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User Friendly Assessment of Stress Intensity Factor in Thermal Shocked Cracked Structures with Finite Boundary Restraint

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Abstract

Evaluation of thermal shock stress intensity factors is a problem of interest in many industries. Many structures experiencing thermal shock can be accurately idealised to comprise finite length elastically / thermo-elastically restrained plates and finite length elastically / thermo-elastically restrained hollow cylinders. This article describes how a 'user-friendly' Compliance Adjusted Weight Function (CAWF) approach can be used to assess mode I stress intensity factor associated with the creep-free thermal shock of such structure with edge and semi-elliptical surface cracks. Based upon a mechanical weight function philosophy, the CAWF approach utilises a mechanical weight function analysis, available mechanical weight functions / geometry factors attributed to an equivalent semi-infinite cracked structure, a crack-free finite element analysis and an elastic Line-Spring analysis of compliance. The need for deriving different weight functions at each configuration of boundary restraint is therefore removed and the results of several verification exercises highlight that the CAWF approach is suitable for estimates within 5-10% of benchmark fracture mechanics finite element values. This suitability of the CAWF approach is valid for a wide range of cracked plate configurations, cracked hollow cylinder configurations and boundary restraint. An exception to this suitability is observed when examining semi-elliptical surface crack configurations and a free or near free boundary restraint condition. Adjustments to correction for this 'free boundary effect' are discussed elsewhere.

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Keywords: Stress intensity factor; Cracked plate; Cracked cylinder; Thermal shock; Finite boundary restraint; Mechanical weight function

1. Introduction

The storage, transportation and processing of thermo-fluids is an essential part of numerous engineering processes. Of the many types of structures used in such process, many can be accurately idealised to comprise finite length elastically / thermo-elastically restrained plates and finite length elastically / thermo-elastically restrained hollow cylinders. The onset of thermal fatigue damage is also common place whereby cracks or crack like flaws develop and the prediction of fitness-for-purpose (i.e. structural integrity and remaining life) under severe thermal shock conditions is an important problem. In creep-free thermal shock conditions a fitness-for-purpose assessment is closely associated with a mode I thermal shock stress intensity factor (K_I^{TS}) evaluation and the ease at which such a fitness-for-purpose can be performed is also dependent on the 'user-friendliness' of the K_I^{TS} evaluation procedure.

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To address this observation, a Compliance Adjusted Weight Function (CAWF) approach is proposed for the ‘user-friendly’ estimation of K_I^{TS} associated with the analysis of two types of creep-free thermal shocked cracked structure. One is an elastically / thermo-elastically restrained edge or semi-elliptical surface cracked plate. The other constitutes an elastically / thermo-elastically restrained edge or semi-elliptical surface cracked hollow cylinder.

2. The CAWF Approach

2.1. Background

Evaluation of mode I stress intensity factor; including those attributed to thermal shock i.e. K_I^{TS} ; are typically evaluated using an analytic solution approach or a numerical methodology such as a finite element [1], thermal weight function [2-3] or mechanical weight function approach [4-8]. Implementation of these analytic methods is all but limited to mathematically simplified infinite or semi-infinite idealizations. In contrast, the finite element and the aforementioned weight function methodologies are suitably generalized and can be used in almost any circumstance provided they are appropriately formulated. However, appropriate formulation of these analysis types is a process that often prohibits the ‘user-friendliness’ of their application in either general circumstances or when analysing K_I or K_I^{TS} associated with realistic structural idealizations such as finite length elastically / thermo-elastically restrained cracked plates and cracked hollow cylinders.

First consider the finite element approach. Application of fracture mechanic finite element analysis requires the specification of K_I evaluation methods and traditionally requires specification of specialised crack tip / front mesh discretisations [9, 10] that become increasing difficult to satisfy when considering cracks that exhibit non planar and non regular characteristics. Implementation of the Energy Domain Integral or J-Integral method [11] into commercially available finite element codes such as ABAQUS [12, 13] and ANSYS [14] have partially resolved the issue of K_I evaluation though the ‘user-friendliness’ of a fracture mechanic finite element analysis is still comprised by the requirement for a case by case specification of specialised crack tip / crack front mesh discretisations. Recent steps having been taken to counteract the difficulty is incorporation of the Extended Finite Element Method (X-FEM) [15-17] into the Abaqus finite element code, though the generality and ‘user-friendliness’ of these X-FEM regimes are not yet confirmed.

