ELSEVIER

CrossMark

Available online at www.sciencedirect.com

ScienceDirect

Procedia Economics and Finance 22 (2015) 655 – 662

Procedia
Economics and Finance

www.elsevier.com/locate/procedia

2nd International Conference 'Economic Scientific Research - Theoretical, Empirical and Practical Approaches', ESPERA 2014, 13-14 November 2014, Bucharest, Romania

Entropy Measures for Assessing Volatile Markets

Muhammad Sheraz[a], Silvia Dedu[b,c,]*, Vasile Preda[a]

[a]*University of Bucharest, Faculty of Mathematics and Computer Science, Academiei nr. 14, Bucharest, 010014, Romania*
[b]*Bucharest University of Economic Studies, Department of Applied Mathematics, Piata Romana nr. 6, Bucharest, 010374, Romania*
[c]*Institute of National Economy, Calea 13 Septembrie nr. 13, Bucharest, 050711, Romania*

Abstract

The application of entropy in finance can be regarded as the extension of information entropy and probability theory. In this article we apply the concept of entropy for underlying financial markets to make a comparison between volatile markets. We consider in the first step Shannon entropy with different estimators, Tsallis entropy for different values of its parameter, Rényi entropy and finally the approximate entropy. We provide computational results for these entropies for weekly and monthly data in the case of four different stock indices.

Selection and/or peer-review under responsibility of the Scientific Committee of ESPERA 2014

Keywords: Entropy measures; Entropy estimators; Volatility; Financial markets.

1. Introduction

The concept of entropy plays a crucial role in extracting the universal features of a system from its microscopic details. In statistical mechanics the entropy is defined as the logarithm of the total number of microstates multiplied by a constant coefficient or alternatively it is written in terms of the probability to occupy the microstates. The Shannon entropy can be used in particular manners to evaluate the entropy corresponding to a probability density distribution around some points, but specific events can be also considered, for example the deviation from the mean or any sudden news in the case of stock market. At this point one needs additional information and the classical

* Corresponding author. Tel.: +4-072-258-0269.
E-mail address: silvia.dedu@csie.ase.ro

Selection and/or peer-review under responsibility of the Scientific Committee of ESPERA 2014
doi:10.1016/S2212-5671(15)00279-8

concept of entropy can be generalized. Tsallis, (1988) proposed an extension of the concept of entropy, which successfully describes the statistical features of complex systems. Some other examples of entropy measures, which depend on power of the probability, were introduced as generalization of the Shannon entropy, include (Kaniadakis, 2001), Rényi, (1961), Ubriaco, (2009) and Shafee, (2007) entropy measures.

The application of entropy in finance can be regarded as the extension of both information entropy and probability theory. Since the last two decades, it has become a very important tool for designing portfolio selection and asset pricing techniques. In mathematical finance, a risk-neutral measure, also called an equivalent martingale measure, is heavily used in the pricing of financial derivatives. In the theory of option pricing the risk-neutral densities play a very important role and stochastic calculus helps us to obtain this framework. The Entropy Pricing Theory was introduced by (Gulko, 1995), as an alternative method for the construction of risk-neutral probabilities without relying on stochastic calculus. The famous Black-Scholes model, 1973 assumes the condition of no arbitrage, which implies the universal risk-neutral probabilities.

Various cantitative techniques are used in solving decision problems which arise in economy, social sciences, engineering and many other domains. We can mention the contributions of (Istudor and Filip, 2014), (Ştefănoiu et al., 2014), (Moinescu and Costea, 2014), (Barik et al., 2012), Filip, (2012), (Costea and Bleotu, 2012), (Nastac et al., 2009) and (Costea et al., 2009). Modeling the trend of financial indices and portfolio selection topics have caught the interest of the researchers, see, for example, Georgescu, (2014), Toma and Dedu, (2014), Tudor, (2012), (Tudor and Popescu-Duţă, 2012), (Şerban et al., 2011) and (Lupu and Tudor, 2008). Recently, (Toma, 2014), Toma, (2012), (Toma and Leoni-Aubin, 2013) investigate different methods for financial data modeling using entropy measures.

