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The possible test of the calculations of nuclear matrix elements of the $(\beta\beta)_{0\nu}$-decay

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Abstract

The existing calculations of the nuclear matrix elements of the neutrinoless double β-decay differ by about a factor three. This uncertainty prevents quantitative interpretation of the results of experiments searching for this process. We suggest here that the observation of the neutrinoless double β-decay of *several* nuclei could allow to test calculations of the nuclear matrix elements through the comparison of the ratios of the calculated lifetimes with experimental data. It is shown that the ratio of the lifetimes is very sensitive to different models.

1. Introduction

The compelling evidences in favour of neutrino oscillations were obtained in the Super-Kamiokande [1], SNO [2] and other atmospheric and solar neutrino experiments. These findings mean that neutrino masses are different from zero and fields of the flavour neutrinos are mixture of the left-handed components of the fields of neutrinos with definite masses. It is a general consensus that small neutrino masses and neutrino mixing is a first evidence for new physics.

There are many unsolved problems in the physics of massive and mixed neutrinos. The most fundamental one is the problem of the *nature* of neutrinos with definite masses: are they Dirac or Majorana particles? The answer to this question cannot be obtained via the investigation of neutrino oscillations. In order to probe the nature of the massive neutrinos it is necessary to study processes in which the total lepton number is not conserved. The most sensitive to the possible violation of the total lepton number process is neutrinoless double β-decay ($(\beta\beta)_{0\nu}$-decay) of even–even nuclei.

The data of many experiments on the search for $(\beta\beta)_{0\nu}$-decay are available at present (see [3,4]). No any indications in favour of $(\beta\beta)_{0\nu}$-decay were obtained up to now.[1]

The strongest limits on the lifetime of the $(\beta\beta)_{0\nu}$-decay were obtained in the Heidelberg–Moscow [8]

[1] The recent claim [5] of some evidence of the $(\beta\beta)_{0\nu}$-decay, obtained from the reanalysis of the data of the Heidelberg–Moscow experiment, was strongly criticised in [6,7].

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and IGEX [9] ^{76}Ge experiments:

$$T^{0\nu}_{1/2}\big(^{76}\mathrm{Ge}\big) \geqslant 1.9\times 10^{25}\ \text{years}\quad (\text{H–M}),$$
$$T^{0\nu}_{1/2}\big(^{76}\mathrm{Ge}\big) \geqslant 1.57\times 10^{25}\ \text{years}\quad (\text{IGEX}). \tag{1}$$

There are several mechanisms of the neutrinoless double β-decay. We will consider here $(\beta\beta)_{0\nu}$-decay in the framework of the Majorana neutrino mixing

$$\nu_{lL} = \sum_i U_{li}\,\nu_{iL}, \tag{2}$$

where U is Pontecorvo–Maki–Nakagava–Sakata unitary mixing matrix and ν_i is the field of the Majorana neutrino with mass m_i. After recent evidences for neutrino oscillations this mechanism appears as the most natural one.[2]

In the case of the Majorana neutrino mixing the matrix element of the $(\beta\beta)_{0\nu}$-decay is proportional to the effective Majorana mass (see [12,13])

$$\langle m\rangle = \sum_i U^2_{ei} m_i. \tag{3}$$

From the results of ^{76}Ge experiments it was found

$$|\langle m\rangle| \leqslant (0.35\text{–}1.24)\ \text{eV}\quad (\text{Heidelberg–Moscow}), \tag{4}$$
$$|\langle m\rangle| \leqslant (0.33\text{–}1.35)\ \text{eV}\quad (\text{IGEX}). \tag{5}$$

In (4) and (5) different calculations of nuclear matrix elements were used.