Now consider the thermal approach [2, 3]. Specific to the analysis of thermo-elastic induced K_I , the thermal weight function approach is a computationally efficient approach that is most practically implemented via an equivalent fracture mechanic finite element formulation [18, 19], and thus shares the prohibition to ‘user-friendliness’ associated with the requirement for a case by case specification of specialised crack tip / crack front mesh discretisations. Furthermore, unless the thermal weight function approach is included as basic functionality in commercial finite element codes it is questionable as to whether it actually offers any appreciable gain after recognising that the time saving associated with using it may be minimal in comparison to the time required for accurate mesh design.

Finally, consider the mechanical weight function approach. Introduced by Bueckner [4] and Rice [5], the mechanical weight function approach was originally proposed to constitute an efficient method with which to evaluate K_I associated with a two or three dimensional structure and the prescription of body force or traction type load. Subsequently, the mechanical weight function methodology has been extended to allow or incorporate K_I evaluations involving mixed traction, displacement prescribed load configurations and mixed fracture modes [6-8]. This in turn means that the mechanical weight function approach can be rationalised to constitute an efficient method which to evaluate the K_I associated with an arbitrary elastic or thermo-elastic response. For instance, it can be shown that a K_I^{TS} evaluation associated with the thermal shock of the finite elastically / thermo-elastically restrained cracked structure can be written,

$$K_I^{TS}(s) = \iint_{A_c} \sigma_{cp}^{TS} h^s dA_c \quad (1)$$

where s describes a specific crack tip / crack front location; a denotes a characteristic crack depth; A_c describes the surface area of the crack face; σ_{cp}^{TS} denotes the component of stress developed at and normal to the prospective crack plane (cp) within an equivalent thermal shocked crack free component; h_s denotes the mechanical weight function associated with the structural boundary conditions and crack front location s .

An equivalent 2D mechanical weight function paradigm can also be written,

$$K_I^{TS}(s) = \int_0^a \sigma_{cp}^{TS} h^s da \tag{2}$$

In either case it is clear that the mechanical method is a potential candidate for ‘user-friendly’ K_I evaluation because it can clearly alleviate the difficulties associated with explicitly modelling the crack tip/crack front if an appropriate h^s is known in priory. It can also be identified that a significant proportion of resource associated with implementation of a MWF problem can be accredited to the acquisition of an appropriate h^s through either retrieval of formularised relations within reference literature or explicit h^s derivation using one of the many techniques [20-32]. Unfortunately, the h^s derivation methodologies adopted within each of these works can be categorised into two types of approach that may or may not be considered particularly ‘user-friendly’, especially when it is recognised that h^s depend on structural boundary conditions and must therefore be evaluated on a case by case basis.

One approach is to rely on complicated fracture mechanic finite element analyses that will inevitably compromise the appeal of a mechanical weight function evaluation.

An obvious ‘user-friendly’ alternative is to acquire a previously documented h^s or use a multiple reference stress intensity factor methodology if the h^s corresponds to the target cracked structure and the multiple reference stress intensity factor correspond to the target cracked structure and are known in priory. This requirement for in prior knowledge of either h^s or multiple reference stress intensity factor implies that a ‘user-friendly’ mechanical weight function evaluation of K_I is at present limited to a variety of cracked infinite structures or structures that are representative of a selection of semi-infinite or finite length rigidly restrained cracked plate, semi-infinite cracked hollow cylinder, semi-infinite plate with cracked fastener holes and semi-infinite cracked joint. The implications associated with this observation are extensive and critically effect the mechanical weight function evaluation of fracture problems that are important and relevant to many industrial processes. An example of such a set of problems corresponds to the mechanical weight function evaluation of K_I^{TS} associated with the thermal shock of finite length elastically/thermo-elastically restrained cracked plates and finite length elastically/thermo-elastically restrained cracked hollow cylinders. That is, it is easily substantiated that there exists only two options if attempting to implement a mechanical weight function philosophy to thermal shocked finite length elastically/thermo-elastically restrained cracked plates and finite length elastically/thermo-elastically restrained cracked hollow cylinders.

