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A complete survey of texture zeros in general and symmetric quark mass matrices

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ABSTRACT

We perform a systematic analysis of all possible texture zeros in general and symmetric quark mass matrices. Using the values of masses and mixing parameters at the electroweak scale, we identify for both cases the maximally restrictive viable textures. Furthermore, we investigate the predictive power of these textures by applying a numerical predictivity measure recently defined by us. With this measure we find no predictive textures among the viable general quark mass matrices, while in the case of symmetric quark mass matrices most of the 15 maximally restrictive textures are predictive with respect to one or more light quark masses.

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One of the most interesting questions of flavor physics is whether mixing angles are related to fermion mass ratios, like in the famous relation [1]

$$\sin\theta_c \simeq \sqrt{\frac{m_d}{m_s}} \tag{1}$$

between the Cabibbo angle θ_c and the ratio of down-quark mass to strange-quark mass. Here and in the following, quark masses are denoted by m_q with $q = d, s, b$ for the down-type quark masses and $q = u, c, t$ for the up-type quark masses. It is an open question if Eq. (1) is only an empirical relation or if there is a deeper reason for it founded in a hitherto undiscovered theory of flavor. Obviously, since the CKM matrix U is defined as

$$U = U_L^{(u)\dagger} U_L^{(d)} \tag{2}$$

with diagonalization matrices $U_L^{(u)}$ and $U_L^{(d)}$ given by

$$U_L^{(d)\dagger} M_d U_R^{(d)} = \mathrm{diag}\,(m_d, m_s, m_b) \quad \text{and}$$
$$U_L^{(u)\dagger} M_u U_R^{(u)} = \mathrm{diag}\,(m_u, m_c, m_t) \tag{3}$$

for the down-type and up-type quarks, respectively, the quark mass matrices M_d and M_u must have some structure in order to deduce relations like Eq. (1). The simplest attempt to achieve such relations is to place texture zeros in the mass matrices [2]. Apart from simplicity, texture zeros have the feature that they are practically synonymous with Abelian symmetries [3]. Unfortunately, even in this limited framework no clear-cut predictive model has emerged—see for instance [4,5] for reviews and [6] for an attempt on a unified texture in both quark and lepton sector. Therefore, it is appropriate to perform a complete study of all possibilities, as was recently done in [7] for the lepton sector (see also [8]).

In the analysis of [7] the notion of "maximally restrictive" textures plays an important role. These have a maximal number of zeros in the pair (M_d, M_u) in the sense that by placing one more zero into this pair it becomes incompatible with experimental data. It turned out that in the lepton sector the predictive power of general mass matrices with texture zeros is rather limited even for maximally restrictive textures. Actually, we find the same in the quark sector, as we will explain in more detail below. In view of this result, we perform an additional analysis with *symmetric* quark mass matrices.[1] In this paper, this is no more than a facile assumption in order to enhance predictivity,[2] however, it could be motivated by left–right symmetric models [10] or by models based on $SO(10)$—see [11] for reviews—with renormalizable

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[1] In many papers the mass matrices are assumed to be Hermitian. It is true that one can always achieve this by separate weak-basis transformations on the right-handed quark fields, however, this is not a valid argument because in the first place it is the texture zeros which define a basis and performing a subsequent basis transformation will in general remove the texture zeros.

[2] For this purpose, other assumptions are possible as well, like for instance a scaling ansatz [9].

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Yukawa couplings to scalar 10-plets and 126-plets, but not to scalar 120-plets which would introduce an antisymmetric component in the mass matrices [12].

Under weak-basis transformations the mass matrices transform as

$$M_d \to V_L^\dagger M_d V_R^{(d)}, \quad M_u \to V_L^\dagger M_u V_R^{(u)} \tag{4}$$

with unitary matrices V_L, $V_R^{(d)}$, $V_R^{(u)}$. Such transformations have no effect on the quark masses and the mixing matrix, *i.e.* they do not change the physical predictions. These weak-basis transformations are used in two ways. Firstly, it is well-known that with weak-basis transformations of Eq. (4) one can generate some zeros in (M_d, M_u) which have, therefore, no predictive power at all [13]. Such cases we exclude a priori from our analysis—for more details on this issue see [7]. Secondly, if one wants to preserve the zeros in the mass matrices, the unitary matrices occurring in Eq. (4) have to be restricted to permutation matrices times diagonal matrices of phase factors. Weak-basis transformations where the unitary matrices in Eq. (4) are pure permutation matrices, *i.e.* "weak-basis permutations" [7], allow to divide the possible patterns of texture zeros in (M_d, M_u) into equivalence classes with identical predictions and it is thus sufficient to treat one representative $(M_d^{(i)}, M_u^{(i)})$ of each equivalence class. Finally, with weak-basis transformations which are diagonal matrices of phase factors, *i.e.* by rephasing, one can then remove redundant phases from each representative $(M_d^{(i)}, M_u^{(i)})$ in order to obtain representatives with the minimal number of parameters.

Another important aspect of our general analysis is that we do not consider model realizations of the textures. Therefore, we cannot treat radiative corrections and we have to assume that the quark masses and the CKM matrix can be reproduced with sufficient accuracy by tree-level mass matrices. Consequently, we take into account only non-singular mass matrices.