Many new experiments on the search for $(\beta\beta)_{0\nu}$-decay of different nuclei are under preparation in different laboratories. In these experiments much higher sensitivities to the effective Majorana mass $|\langle m\rangle|$ than the present-day ones are expected (see [3]). For example, the sensitivities to $|\langle m\rangle|$ which are planned to be reached in the experiments CUORE (^{130}Te) [14], GENIUS (^{76}Ge) [15], MAJORANA (^{76}Ge) [16], EXO (^{136}Xe) [17], MOON (^{100}Mo) [18] are, respectively, equal to 2.7×10^{-2} eV, 1.5×10^{-2} eV, 2.5×10^{-2} eV, 5.2×10^{-2} eV, 3.6×10^{-2} eV.[3]

The observation of the $(\beta\beta)_{0\nu}$-decay would be a proof that neutrinos with definite masses are Majorana particles. It was shown in many papers (see [20] and references therein) that *the measurement of the effective Majorana mass* $|\langle m\rangle|$ would allow to obtain an unique information about neutrino mass spectrum and Majorana CP phase.

There exist, however, a serious problem of the determination of $|\langle m\rangle|$ from experimental data. It is connected with nuclear matrix elements: the calculated matrix elements vary within factor three.

In this Letter we would like to propose a possible test of the calculations of the nuclear matrix elements, based on the comparison of the results of the calculations with the experimental data. In order to realize the proposed test it is necessary to observe $(\beta\beta)_{0\nu}$-decay of *several nuclei*.

2. Possible test of nuclear matrix elements calculations

In the framework of the Majorana neutrino mixing (2) the total probability of the $(\beta\beta)_{0\nu}$-decay has the following general form (see [12,13]):

$$\Gamma^{0\nu}(A,Z) = |\langle m\rangle|^2 |M(A,Z)|^2 G^{0\nu}(E_0,Z), \tag{6}$$

where $M(A,Z)$ is the nuclear matrix element and $G^{0\nu}(E_0,Z)$ is known phase-space factor (E_0 is the energy release). Thus, in order to determine $|\langle m\rangle|$ from the experimental data we need to know the nuclear matrix element $M(A,Z)$. This last quantity must be calculated.

There exist at present large uncertainties in the calculations of the nuclear matrix elements of the $(\beta\beta)_{0\nu}$-decay (see [21–23]). Two basic approaches to the calculation are used: quasiparticle random phase approximation and the nuclear shell model. Different calculations of the lifetime of the $(\beta\beta)_{0\nu}$-decay differ by about one order of magnitude. For example, for the lifetime of the $(\beta\beta)_{0\nu}$-decay of ^{76}Ge it was obtained the range [23] [4]

$$6.8\times10^{26} \leqslant T^{0\nu}_{1/2}\big(^{76}\mathrm{Ge}\big) \leqslant 70.8\times10^{26}\ \text{years}. \tag{7}$$

[2] Other mechanisms of the $(\beta\beta)_{0\nu}$-decay are based on SUSY R-parity violating models [10], on a model with admixture of heavy neutrinos to the light ones [11] etc. In [11] possibilities to distinguish different mechanisms are considered. The proposed tests require detection of the $(\beta\beta)_{0\nu}$-transition into excited states and precise calculations of the transition probabilities.

[3] In the calculation of these sensitivities the nuclear matrix elements, given in [19], were used.

[4] The values given in (7) were calculated under the assumption that $|\langle m\rangle| = 5\times10^{-2}$ eV.

The problem of the calculation of the nuclear matrix elements of the neutrinoless double β-decay is a real theoretical challenge. It is obvious that without solution of this problem the effective Majorana neutrino mass $|\langle m\rangle|$ cannot be determined from the experimental data with reliable accuracy (see discussion in [24]).

We will propose here a method which allows to check the results of the calculations of the nuclear matrix elements of the $(\beta\beta)_{0\nu}$-decay of different nuclei by confronting them with experimental data. We will take into account the following.

(1) for small neutrino masses ($m_i \lesssim 10$ MeV) the nuclear matrix elements do not depend on neutrino masses [12,13];
(2) the sensitivity $|\langle m\rangle| \simeq$ a few 10^{-2} eV is planned to be reached in experiments on the search for neutrinoless double β-decay of *different* nuclei.

From (6) we have

$$R(A,Z/A',Z') = \frac{T^{0\nu}_{1/2}(A,Z)}{T^{0\nu}_{1/2}(A',Z')} = \frac{|M(A',Z')|^2 G^{0\nu}(E'_0,Z')}{|M(A,Z)|^2 G^{0\nu}(E_0,Z)}. \quad (8)$$

Thus, if the neutrinoless double β-decay of different nuclei will be observed, the calculated ratios of the corresponding nuclear matrix elements-squared can be confronted with the experimental values.