The first option is to derive a mechanical weight function for the appropriate /thermo-elastically restrained cracked plate/cracked hollow cylinder configuration and then use Eq.1 or Eq.2. The drawback to this approach is attributed to an observance that impractical complex curve fitting regimes and a fracture mechanic finite element analysis is often required for this task. Furthermore, if a fracture mechanic finite element analysis was to be used to it would be far simpler in most cases just to solve the K_I^{TS} rather than endeavouring to construct a suitable mechanical weight function.

A more ‘user-friendly’ alternative would be to adopt an approximation such as that indicated Eq.(3a) and Eq.(3b).

$$K_I^{TS}(s) = \begin{cases} \iint_{A_c} \sigma_{cp}^{TS} h_f^s dA_c & \dots \text{if boundary restraint stiffnes is low} & (3a) \\ \iint_{A_c} \sigma_{cp}^{TS} h_r^s dA_c & \dots \text{if boundary restraint stiffness is high} & (3b) \end{cases}$$

In this case, a traditional mechanical weight function philosophy can be seen to approximate the evaluation process by utilising (1) the h_f^s and h_r^s for an equivalent semi-infinite / free (f) and rigidly (r) restrained cracked plate / cracked hollow cylinder (2) the σ_{cp}^{TS} associated with an equivalent finite length elastically/thermo-elastically restrained crack-free plate / hollow cylinder. In the case of a general crack and load configuration this approach is denoted by Eq.3a and Eq.3b.

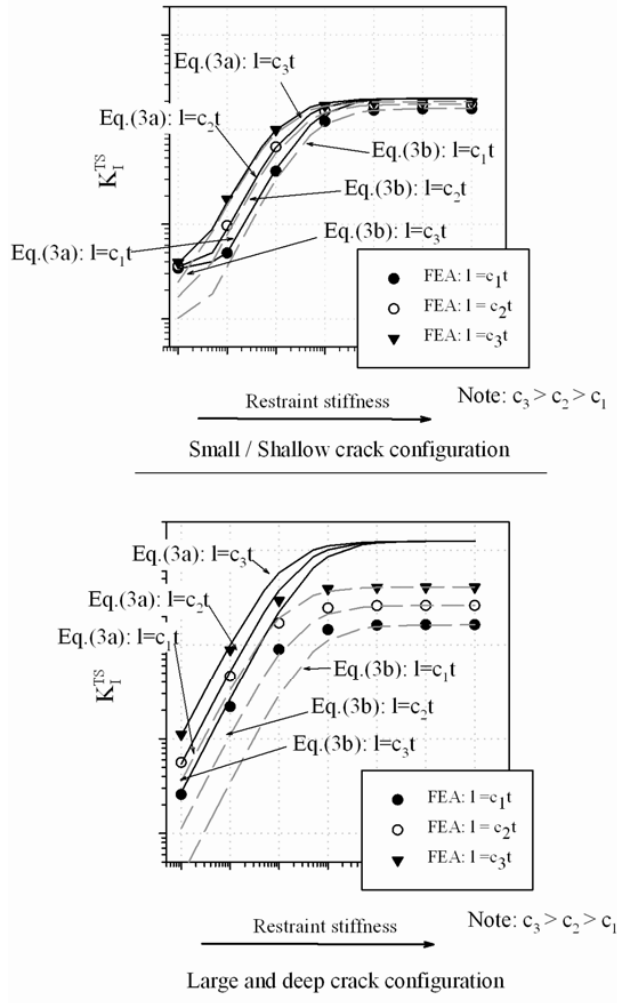


Figure 1: Typical evaluation characteristics if Eq.(3a) and Eq.(3b) are used for estimating K_I^{TS} associated with the creep-free thermal shock of finite length elastically/thermo-elastically restrained cracked plates / cracked hollow cylinders. Note that t is used to denote wall thickness and l denotes hollow cylinder length.