The Principle of Maximum Entropy was used by Guo, 2001, in order to estimate the distribution of an asset from a set of option prices. Beside this work, the maximum entropy principle was used to retrieve the risk-neutral density of future stock risks or other asset risks by Rompolis, (2010). (Preda and Sheraz, 2014) have used recently the Shafee entropy measure to obtain risk-neutral densities. Recently (Preda et al., 2014), used Tsallis and Kaniadakis entropy measures for the case of semi-Markov regime switching interest rate models.

This paper is organized as follows. In Section 2 we discuss underlying types of entropy measures. In Section 3 we present our computational results for analyzing stock market volatility by considering, the Shannon, Tsallis, Rényi entropy measures and approximate entropy. Section 4 concludes our results.

2. Preliminaries

In this section we present the introductory framework, some important definitions and mathematical tools. The mathematical definitions of underlying entropy measures are discussed.

The Shannon entropy corresponding to a discrete random variable of probability measure $P = \{p_1, p_2, \ldots, p_n\}$ is given by:

$$H(P) = -\sum_{i=1}^{n} p_i \ln p_i,$$

with $p_i \geq 0, i = \overline{1,n}, \sum_{i=1}^{n} p_i = 1$, where $0 \ln 0 = 0$.

The Tsallis entropy is a generalization of the standard Boltzmann--Gibbs entropy. It was introduced as a basis for generalizing the standard statistical mechanics. Let $q \in \mathbf{R}$. If $P = \{p_1, p_2, \ldots, p_n\}$ is the discrete set of probabilities, then the Tsallis entropy of order q is given by:

$$H_q(P) = \begin{cases} \frac{1}{q-1}\left(1-\sum_{i=1}^{n} p_i^q\right), \text{ if } q \neq 1 \\ -\sum_{i=1}^{n} p_i \ln p_i, \quad \text{if } q \neq 1 \end{cases}$$

In the field of information theory, the Rényi entropy, named after Alfréd Rényi, generalizes the Shannon entropy. The relative entropy minimization has been used extensively for the calibration of the financial models. Let $r \geq 0,\ r \neq 1$. The Rényi entropy of order r is given by:

$$H_r(P) = \frac{1}{1-r}\log\left(\sum_{i=1}^{n} p_i^r\right).$$

Note that for $r \to 0$ the Rényi entropy is just the logarithm of the size of the support of the random variable and for $r \to 1$ we obtain Shannon entropy.

3. Entropy measures and stock market volatility

We will apply the concept of entropy for underlying financial markets to calculate Shannon, Tsallis, Rényi and approximate entropies. We consider the weekly and monthly closing prices of CAC 40 (Paris Index), Hang-Seng (Hong Kong Index), FTSCST China (Singapore Index) and FTSE.MI (Milan Index) for the period 2000-2012. In the first step we have used different estimators for the Shannon entropy measure in order to asses the consistency of the resulted values. In tables 1-4 we consider maximum likelihood (ML), Biased corrected maximum likelihood (MM), Jeffreys (entropy.Dirichlet with $a = 1/2$), Laplace (entropy.Dirichlet with $a = 1$), SG (entropy.Dirichlet with $a = 1$/length underlying financial time series), CS (Chao Shen entropy) and Shrink entropy estimators, to calculate the Shannon entropy. In the next step we have considered the Tsallis entropy measure for different values of Tsallis parameter q. In the third step we switch our self to consider the Rényi entropy measure for the same stock indices and we calculate the entropy value for different values of Rényi parameter r. Finally we calculate the approximate entropy. The main purpose of using diffrent entropy measures is to compare their performance and variation between resulted values. The results obtained are presented in tables 1-8.

Table 1. Entropy results weekly for selected indices: Paris Index

Shannon		Tsallis		Rényi		Approximate entropy
Method	Value	q	Value	r	Value	Value
ML	6.42749	0	635.000	0	6.45519	0.3027921
MM	6.42760	0.2	216.398	0.25	6.44828	
Jeffreys	6.42749	0.4	77.9564	0.5	6.49135	
Laplace	6.42740	0.6	30.3425	1	6.42749	
SG	6.42749	0.8	13.1024	2	6.40009	
Minimax	6.45260	1	6.42749	4	6.34914	
CS	6.42749	1.2	3. 61590	8	6.27154	
Shrink	6.42771	1.4	2.30800	16	6.18923	
		1.6	1.63100	32	6.12355	
		1.8	1.24250	64	6.07705	
		2	0.99830	Infinite	5.99893	