In the Table 1 we present the ratios of lifetimes of the $(\beta\beta)_{0\nu}$-decay of several nuclei, calculated in six different models. For the lifetimes we used the values given in [23]. As it is seen from Table 1, the calculated ratios are very sensitive to the model: they vary within about one order of magnitude.

As we can see from the Table 1, the ratio $R(^{76}\text{Ge}/^{130}\text{Te})$, calculated in [19] and [28] is equal, correspondingly, 4.6 and 3.6. It is clear that it will be difficult to distinguish models [19] and [28] by the observation of the neutrinoless double β-decay of ^{76}Ge and ^{130}Te. However, it will be no problem to distinguish the corresponding models via the observation of the $(\beta\beta)_{0\nu}$-decay of ^{76}Ge and ^{100}Mo (the corresponding ratio is equal 1.8 and 10.7, respectively). This example illustrates the importance of the investigation of $(\beta\beta)_{0\nu}$-decay of more than two nuclei.

The nuclear part of the matrix element of the $(\beta\beta)_{0\nu}$-decay is determined by the matrix element of the T-product of two hadronic charged currents connected by the propagator of massless boson. This matrix element cannot be connected with matrix element of any observable hadronic process. We believe that the method, proposed here, which is based on the factorisation of neutrino and nuclear parts of the matrix element of the $(\beta\beta)_{0\nu}$-decay, is the only possibility to test the calculations of the nuclear matrix elements in a model independent way.

We would like to finish with the following remark. If the ratio (8), calculated in some model, is in agreement with experimental data this could only mean that the model is correct up to a possible factor, which does not depend on A and Z (and drops out from the ratio (8)). Such factor was found and calculated in Ref. [30]. In that paper in addition to the usual axial and vector terms in the nucleon matrix element pseudoscalar and weak magnetic form factors were taken into account. It was shown that in the case of the light Majorana neutrinos these additional terms lead to a universal $\simeq 30\%$ reduction of the nuclear matrix elements of the $(\beta\beta)_{0\nu}$-decay, which practically does not depend on the type of the nuclei. This reduction will cause the corresponding raise of the value of the effective Majorana mass $|\langle m\rangle|$ that could be obtained from the results of the future experiments.

Table 1
The results of the calculation of the ratios of the lifetime of $(\beta\beta)_{0\nu}$-decay of several nuclei in six different models. The references to the corresponding papers are given in brackets

Lifetime ratios	[25]	[26]	[27]	[19]	[28]	[29]
$R(^{76}\text{Ge}/^{130}\text{Te})$	11.3	3	20	4.6	3.6	4.2
$R(^{76}\text{Ge}/^{136}\text{Xe})$		1.5	4.2	1.1	0.6	2
$R(^{76}\text{Ge}/^{100}\text{Mo})$			14	1.8	10.7	0.9

3. Conclusion

The observation of the neutrinoless double β-decay would have a great impact on the understanding of the origin of neutrino masses and mixing. The accurate measurement of the effective Majorana mass $|\langle m\rangle|$ would allow to make important conclusions on the neutrino mass spectrum and Majorana CP phase (see [20] and references therein).

Let us consider the minimal scheme of three-neutrino mixing and label neutrino masses in such a way that $m_1 < m_2 < m_3$.[5] From the results of the neutrino oscillation experiments only neutrino mass-squared differences $\Delta m^2_{21} = m^2_2 - m^2_1$ and $\Delta m^2_{32} = m^2_3 - m^2_2$ can be inferred. In order to illustrate the importance of the measurement of $|\langle m\rangle|$ we will consider three typical neutrino mass spectra, compatible with the results of neutrino oscillation experiments.