However, the limitations of these approximations have been investigated [33-35] and it can be shown by virtue of Fig.1 that the approximations attributed to Eq.3a and Eq.3b are either excessively conservative or non-

conservative under a wide range of intermediate boundary restraint stiffness. More importantly, the lack of an overlap between Eq.3a and Eq.3b as well as their observed conservatism or non-conservative tends to imply that the indicated adaptation cannot be easily defined and a confident and 'user-friendly' MWF analysis must use Eq.3a or an equivalent two dimensional variant. The difficulty with such a realisation lies in the previously noted observation that the evaluations attributed to Eq.3a (or an equivalent two dimensional variant) can be drastically over conservative to a point where the evaluations become unsuitable for implementation to fitness-for-purpose assessment. It is therefore reasonable to perceive that the mechanical weight function approach does not constitute a 'user-friendly' approach with which to evaluate K_I^{TS} associated with the creep-free thermal shock of finite length elastically / thermo-elastically restrained crack plates and finite length elastically / thermo-elastically restrained cracked hollow cylinders. In fact, this viewpoint can be extended to more general structures and it is reasonable to perceive that the mechanical weight function approach does not constitute a 'user-friendly' method with which to evaluate K_I for any particular structure unless the mechanical weight function or reference stress intensity factors for that precise structure are known in priory.

The aim of this article is to demonstrate how a CAWF approach can provide both 'user-friendly' and confident assessments of the K_I^{TS} associated with the creep-free thermal shock of finite length elastically / thermo-elastically restrained crack plates and finite length elastically / thermo-elastically restrained cracked hollow cylinders. Based upon mechanical weight function fundamentals, the CAWF approach incorporates a mechanical weight function analysis, a crack-free finite element analysis and an elastic Line-Spring analysis of compliance. Significantly, the CAWF approach is advantageous over a fracture mechanic finite element analysis, a thermal weight function analysis and a mechanical weight function analysis for several predominant reasons. First and foremost, the mechanical weight function component of the CAWF evaluation utilises well known and easily documented mechanical weight functions and geometry factors for an equivalent semi-infinite cracked structure. Second, the requirement for finite element analysis only extends to crack-free analyses that are significantly more simplified and do not require implementation of complex crack tip / crack front mesh discretisations. Finally, the ease at which the Line-Spring compliance analysis can be documented further advantages the CAWF approach. More specific details concerning the CAWF approach and current CAWF formulations are presented below.

2.2. The CAWF Formulation for Thermal Shocked Cracked Plates and Cracked Hollow Cylinders

Two distinct CAWF formulations are presented here. One formulation can be used as a method in which to estimate K_I^{TS} associated with the thermal shock of finite length elastically/thermo-elastically restrained cracked plates. A description of the said cracked plates is provided in Fillery and Hu [36] and this plate associative CAWF formulation can be written for an edge or semi-elliptical surface crack configuration as,

$$\begin{aligned}
 K_I^{TS}(s) \approx & \int_0^a a \sigma_{tt}^{cp}(\xi, 0) h_{tt}^s - f_p^s d\xi + f_p^s \sqrt{\frac{\pi a}{Q}} \frac{P_{TS}^{cp}}{A_{cp}} + f_{gb}^s \sqrt{\frac{\pi a}{Q}} \frac{6 M_{TS}^{cp}}{W t^2} \\
 & + \left(\mathbf{A}_{cpr} \mathbf{C}_r^{-1} \mathbf{A} \left(\mathbf{C}^{-1} - \mathbf{C}_f^{-1} \right) \mathbf{F} \right)^T \times \left[\begin{array}{c} \frac{f_p^s \sqrt{\frac{\pi a}{Q}}}{A_{eff}} \\ f_{gb}^s \sqrt{\frac{\pi a}{Q}} \frac{t}{2 I_{eff}} - f_p^s \sqrt{\frac{\pi a}{Q}} \frac{x_{eff}}{I_{eff}} \end{array} \right] \tag{5}
 \end{aligned}$$

where s denotes a crack front location; a denotes the crack depth; $\sigma_{tt}^{cp}(\xi, 0)$ denotes a prospective crack plane stress within an equivalent crack-free plate; $\left(\mathbf{A}_{cpr} \mathbf{C}_r^{-1} \mathbf{A} \left(\mathbf{C}^{-1} - \mathbf{C}_f^{-1} \right) \mathbf{F} \right)^T$ denotes an operation of elastic Line-Spring

compliance; P_{TS}^{cp} , M_{TS}^{cp} denote principle tensile force and bending moment; x_{eff} , A_{eff} and I_{eff} denote cross sectional area properties attributed to the elastic Line-Spring compliance; f_p^s , f_{gb}^s and h_{axi-f}^s denote geometry factors and a mechanical weight function associated with an equivalent semi-infinite cracked plate; Q denotes an elliptical crack shape factor.