Table 2. Entropy results weekly for selected indices: Hong Kong Index

Shannon		Tsallis		Rényi		Approximate entropy
Method	Value	*q*	Value	*r*	Value	Value
ML	6.41303	0	635.0000	0	6.45519	0.2855268
MM	6.41064	0.2	215.8622	0.25	6.44444	
Jeffreys	6.41303	0.4	77.66670	0.5	6.43382	
Laplace	6.41304	0.6	30.22485	1	6.41303	
SG	6.41304	0.8	13.05985	2	6.37385	
Minimax	6.41306	1	6.413030	4	6.30703	
CS	6.41303	1.2	3. 611180	8	6.21319	
Shrink	6.41309	1.4	2.306500	16	6.10259	
		1.6	1.630600	32	5.99881	
		1.8	1.242410	64	5.92947	
		2	0.998290	Infinite	5.84849	

Table 3. Entropy results weekly for selected indices: Milan Index

Shannon		Tsallis		Rényi		Approximate entropy
Method	Value	*q*	Value	*r*	Value	Value
ML	6.41080	0	648.0000	0	6.45517	0.2174744
MM	6.41082	0.2	215.7645	0.25	6.44375	
Jeffreys	6.41081	0.4	77.61600	0.5	6.43252	
Laplace	6.41081	0.6	30.20510	1	6.41080	
SG	6.41080	0.8	13.05300	2	6.37075	
Minimax	6.41083	1	6.410800	4	6.30520	
CS	6.41080	1.2	3. 610480	8	6.22076	
Shrink	6.41084	1.4	2.306290	16	6.13919	
		1.6	1.630540	32	6.07488	
		1.8	1.243990	64	6.03039	
		2	0.998280	Infinite	5.95829	

Table 4. Entropy results weekly for selected indices: Singapore Index

Shannon		Tsallis		Rényi		Approximate entropy
Method	Value	*q*	Value	*r*	Value	Value
ML	5.47276	0	248.0000	0	5.51745	0.2315494
MM	5.47450	0.2	101.2816	0.25	5.50659	
Jeffreys	5.47292	0.4	43.52079	0.5	5.59534	
Laplace	5.47307	0.6	19.98199	1	5.47276	
SG	5.47276	0.8	9.986681	2	5.42428	
Minimax	5.47309	1	5.472760	4	5.31731	
CS	5.47276	1.2	3. 323410	8	5.12064	
Shrink	5.47584	1.4	2.217830	16	4.91832	

		1.6	1.603100	32	4.77948	
		1.8	1.233820	64	4.40446	
		2	0.995590	Infinite	4.63092	

Table 5. Entropy results monthly for selected indices: Paris Index

Shannon		**Tsallis**		**Rényi**		**Approximate entropy**
Method	**Value**	***q***	**Value**	***r***	**Value**	**Value**
ML	4.96272	0	146.0000	0	4.99043	0.5229593
MM	4.96283	0.2	66.17848	0.25	4.98352	
Jeffreys	4.96273	0.4	31.39729	0.5	4.97659	
Laplace	4.96273	0.6	15.78017	1	4.96272	
SG	4.96272	0.8	8.505422	2	4.93533	
Minimax	4.96279	1	4.962724	4	4.88446	
CS	4.96272	1.2	3. 144788	8	4.80753	
Shrink	4.96294	1.4	2.155059	16	4.72727	
		1.6	1.580966	32	4.66493	
		1.8	1.225994	64	4.62299	
		2	0.992811	Infinite	4.56228	

Table 6. Entropy results monthly for selected indices: Milan Index

Shannon		**Tsallis**		**Rényi**		**Approximate entropy**
Method	**Value**	***q***	**Value**	***r***	**Value**	**Value**
ML	4.94603	0	146.0000	0	4.99043	0.4656637
MM	4.94605	0.2	65.98193	0.25	4.97898	
Jeffreys	4.94603	0.4	31.25580	0.5	4.96775	
Laplace	4.94603	0.6	15.70362	1	4.94603	
SG	4.94603	0.8	8.468545	2	4.90604	
Minimax	4.94607	1	4.946035	4	4.84086	
CS	4.94603	1.2	3. 137523	8	4.75743	
Shrink	4.94606	1.4	2.151979	16	4.67681	
		1.6	1.579684	32	4.61285	
		1.8	1.225468	64	4.56947	
		2	0.992598	Infinite	4.51116	