(1) The hierarchy of neutrino masses $m_1 \ll m_2 \ll m_3$. For the effective Majorana mass we have in this case the bound

$$|\langle m\rangle| \leqslant \sin^2\theta_{\rm sol}\sqrt{\Delta m^2_{\rm sol}} + |U_{e3}|^2\sqrt{\Delta m^2_{\rm atm}}. \qquad (9)$$

Using the best-fit values of the neutrino oscillation parameters in the most favourable MSW LMA region [2] $\Delta m^2_{\rm sol} = 5.0 \times 10^{-5}$ eV^2; $\tan^2\theta_{\rm sol} = 0.34$, the value of the atmospheric neutrino mass-squared difference $\Delta m^2_{\rm atm} = 2.5 \times 10^{-3}\,{\rm eV}^2$, obtained from the analysis of the data of the Super-Kamiokande atmospheric neutrino experiment [1], and the CHOOZ [33] bound

$$|U_{e3}|^2 \leqslant 4 \times 10^{-2} \qquad (10)$$

for the effective Majorana mass we have

$$|\langle m\rangle| \leqslant 3.8 \times 10^{-3}\ {\rm eV}. \qquad (11)$$

This bound is significantly smaller than the expected sensitivity of the future $(\beta\beta)_{0\nu}$-experiments.[6]
Thus, the observation of the $(\beta\beta)_{0\nu}$-decay in the experiments of the next generation would presumably create a problem for the hierarchy of neutrino mass, motivated by the famous see-saw mechanism of neutrino mass generation.

(2) Inverted hierarchy of neutrino masses: $m_1 \ll m_2 < m_3$. The effective Majorana mass is given in this case by

$$|\langle m\rangle| \simeq \left(1 - \sin^2 2\,\theta_{\rm sol}\,\sin^2\alpha\right)^{1/2}\sqrt{\Delta m^2_{\rm atm}}, \qquad (12)$$

where $\alpha = \alpha_3 - \alpha_2$ is the difference of the Majorana CP phases. Using the best-fit value of the parameter $\tan^2\theta_{\rm sol}$ we have

$$\frac{1}{2}\sqrt{\Delta m^2_{\rm atm}} \lesssim |\langle m\rangle| \lesssim \sqrt{\Delta m^2_{\rm atm}}. \qquad (13)$$

Thus, in the case of the inverted mass hierarchy the scale of $|\langle m\rangle|$ is determined by $\sqrt{\Delta m^2_{\rm atm}} \simeq 5 \times 10^{-2}$ eV. If the value of $|\langle m\rangle|$ is in the range (12), which can be reached in the future experiments on the search for $(\beta\beta)_{0\nu}$-decay, it will be a signature of inverted neutrino mass hierarchy.

(3) Practically degenerate neutrino mass spectrum. If $m_1 \gg \sqrt{\Delta m^2_{\rm atm}}$ for neutrino masses we have $m_2 \simeq m_3 \simeq m_1$. Effective Majorana mass in this case is equal to

$$|\langle m\rangle| \simeq m_1\left|\sum_{i=1}^{3} U^2_{ei}\right|. \qquad (14)$$

Taking into account the CHOOZ bound (10), in the case of the LMA solution of the solar neutrino problem we have $|U_{e3}|^2 \ll |U_{e1}|^2, |U_{e2}|^2$. Hence, we can neglect the contribution of $|U_{e3}|^2$ to the effective Majorana mass. From (14) we have

$$m_1 \simeq \frac{|\langle m\rangle|}{(1 - \sin^2 2\theta_{\rm sol}\sin^2\alpha)^{1/2}}. \qquad (15)$$

For the best-fit LMA value $\tan^2\theta_{\rm sol} = 0.34$ from (15) we obtain the bounds

$$|\langle m\rangle| \leqslant m_1 \lesssim 2|\langle m\rangle|. \qquad (16)$$

Thus, if it will occur that the effective Majorana mass $|\langle m\rangle|$ is significantly larger than $\sqrt{\Delta m^2_{\rm atm}}$ it will be an evidence for the practically degenerate neutrino mass spectrum.

The measurement of the effective Majorana mass $|\langle m\rangle|$ could allow to obtain an information about the Majorana CP phase difference α. In fact we have [35].

$$\sin^2\alpha \simeq \left(1 - \frac{|\langle m\rangle|^2}{m_0^2}\right)\frac{1}{\sin^2 2\theta_{\rm sol}}, \qquad (17)$$

where $m_0 = \sqrt{\Delta m^2_{\rm atm}}$ in the case of the inverted neutrino mass hierarchy and $m_0 = m_1$ in the case of practically degenerate mass spectrum. In the case of

[5] The LSND result [31], which requires more than three massive and mixed neutrinos, needs confirmation. The MiniBooNE experiment [32], started recently, aims to check the LSND claim.

