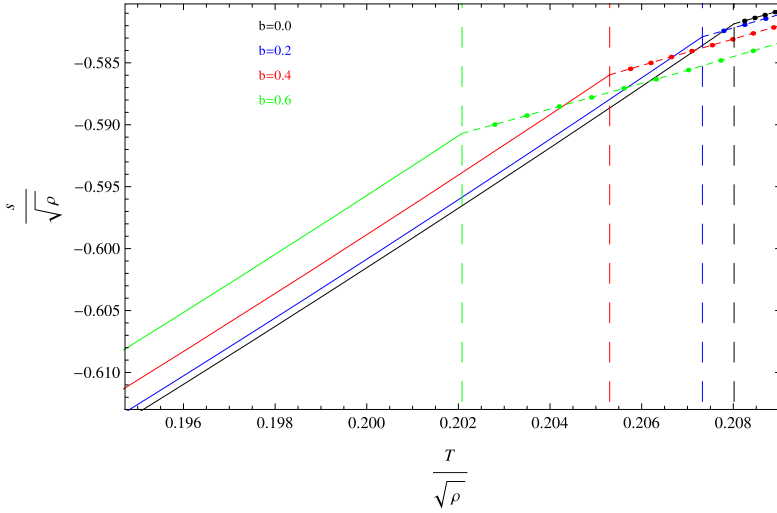


Fig. 2. (Color online.) The entanglement entropy of the operator $\langle O_- \rangle$ as a function of the temperature T and the Born–Infeld factor b for $\ell\sqrt{\rho} = 1$. The four dot-dashed lines from top to bottom correspond to $b = 0$ (black), $b = 0.2$ (blue), $b = 0.4$ (red), $b = 0.6$ (green), but the four solid lines from bottom to top are for $b = 0$ (black), $b = 0.2$ (blue), $b = 0.4$ (red), and $b = 0.6$ (green), respectively.

entropy is a good probe to study the properties of the phase transition. The figure also shows that the entanglement entropy decreases as the Born–Infeld parameter increases for the normal phase, but increases as the Born–Infeld parameter increases for the superconducting state. Moreover, the entanglement entropy in superconducting phase is less than the one in the normal case and drops as the temperature decreases, and this property holds for different values of the parameter b . This behavior of the entanglement entropy is due to the fact that the metal phase can be thought of as the one filled with free charge carriers, such as electrons. The condensate turns on at the critical temperature and the free charge carriers are continuously condensed to Cooper pairs as temperature decreases. Therefore, the formation of Cooper pairs make the degrees of freedom decrease in the superconducting phase.

The entanglement entropy as a function of the parameter b for different ℓ in the superconducting phase is shown in Fig. 3. For fixed belt width ℓ , the entanglement entropy becomes smaller as the Born–Infeld factor b decreases. This is because that the condensation becomes stronger with higher condensation gap for smaller parameter b at low temperature so that the number of Cooper pairs is increased, which results in less degree of freedom. On the other hand, for a given Born–Infeld factor b , with the decrease of belt width ℓ the entanglement entropy also decreases.

4.2. Holographic entanglement entropy of the operator $\langle O_+ \rangle$

The behavior of the entanglement entropy of the operator $\langle O_+ \rangle$ as a function of temperature T and the Born–Infeld factor b is described by Fig. 4. It is shown that the critical temperature T_c of the phase transition decreases as the Born–Infeld factor b increases. That is to say, the stronger Born–Infeld electrodynamics correction makes the scalar condensation harder to form. We also find that the change of the entanglement entropy is discontinuous at T_c and the entanglement entropy in the hair phase is less than the one in the normal phase. Interestingly, in the superconducting phase, the dependence of the entanglement entropy on the Born–Infeld factor b