The second CAWF formulation corresponds to a method in which to estimate K_I^{TS} associated with the thermal shock of finite length elastically/thermo-elastically restrained cracked hollow cylinders. A description of the said cracked plates is provided in Fillery and Hu [37] and this hollow cylinder associative CAWF formulation can be written for an edge or semi-elliptical surface crack configuration as,

$$K_I^{TS} \approx \int_0^a \sigma_{axi}^{cp}(\eta, 0) h_{axi-f}^s d\eta + f_p^s \sqrt{\frac{\pi a}{Q}} \frac{P_{TS}^{cp}}{A_{cp}} + f_{gb}^s \sqrt{\frac{\pi a}{Q}} \frac{4M_{TS}^{cp}}{R_o^4 - R_i^4} + \left(\mathbf{A}_{cpr} \mathbf{C}_r^{-1} \mathbf{A} (\mathbf{C}^{-1} - \mathbf{C}_f^{-1}) \mathbf{F} \right)^T \times \begin{bmatrix} \frac{f_p^s \sqrt{\frac{\pi a}{Q}}}{A_{eff}} \\ f_{gb}^s \sqrt{\frac{\pi a}{Q}} \frac{R_o}{I_{eff}} - f_p^s \sqrt{\frac{\pi a}{Q}} \frac{x_{eff}}{I_{eff}} \end{bmatrix} \tag{6}$$

where $\sigma_{axi}^{cp}(\eta, 0)$ denotes a prospective crack plane stress within an equivalent crack-free hollow cylinder; f_p^s , f_{gb}^s and h_{axi-f}^s denote geometry factors and a mechanical weight function associated with an equivalent semi-infinite cracked hollow cylinder.

2.3. The CAWF's Crack-Free Finite Element Formulation

A most desirable attribute of the CAWF approach is its ability to constitute a 'user-friendly' method for estimating K_I^{TS} associated with the creep-free thermal shock of a finite length elastically / thermo-elastically restrained cracked plate and a finite length elastically / thermo-elastically restrained cracked hollow cylinder. Although the mathematics of the CAWF approach has been demonstrated to be extensive, there are two reasons for affixing the 'user-friendly' description. First, the K_I^{TS} evaluation is performed by using a simple crack-free stress analysis and mechanical weight functions / geometry factors that are either currently available or easily documented within published literature. Second, the elastic compliance analysis that comprises the CAWF approach is suitable for tabulation where it is expected the cracked structural compliance associated with \mathbf{C} , \mathbf{C}_r and \mathbf{C}_f will be documented for future references. Thus, the CAWF approach can prove a systematic and 'user-friendly' method if:

1. Boundary restraint compliance terms contributing to \mathbf{C} , \mathbf{C}_r and \mathbf{C}_f can be efficiently ascertained in a 'user friendly' manner.
2. Prospective crack plate stress within equivalent crack-free plates and hollow cylinders can be efficiently ascertained in a 'user friendly' manner.

Assuming cracked structural compliance associated with \mathbf{C} , \mathbf{C}_r and \mathbf{C}_f are appropriately tabulated, it is easily identified that such a 'user friendly' evaluation is possible if the CAWF approach is embedded into a crack-free finite element procedure that comprises two distinct steps. For brevity, specifics concerning this finite element formulation are not presented here and readers are referred to Fillery and Hu [36, 37].

3. Application of the CAWF Approach to Thermal Shocked Cracked Plates / Cracked Hollow Cylinders

This section presents a validation for the CAWF formulations presented above by evaluating K_I^{TS} associated with the thermal shock of finite length elastically restrained cracked plates and finite length elastically restrained cracked

hollow cylinders. Proof of validation is provided via Fig.2 and Fig.3. Both figures indicate that three types of analysis approach are used to establish CAWF validation. These approaches are:

1. The CAWF approach,
2. A fracture mechanic finite element analysis.
3. A comparative mechanical weight function evaluation which comprises Eq.3a or equivalently the CAWF formulations of Eq.5 or Eq.6 with $C = C_f$.