Table 7. Entropy results monthly for selected indices: Singapore Index

Shannon		Tsallis		Rényi		Approximate entropy
Method	Value	*q*	Value	*r*	Value	Value
ML	4.01730	0	57.00000	0	4.06044	0.5148562
MM	4.01904	0.2	30.71663	0.25	4.04980	

Jeffreys	4.01746	0.4	17.18966	0.5	4.03906
Laplace	4.01760	0.6	10.05588	1	4.01730
SG	4.01731	0.8	6.185851	2	3.97268
Minimax	4.01796	1	4.017309	4	3.88185
CS	4.01730	1.2	2.757170	8	3.73275
Shrink	4.02047	1.4	1.995183	16	3.59674
		1.6	1.514623	32	3.51340
		1.8	1.198296	64	3.46680
		2	0.981177	Infinite	3.41492

Table 8. Entropy results monthly for selected indices: Hong Kong Index

Shannon		Tsallis		Rényi		Approximate entropy
Method	Value	q	Value	r	Value	Value
ML	4.94744	0	146.0000	0	4.99043	0.5076282
MM	4.94747	0.2	66.00368	0.25	4.97947	
Jeffreys	4.94744	0.4	31.27056	0.5	4.96864	
Laplace	4.94745	0.6	15.71112	1	4.94744	
SG	4.94744	0.8	8.471911	2	4.90739	
Minimax	4.94750	1	4.947445	4	4.83853	
CS	4.94744	1.2	3. 138080	8	4.73841	
Shrink	4.94750	1.4	2.152190	16	4.61130	
		1.6	1.579760	32	4.49542	
		1.8	1.225490	64	4.42676	
		2	0.992600	Infinite	4.35765	

The term entropy can be viewed as the measure of disorder, uncertainity or ignorance of a system which also resembles with the features of stock market volatility. The entropy attains the maximum value when all likely events have same probability of occurrences. As it is quite evident from our empirical results, volatility shows different patterns for selected indices. These patterens exhibit linear and nonlinear dynamics. The entropy captures the overall linear and nonlinear dispersion (volatility) observed in the data series. We can see form the computational results that all entropies are positive, which conclude that data series are nonlinear. Using the values from tables 1-4 now we analyze this behaviour. As we have discussed regarding the concept of entropy and some of its properties, it results that it can be used to capture the linear and nonlinear trends of volatility for the underlying data sets. We remark that all the values of the entropies for the selected indices are positive, which concludes the nonlinear character of the financial series. In the weekly and monthly data series we can see that Paris Index has larger value of approximate entropy, but on other hand for Milan Index the value is smaller for weekly and monthly data, as compared to other indices. It results in this case that Paris Index is more volatile and Milan Index is less volatile. In the case of Shannon entropy estimators it is obious again the Paris Index has larger value but there is a little difference between Paris, Hong Kong and Milan indices. In the case of Tsallis and Rényi entropy measures, we remark that as q and r approaches to 1 we obtain the Shannon entropy. The behavior of stock indices volatility depends on values of q and r.

4. Conclusions

In this article we have used the entropic approach in order to asses the volatile stock index. The term entropy can be viewed as the measure of disorder, uncertainty or ignorance of a system which also resembles with the features of the stock market volatilty. We have used the Tsallis, Shannon and Rényi entropy measures and the approximate entropy as well as an alternative way to feature the stock market volatility. Our computational results show that Paris Index for the period 2000 to 2012 is more volatile then other underlying indices in both weekly and monthly data series. Some numerical results have been discussed. The entropic aproach for stock market volatility is a new approach and it explores the new horizons for future research in this domain.

Acknowledgements

This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS – UEFISCDI, project number PN-II-RU-TE-2012-3-0007.

References

Black, F. & Scholes, M., 1973. The pricing of options and corporate liabilities, Journal of Political Economy. 81,637-659.

Costea, A., Bleotu, V., 2012. A new fuzzy clustering algorithm for evaluating the performance of non-banking financial institutions in Romania, Economic Computation and Economic Cybernetics Studies and Research, 46, 4, 179-199.

Costea, A., Eklund, T., Karlsson, J., Voineagu, V., 2009. Financial Performance Analysis of the Scandinavian Telecommunications Companies using Statistical and Neural Networks Techniques, Economic Computation and Economic Cybernetics Studies and Research, 43, 4,87-104.