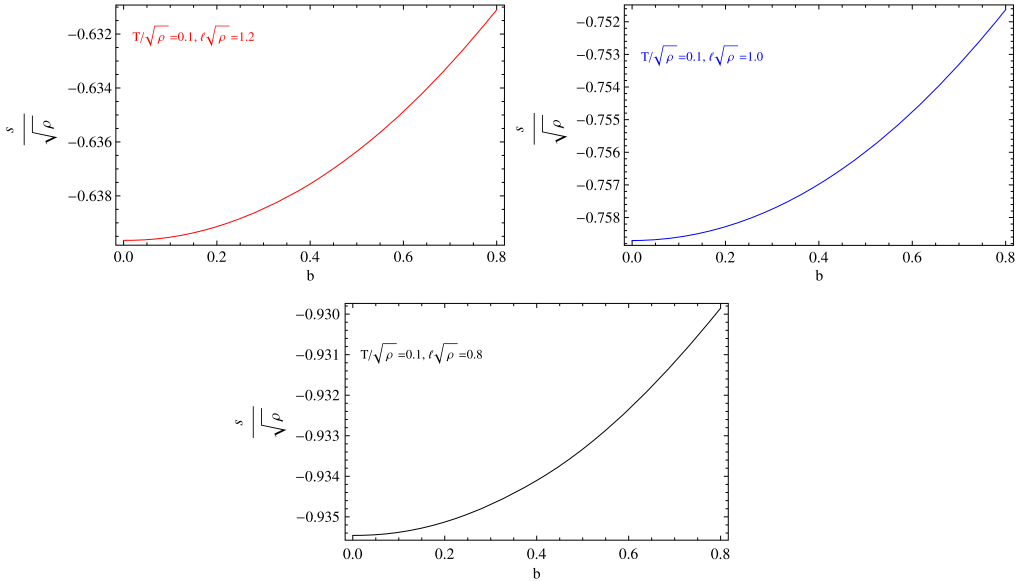


Fig. 3. (Color online.) The entanglement entropy of the operator $\langle O_- \rangle$ as a function of the Born–Infeld factor b for different widths ℓ as $T/\sqrt{\rho} = 0.10$. The top left plot (red) is for $\ell\sqrt{\rho} = 1.2$, the top right one (blue) for $\ell\sqrt{\rho} = 1.0$, and the bottom one (black) for $\ell\sqrt{\rho} = 0.8$.

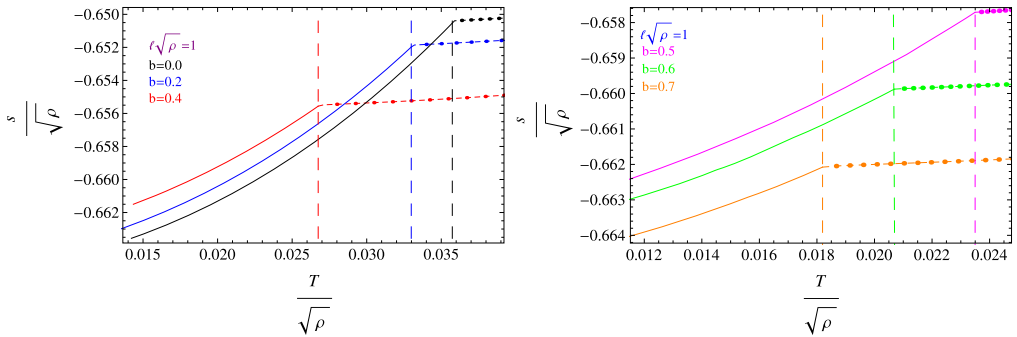


Fig. 4. (Color online.) The entanglement entropy of the operator $\langle O_+ \rangle$ as a function of the temperature T and the Born–Infeld factor b with $\ell\sqrt{\rho} = 1$. The vertical dashed lines represent the critical temperature of the phase transition for the different values of the Born–Infeld factor. The dot-dashed lines are from the normal phase and the solid ones are from the superconducting cases. In the left plot, the three lines from bottom to top correspond to $b = 0$ (black), $b = 0.2$ (blue) and $b = 0.4$ (red), but in the right one are for $b = 0.7$ (orange), $b = 0.6$ (green), $b = 0.5$ (magenta), respectively.

is non-monotonic. In the left plot of Fig. 4, we find that the entanglement entropy increases with the increase of the factor b when $b < b_T$. However, in the right plot of Fig. 4, we can see that the entanglement entropy decreases with the increase of the factor b when $b > b_T$.

To further illustrate the effect of the Born–Infeld factor b on the entanglement entropy of the operator $\langle O_+ \rangle$ in the superconducting phase, we plot the entanglement entropy of the operator $\langle O_+ \rangle$ as a function of Born–Infeld factor b for different widths ℓ with $T/\sqrt{\rho} = 0.010$ in Fig. 5. Obviously, with the increase of the factor b , the entanglement entropy first rises and arrives at its maximum as $b = b_T$, then decreases monotonously. This process implies that there is some kind