Inspection of Fig.3 and Fig.4 will also summarize several key qualitative and quantitative attributes of the CAWF evaluations that are notable. For example, collective assessment of Fig.2 and Fig.4 demonstrates:

1. A good to excellent correlation between the CAWF approach and the benchmark fracture mechanic finite element (FEA) analyses.
2. A difference between the CAWF approach and the benchmark fracture mechanic finite element analyses is noticeable when considering a surface crack configuration, small plate or hollow cylinder aspect ratio and a low or near free boundary restraint; herein denoted a free boundary effect.

Key attributes of the CAWF approach that are not shown can additionally be listed as follows:

1. The difference between the CAWF approach and Eq.3a (i.e. what is considered the next most 'user-friendly' evaluation) is most pronounced at small plate or hollow cylinder aspect ratio (i.e. $l/t < 10$) where compliance change due to crack inclusion is most pronounced.
2. The difference between the CAWF approach and Eq.3a (i.e. what is considered the next most 'user-friendly' evaluation) is at times very significant and is most pronounced at large crack depth to thickness ratios $a/t > 0.5$ where compliance change due to crack inclusion is most pronounced.
3. The difference between the CAWF approach and Eq.3a is most pronounced at large crack aspect ratio (i.e. $c/a > 5-8$) where compliance change due to crack inclusion is most pronounced.
4. The difference between the CAWF approach and Eq.3a is lessened at small crack depth to thickness ratios $a/t < 0.4$ where compliance change due to crack inclusion is negligible.
5. The difference between the CAWF approach and Eq.3a is lessened at small plate or hollow cylinder aspect ratios where compliance change due to crack inclusion is negligible.
6. The difference between the CAWF approach and Eq.3a is lessened at small crack aspect ratio where compliance change due to crack inclusion is negligible.

Readers are referred to Fillery and Hu [38, 39] for more specific details concerning these validation exercises. Note that treatment for the aforementioned free boundary effect (2nd list point) has also been addressed and a CAWF adjustment proposed in Fillery and Hu [40].

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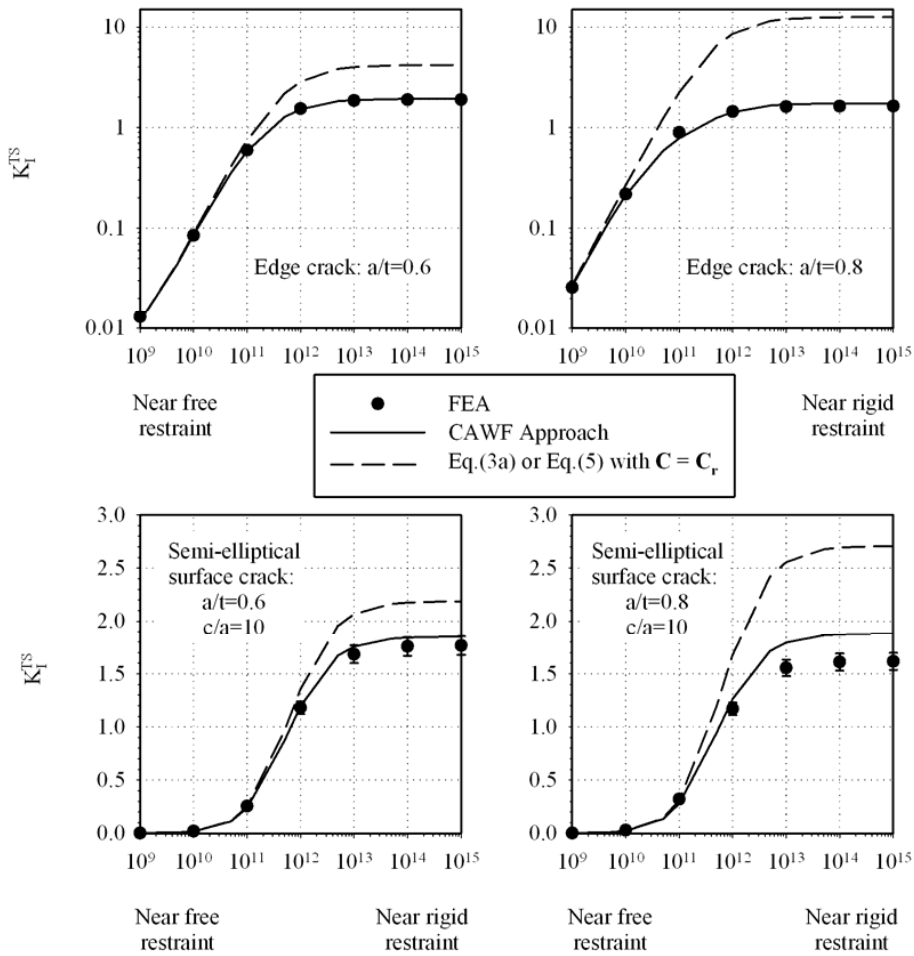