[6] Let us note, however, that at the next stage of the GENIUS experiment (10 tons of enriched ^{76}Ge) the bound (11) is expected to be reached [34].

the CP conservation in the lepton sector $\sin^2\alpha = 0$ ($\sin^2\alpha = 1$) for equal (opposite) CP parities of ν_2 and ν_3.

Thus, accurate measurement of $|\langle m\rangle|$ ($\Delta m^2_{\mathrm{atm}}$ and $\sin^2 2\theta_{\mathrm{sol}}$) would allow to determine Majorana CP phase difference α in the case of the inverted hierarchy of neutrino masses.

In order to determine the parameter $\sin^2\alpha$ in the case of the degenerate neutrino mass spectrum we need to know m_1. The mass m_1 can be inferred from experiments on the measurement of the high-energy part of the β-spectra. From the latest data of Mainz [36] and Troitsk [37] tritium experiments the bound $m_1 \leqslant 2.2$ eV was obtained. In the future tritium experiment KATRIN [38] the sensitivity $m_1 \simeq 0.35$ eV is expected.

An information about absolute values of neutrino masses can be obtained also from cosmological data. From 2dF Galaxy Redshift survey and CMB data it was found that [39] $\sum_i m_i \leqslant (1.8–2)$ eV. The future MAP/PLANK CMB data and high precision Sloan Digital Sky Survey could render [40] $\sum_i m_i \simeq 0.3$ eV.

In conclusion we would like to stress that in order to obtain an unique information on neutrino mass spectrum and Majorana CP phase from the observation of the neutrinoless double β-decay we need to have a possibility to control the calculations of the nuclear matrix elements. We have shown here that the observation of the $(\beta\beta)_{0\nu}$-decay of *several* nuclei would allow to test in a model independent way the results of calculations of the nuclear matrix elements.

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References

[1] Super-Kamiokande Collaboration, S. Fukuda, et al., Phys. Rev. Lett. 81 (1998) 1562;
S. Fukuda, et al., Phys. Rev. Lett. 82 (1999) 2644;
S. Fukuda, et al., Phys. Rev. Lett. 85 (2000) 3999.

[2] SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 87 (2001) 071301;
Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011301, nucl-ex/0204008;
Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011302, nucl-ex/0204009.

[3] O. Cremonesi, in: Proceedings of the 20th International Conference on Neutrino Physics and Astrophysics, Neutrino 2002, Munich, Germany, May 25–30, 2002.

[4] V.L. Tretyak, Y.G. Zdesenko, At. Data Nucl. Data Tables 80 (2002) 83.

[5] H.V. Klapdor-Kleingrothaus, et al., Mod. Phys. Lett. 16 (2001) 2409.

[6] F. Feruglio, A. Strumia, F. Vissani, hep-ph/0201291.

[7] C.E. Aalseth, et al., Mod. Phys. Lett. 17 (2002) 1475, hep-ex/0202018.

[8] Heidelberg–Moscow Collaboration, H.V. Klapdor-Kleingrothaus, et al., Eur. Phys. J. A 12 (2001) 147.

[9] IGEX Collaboration, C.E. Aalseth, et al., Phys. At. Nucl. 63 (2000) 1225, hep-ex/0202026.

[10] R. Mohapatra, Phys. Rev. D 34 (1986) 3457;
J.D. Vergados, Phys. Lett. B 184 (1987) 55.

[11] F. Šimkovic, A. Faessler, Prog. Part. Nucl. Phys. 48 (2002) 201.

[12] M. Doi, T. Kotani, E. Takasugi, Progr. Theor. Phys. Suppl. 53 (1985) 1.

[13] S.M. Bilenky, S.T. Petcov, Rev. Mod. Phys. 59 (1987) 671.

[14] E. Fiorini, Phys. Rep. 307 (1998) 309.

[15] H.V. Klapdor-Kleingrothaus, et al., J. Phys. G 24 (1998) 483.

[16] L. De Braeckeleer, et al., in: Proceedings of the Carolina Conference on Neutrino Physics, Columbia, SC, USA, March 2000;
C.E. Aalseth, et al., hep-ex/0201021.