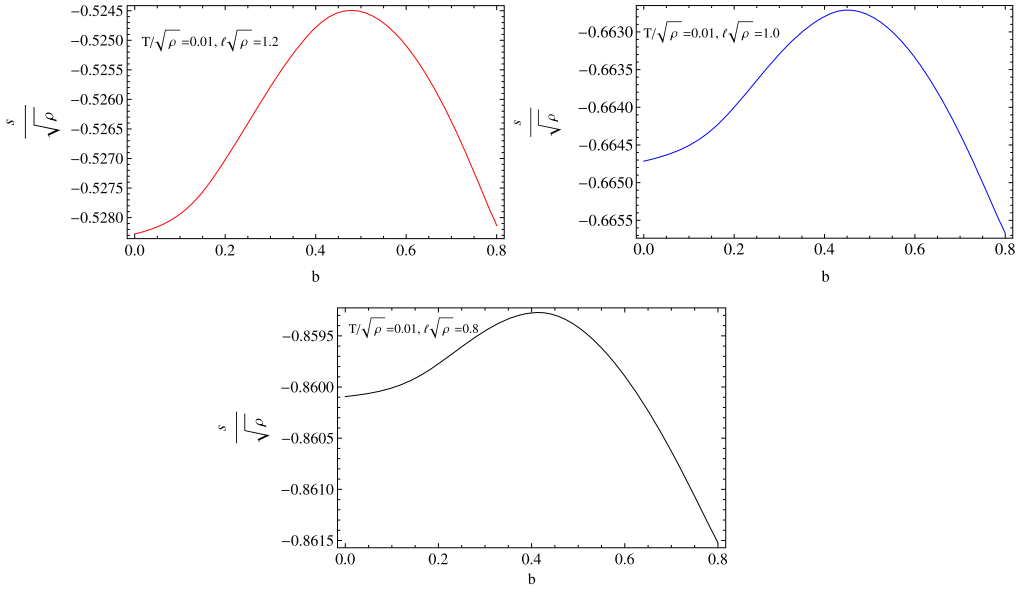


Fig. 5. (Color online.) The entanglement entropy of the operator $\langle O_+ \rangle$ as a function of the Born–Infeld factor b for different widths ℓ with $T/\sqrt{\rho} = 0.010$. The top left plot (red) is for $\ell\sqrt{\rho} = 1.2$, the top right one (blue) for $\ell\sqrt{\rho} = 1.0$, and the bottom one (black) for $\ell\sqrt{\rho} = 0.8$.

of the significant reorganization of the degrees of freedom. And the threshold b_T becomes smaller as the width of the subsystem ℓ decreases. For the fixed Born–Infeld factor b , the entanglement entropy decreases with the decrease of belt width ℓ .

5. Summary

We studied the behaviors of the holographic entanglement entropy in the metal/superconductor phase transition for the Born–Infeld electrodynamics with full backreaction. By calculating the entanglement entropy of the system, we noted that the critical temperature of the condensation for the operators $\langle O_- \rangle$ and $\langle O_+ \rangle$ becomes smaller with the increase of the Born–Infeld parameter b , which implies that the Born–Infeld factor makes the scalar condensation harder to form. Both for the operators $\langle O_- \rangle$ and $\langle O_+ \rangle$, we found that the entanglement entropy in the superconducting phase is less than the one in the normal phase, and drops as the temperature decreases for the fixed parameter b and belt width ℓ . This is due to the fact that the formation of Cooper pairs makes the degrees of freedom decrease in the hair phase. For a given temperature, we observed that the entanglement entropy of the operator $\langle O_- \rangle$ in the metal (or superconducting) phase decreases (or increases) with the increase of the Born–Infeld factor b for the fixed ℓ . Interestingly, the influence of the Born–Infeld factor b on the entanglement entropy of the operator $\langle O_+ \rangle$ is nontrivial. The entanglement entropy of the operator $\langle O_+ \rangle$ in the metal phase decreases with the increase of the Born–Infeld parameter. However, with the increase of the Born–Infeld parameter b for the fixed belt width ℓ of subsystem A , the entanglement entropy of the operator $\langle O_+ \rangle$ in the superconducting phase first increases and reaches the maximum at some threshold b_T , then decreases monotonously. The threshold b_T becomes smaller as the width of the subsystem A

decreases. This process implies that there is some kind of the significant reorganization of the degrees of freedom which should be further studied in the future.