Filip, F.G., 2012. Decision-Making Perspective for Designing and Building Information Systems. International Journal of Computers, Communications and Control 7, 2, 264-272.

Georgescu, G., 2014. Public Debt, Sovereign Risk and Sustainable Development of Romania. Procedia Economics and Finance 8, 353-361, Elsevier.

Gulko, L., 1995. The Entropy Theory of Bond Option Pricing, Working Paper, Yale School of Management, October, New Haven, CT.

Guo, W.Y., 2001. Maximum entropy in option pricing: a convex-spline smoothing method, Journal of Futures Markets 21, 819-832.

Istudor, I., Filip, F.G., 2014. The Innovator Role of Technologies in Waste Management towards the Sustainable Development. Procedia Economics and Finance 8, 420-428.

Kaniadakis, G., 2001. Non-linear kinetics underlying generalized statistics, Physica A: Statistical Mechanics and its Applications 296(3), 405—425.

Lupu, R., Tudor, C., 2008. Direction of change at the Bucharest Stock Exchange. Romanian Economic Journal 11 (27), 165-185.

Moinescu, B., Costea, A., 2014. Towards an early-warning system of distressed non-banking financial institutions, Economic Computation and Economic Cybernetics Studies and Research, 48, 2, 75-90.

Nastac, I., Bacivarov, A., Costea, A., 2009. A Neuro-Classification Model for Socio-Technical Systems, Romanian Journal of Economic Forecasting 11, 3, 100-109.

Preda, V., Dedu, S., Sheraz, M., 2014. New measure selection for Hunt and Devolder's semi-Markov regime switching interest rate model , Physica A 407, 350-359.

Preda,V., Sheraz, M., 2014. Risk-neutral densities in entropy theory of stock options using Lambert function ana a new approach, to be published in Proceedings of the Romanian Academy.

Rényi, A., 1961. On measures of entropy and information, In Proceedings of the 4th Berkely Sympodium on Mathematics of Statistics and Probability., 1,.547-561.

Rompolis,L.S., 2010. Retrieving risk neutral densities from European option prices based on the principle of maximum entropy. Journal of Empirical Finance 17, 918---937.

Şerban, F., Ştefănescu, M.V., Dedu, S., 2011. Building an Optimal Portfolio Using a Mean-VaR Framework. Mathematical Methods and Techniques in Engineering and Environmental Science, 126-131.

Şerban, F., Ştefănescu M.V., Dedu, S., 2011. The Relationship Profitability - Risk for an Optimal Portfolio Building with Two Risky Assets and a Risk-Free Asset. International Journal of Applied Mathematics and Informatics, 5, 4, 299 – 306.

Shafee, F., 2007. Lambert function and a new non-extensive form of entropy, Journal of Applied Mathematics., 72, 785-800.

Ştefănoiu, D., Borne, P., Popescu, D., Filip, F.G., El Kamel, A., 2014. Optimisation en science de l'ingénieur; méthaheuristiques, méthods stochastiques et aide à la décision. Hermes Science - Lavoisier, Paris.

Toma, A., 2012. Robust Estimations for Financial Returns: An approach based on pseudodistance minimization, Economic Computation and Economic Cybernetics Studies and Research, 46, 1, p. 117.

Toma, A., 2014. Model selection criteria using divergences, Entropy 16(5), 2686-2698, 2014.

Toma, A., Dedu, S., 2014. Quantitative techniques for financial risk assesment: a comparative approach using different risk measures and estimation methods. Procedia Economics and Finance 8, 712-719, Elsevier.
Toma, A., Leoni-Aubin, S., 2013. Portfolio selection using minimum pseudodistance estimators, Economic Computation and Economic Cybernetics Studies and Research, 47, 1, 97-113.
Tsallis, C., 1988. Possible generalization of Boltzmann-Gibbs statistics, Journal of Statistical Physics 52,479-487.
Tudor, C., Popescu-Duță, C., 2012. On the causal relationship between stock returns and exchange rates changes for 13 developed and emerging markets. Procedia-Social and Behavioral Sciences 57, 275-282.
Tudor, C., 2012. Active portfolio management on the Romanian Stock Market, Procedia-Social and Behavioral Sciences 58, 543-551.
Ubriaco, M.R., 2009. Entropies based on fractional calculus, Physics Letters A 373, 2516-2519.