[17] M. Danilov, et al., Phys. Lett. B 480 (2000) 12.

[18] H. Ejiri, et al., Phys. Rev. Lett. 85 (2000) 2917.

[19] A. Staudt, K. Muto, H.V. Klapdor-Kleingrothaus, Europhys. Lett. 13 (1990) 31.

[20] S.M. Bilenky, C. Giunti, W. Grimus, B. Kayser, S.T. Petcov, Phys. Lett. B 465 (1999) 193;
F. Vissani, JHEP 06 (1999) 022;
M. Czakon, et al., hep-ph/0003161;
H.V. Klapdor-Kleingrothaus, H. Pas, A.Yu. Smirnov, Phys. Rev. D 63 (2001) 073005;
S.M. Bilenky, S. Pascoli, S.T. Petcov, Phys. Rev. D 64 (2001) 053010;
S. Pascoli, S.T. Petcov, hep-ph/0205022.

[21] A. Faessler, F. Simkovic, J. Phys. G 24 (1998) 2139.

[22] J. Suhonen, O. Civitarese, Phys. Rep. 300 (1998) 123.

[23] S.R. Elliott, P. Vogel, Annu. Rev. Nucl. Part. Sci. 52 (2002), hep-ph/0202264.

[24] V. Barger, S.L. Glashow, P. Langacker, D. Marfatia, Phys. Lett. B 540 (2002) 247;
S. Pascoli, S.T. Petcov, W. Rodejhann, hep-ph/0209059.

[25] W.C. Haxton, G.J. Stephenson Jr., Progr. Part. Nucl. Phys. 12 (1984) 409.

[26] E. Caurier, F. Nowacki, A. Poves, J. Retamosa, Phys. Rev. Lett. 77 (1996) 1954;
E. Caurier, F. Nowacki, A. Poves, J. Retamosa, Nucl. Phys. A 654 (1999) 973.

[27] J. Engel, P. Vogel, M. Zirnbauer, Phys. Rev. C 37 (1988) 731.

[28] J. Toivanen, J. Suhonen, Phys. Rev. Lett. 75 (1995) 410.
[29] G. Pantis, F. Šimkovic, J.D. Vergados, A. Faessler, Phys. Rev. C 53 (1996) 695;
F. Šimkovic, G. Pantis, J.D. Vergados, A. Faessler, Phys. Rev. C 60 (1999) 055502;
F. Šimkovic, M. Nowak, W.A. Kaminski, A.A. Raduta, A. Faessler, Phys. Rev. C 64 (2001) 035501.
[30] F. Šimkovic, G. Pantis, J.D. Vergados, A. Faessler, Phys. Rev. C 60 (1999) 055502.
[31] LSND Collaboration, G. Mills, in: Proceedings of the 19th International Conference on Neutrino Physics and Astrophysics, Neutrino 2000, Sudbury, Canada, June 16–21, 2000.
[32] MiniBooNE Collaboration, R. Tayloe, in: Proceedings of the 20th International Conference on Neutrino Physics and Astrophysics, Neutrino 2002, Munich, Germany, May 25–30, 2002.
[33] CHOOZ Collaboration, M. Apollonio, et al., Phys. Lett. B 466 (1999) 415.
[34] H.V. Klapdor-Kleingrothaus, hep-ph/0206249.
[35] S.M. Bilenky, C. Giunti, C.W. Kim, S.T. Petcov, Phys. Rev. D 54 (1996) 4432.
[36] Ch. Weinheimer, in: Proceedings of the 20th International Conference on Neutrino Physics and Astrophysics, Neutrino 2002, Munich, Germany, May 25–30, 2002.
[37] V. Lobashev, et al., Nucl. Phys. Proc. Suppl. 91 (2001) 280.
[38] KATRIN Collaboration, A. Osipowicz, et al., astro-ph/0109033.
[39] O. Elgaroy, et al., astro-ph/0204152.
[40] W. Hu, D.J. Eisenstein, M. Tegmark, Phys. Rev. Lett. 80 (1998) 5255;
W. Hu, M. Tegmark, Astrophys. J. Lett. 514 (1999) 65.