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References

- [1] G. 't Hooft, Dimensional reduction in quantum gravity, THU-93/26, arXiv:gr-qc/9310026, 1993.
- [2] L. Susskind, The world as a hologram, *Math. Phys.* 36 (1995) 6377–6396, arXiv:hep-th/9409089.
- [3] J.M. Maldacena, The large N limit of superconformal field theories and supergravity, *Adv. Theor. Math. Phys.* 2 (1998) 231–252, arXiv:hep-th/9711200.
- [4] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Gauge theory correlators from noncritical string theory, *Phys. Lett. B* 428 (1998) 105–114, arXiv:hep-th/9802109.
- [5] E. Witten, Anti-de Sitter space and holography, *Adv. Theor. Math. Phys.* 2 (1998) 253–291, arXiv:hep-th/9802150.
- [6] X.H. Ge, B. Wang, S.F. Wu, G.H. Yang, Analytical study on holographic superconductors in external magnetic field, *J. High Energy Phys.* 08 (2010) 108.
- [7] R. Gregory, S. Kanno, J. Soda, Holographic superconductors with higher curvature corrections, *J. High Energy Phys.* 0910 (2009) 010, arXiv:0907.3203 [hep-th].
- [8] E. Nakano, W.Y. Wen, Critical magnetic field in a holographic superconductor, *Phys. Rev. D* 78 (2008) 046004.
- [9] I. Amado, M. Kaminski, K. Landsteiner, Hydrodynamics of holographic superconductors, *J. High Energy Phys.* 05 (2009) 021.
- [10] G. Koutsoumbas, E. Papantonopoulos, G. Siopsis, Exact gravity dual of a gapless superconductor, *J. High Energy Phys.* 07 (2009) 026.
- [11] O.C. Umeh, Scanning the parameter space of holographic superconductors, *J. High Energy Phys.* 08 (2009) 062.
- [12] J. Sonner, Rotating holographic superconductor, *Phys. Rev. D* 80 (2009) 084031.
- [13] S. Franco, A.M. Garcia-Garcia, D. Rodriguez-Gomez, Holographic approach to phase transitions, *Phys. Rev. D* 81 (2010) 041901(R).
- [14] C.P. Herzog, Analytic holographic superconductor, *Phys. Rev. D* 81 (2010) 126009, arXiv:1003.3278 [hep-th].
- [15] G.T. Horowitz, M.M. Roberts, Holographic superconductors with various condensates, *Phys. Rev. D* 78 (2008) 126008.
- [16] R.A. Konoplya, A. Zhidenko, Holographic conductivity of zero temperature superconductors, *Phys. Lett. B* 686 (2010) 199.
- [17] G. Siopsis, J. Therrien, Analytic calculation of properties of holographic superconductors, *J. High Energy Phys.* 05 (2010) 013.
- [18] K. Maeda, M. Natsuume, T. Okamura, Universality class of holographic superconductors, *Phys. Rev. D* 79 (2009) 126004.
- [19] R.G. Cai, Z.Y. Nie, H.Q. Zhang, Holographic p-wave superconductors from Gauss–Bonnet gravity, *Phys. Rev. D* 82 (2010) 066007.
- [20] Q.Y. Pan, B. Wang, General holographic superconductor models with Gauss–Bonnet corrections, *Phys. Lett. B* 693 (2010) 159.
- [21] Oriol Domènech, Marc Montull, Alex Pomarol, Alberto Salvio, Pedro J. Silva, Emergent gauge fields in holographic superconductors, *J. High Energy Phys.* 033 (2010) 1008.
- [22] J.L. Jing, L.C. Wang, Q.Y. Pan, S.B. Chen, Holographic superconductors in Gauss–Bonnet gravity with Born–Infeld electrodynamics, *Phys. Rev. D* 83 (2011) 066010.
- [23] M. Born, L. Infeld, Foundations of the new field theory, *Proc. R. Soc. A* 144 (1934) 425.
- [24] G.W. Gibbons, D.A. Rasheed, Electric–magnetic duality rotations in non-linear electrodynamics, *Nucl. Phys.* 454 (1995) 185.
- [25] B. Hoffmann, Gravitational and electromagnetic mass in the Born–Infeld electrodynamics, *Phys. Rev.* 47 (1935) 877.
- [26] W. Heisenberg, H. Euler, Folgerungen aus der Diracschen Theorie des Positrons, *Z. Phys.* 98 (1936) 714.