Figure 2: Maximum normalised K_I^{TS} attributed to the creep-free thermal shock of an elastically restrained edge/ semi-elliptical surface cracked plate; a plate aspect ratio $l/t=5$. Note that a denotes crack depth, c denotes crack aspect for a semi-elliptical surface crack, the horizontal axes are logarithmic, vertical axes of top charts are logarithmic and error bars (where visible) illustrate a $\pm 5\%$ error margin about target fracture mechanic finite element results (FEA).

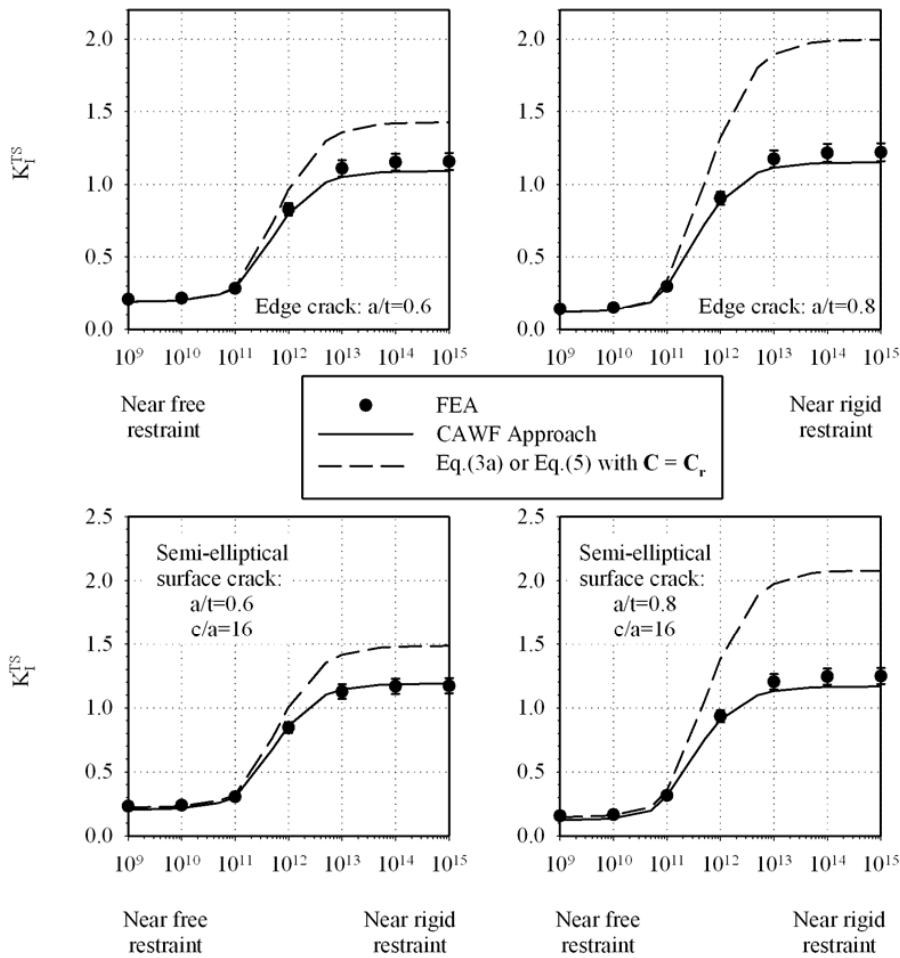


Figure 3: Maximum normalised K_I^{TS} attributed to the creep-free thermal shock of an elastically restrained edge/ semi-elliptical surface cracked hollow cylinder; a hollow cylinder aspect ratio $l/t=5$. Note that a denotes crack depth, c denotes crack aspect for a semi-elliptical surface crack, the horizontal axes are logarithmic, vertical axes of top charts are logarithmic and error bars (where visible) illustrate a $\pm 5\%$ error margin about target fracture mechanic finite element results (FEA).

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