- [27] H.P. de Oliveira, Non-linear charged black holes, *Class. Quantum Gravity* 11 (1994) 1469.
- [28] O. Miskovic, R. Olea, Conserved charges for black holes in Einstein–Gauss–Bonnet gravity coupled to nonlinear electrodynamics in AdS space, *Phys. Rev. D* 83 (2011) 024011.
- [29] J.L. Jing, S.B. Chen, Holographic superconductors in the Born–Infeld electrodynamics, *Phys. Lett. B* 686 (2010) 68.
- [30] Sunandan Gangopadhyay, Dibakar Roychowdhury, Analytic study of properties of holographic superconductors in Born–Infeld electrodynamics, *J. High Energy Phys.* 05 (2012) 002.
- [31] S. Ryu, T. Takayanagi, Entanglement entropy and the Berry phase in the solid state, *Phys. Rev. Lett.* 96 (2006) 181602.
- [32] S. Ryu, T. Takayanagi, Aspects of holographic entanglement entropy, *J. High Energy Phys.* 0608 (2006) 045.
- [33] T. Nishioka, S. Ryu, T. Takayanagi, Holographic entanglement entropy: an overview, *J. Phys. A* 42 (2009) 504008.
- [34] T. Albash, C.V. Johnson, Holographic entanglement entropy and renormalization group flow, *J. High Energy Phys.* 1202 (2012) 095.
- [35] R.C. Myers, A. Singh, Comments on holographic entanglement entropy and RG flows, *J. High Energy Phys.* 1204 (2012) 122.
- [36] J. de Boer, M. Kulaxizi, A. Parnachev, Holographic entanglement entropy in Lovelock gravities, *J. High Energy Phys.* 1107 (2011) 109.
- [37] L.Y. Hung, R.C. Myers, M. Smolkin, On holographic entanglement entropy and higher curvature gravity, *J. High Energy Phys.* 1104 (2011) 025.
- [38] T. Nishioka, T. Takayanagi, AdS bubbles, entropy and closed string tachyons, *J. High Energy Phys.* 0701 (2007) 090.
- [39] I.R. Klebanov, D. Kutasov, A. Murugan, Entanglement as a probe of confinement, *Nucl. Phys. B* 796 (2008) 274.
- [40] A. Pakman, A. Parnachev, Topological entanglement entropy and holography, *J. High Energy Phys.* 0807 (2008) 097.
- [41] Raúl E. Arias, Ignacio Salazar Landea, Backreacting p-wave Superconductors, *J. High Energy Phys.* 01 (2013) 157.
- [42] T. Albash, C.V. Johnson, Holographic studies of entanglement entropy in superconductors, *J. High Energy Phys.* 1205 (2012) 079, arXiv:1202.2605 [hep-th].
- [43] N. Ogawa, T. Takayanagi, Higher derivative corrections to holographic entanglement entropy for AdS solitons, *J. High Energy Phys.* 1110 (2011) 147.
- [44] Xi Dong, Holographic entanglement entropy for general higher derivative gravity, arXiv:1310.5713 [hep-th].
- [45] R.G. Cai, S. He, L. Li, Y.L. Zhang, Holographic entanglement entropy on P-wave superconductor phase transition, *J. High Energy Phys.* 1207 (2012) 027.
- [46] Li-Fang Li, Rong-Gen Cai, Li Li, Chao Shen, Entanglement entropy in a holographic p-wave superconductor model, arXiv:1310.6239 [hep-th].
- [47] R.G. Cai, S. He, L. Li, Y.L. Zhang, Holographic entanglement entropy in insulator/superconductor transition, *J. High Energy Phys.* 1207 (2012) 088.
- [48] R.G. Cai, S. He, L. Li, L.F. Li, Entanglement entropy and Wilson loop in Stückelberg holographic insulator/superconductor model, *J. High Energy Phys.* 1210 (2012) 107.
- [49] X.M. Kuang, E. Papantonopoulos, B. Wang, Entanglement entropy as a probe of the proximity effect in holographic superconductors, *J. High Energy Phys.* 05 (2014) 130.
- [50] Yan Peng, Qiyuan Pan, Holographic entanglement entropy in general holographic superconductor models, *J. High Energy Phys.* 06 (2014) 011.
- [51] Weiping Yao, Jiliang Jing, Holographic entanglement entropy in insulator/superconductor transition with Born–Infeld electrodynamics, *J. High Energy Phys.* 05 (2014) 058.
- [52] S.A. Hartnoll, C.P. Herzog, G.T. Horowitz, Building a holographic superconductor, *Phys. Rev. Lett.* 101 (2008) 031601.
- [53] S.A. Hartnoll, C.P. Herzog, G.T. Horowitz, Holographic superconductors, *J. High Energy Phys.* 12 (2008) 015.