To test if a texture is compatible with the observations, we perform a χ^2-analysis. We have ten physical observables: the six quark masses, the three mixing angles and the CP-violating phase. We have to check for each texture $(M_d^{(i)}, M_u^{(i)})$ whether it can reproduce the input data within experimental errors. Actually, for the mixing matrix U we prefer to use the Wolfenstein parameters [14] λ, A, $\bar{\rho}$ and $\bar{\eta}$, as defined in [15]. In order to have a consistent set of input data, we have to fix a common energy scale μ at which the quark masses and mixing parameters are taken. We settle on the scale $\mu = M_Z$, the mass of the Z gauge boson, which means that, if the texture zeros have a symmetry realization, then this symmetry is effective at the electroweak scale.[3] This scale has the advantage that all observables used for the input, with the exception of the top quark mass, are measured at energies below M_Z and, therefore, are evolved by the renormalization group equations of the Standard Model[4] to the scale $\mu = M_Z$. Concretely, we take the input values at M_Z from [17]; since in that paper the mixing angles and the CKM phase are given, we have to convert these into the Wolfenstein parameters. We display our input in Table 1.

Table 1
The quark masses and the parameters of the CKM matrix in the $\overline{\text{MS}}$ scheme at $\mu = M_Z$ computed within the Standard Model in [17]. The quark masses are given in units of $v = 174.104$ GeV. We have transformed the mixing parameters $\theta_{12}, \theta_{23}, \theta_{13}$ and δ given in Table 2 of [17] to the four Wolfenstein parameters [15] λ, A, $\bar{\rho}$ and $\bar{\eta}$ assuming Gaussian error propagation.

$m_u/(10^{-6}v)$	7.4	$^{+1.5}_{-3.0}$
$m_d/(10^{-5}v)$	1.58	$^{+0.23}_{-0.10}$
$m_s/(10^{-4}v)$	3.12	$^{+0.17}_{-0.16}$
$m_c/(10^{-3}v)$	3.60	± 0.11
$m_b/(10^{-2}v)$	1.639	± 0.015
$m_t/(10^{-1}v)$	9.861	$^{+0.086}_{-0.087}$
λ	0.22540	± 0.00070
A	0.828	± 0.014
$\bar{\rho}$	0.133	± 0.020
$\bar{\eta}$	0.350	± 0.015

Now we formulate a criterion that a texture $(M_d^{(i)}, M_u^{(i)})$ is compatible with the data. We stipulate that the contribution of each observable to $\chi^2_{\min}$, the minimum of χ^2, is at most 25; this means that the deviation of the observable from its experimental value is at most 5σ [7]. Since we have ten input values, this implies $\chi^2_{\min} \le 250$.

Even if we know that a texture $(M_d^{(i)}, M_u^{(i)})$ is compatible with the data, we do not know whether it has any predictive power or not. In order to discuss this question, we apply the numerical method developed in [7] which we repeat here briefly. The method is completely general, independent of the problem under discussion.[5] Consider a model with a set of parameters x making predictions $P_j(x)$ for the observables $\mathcal{O}_j$ with experimental mean values $\overline{\mathcal{O}}_j$ and errors σ_j. Then the χ^2-function of the model is given by[6]

$$\chi^2(x) = \sum_j \chi_j^2(x) \quad \text{with} \quad \chi_j^2(x) = \left(\frac{P_j(x) - \overline{\mathcal{O}}_j}{\sigma_j} \right)^2. \tag{5}$$

Let us assume that the model gives a good fit to the observables, *i.e.* $\chi^2_{\min}$ is sufficiently small. Now we want to pose the question whether the model is predictive with respect to the observable $\mathcal{O}_i$. Loosely speaking, this means we want to investigate how much the prediction for $\mathcal{O}_i$ can deviate from its mean value while varying x such that all $P_j(x)$ with $j \ne i$ remain close to $\overline{\mathcal{O}}_j$. The numerical implementation is done in two steps [7]:

1. We define

$$\widetilde{\chi}_i^2(x) = \chi^2(x) - \chi_i^2(x), \tag{6}$$

where we have removed the χ^2-contribution of the observable whose predictivity we want to investigate. With this $\widetilde{\chi}_i^2(x)$ we define a region in the parameter space via

$$B_i = \left\{ x \,|\, \widetilde{\chi}_i^2(x) \le \chi^2_{\min} + \delta\chi^2 \quad \text{and} \quad \chi_j^2(x) \le 25 \;\; \forall j \ne i \right\}, \tag{7}$$

where $\chi^2_{\min}$ is the minimal value of the *total* $\chi^2(x)$ of Eq. (5) and $\delta\chi^2$ is a fixed parameter in the range $0 \le \delta\chi^2 \lesssim 1$.

[3] So we have in mind that the symmetry responsible for the texture zeros is broken at the electroweak scale. If it were broken at a higher scale Λ, then in general the texture zeros would not be stable under the renormalization group evolution of the Yukawa coupling matrices between Λ and the electroweak scale.

[4] Within the Standard Model the relevant mass ratios m_d/m_b, m_s/m_b, m_u/m_t and m_c/m_t do not significantly change due to renormalization group running of the quark masses at scales from $\mu \sim 2$ GeV to $\mu \sim m_t$. This can be easily checked using the results of [16]. Moreover, within the Standard Model also the mixing angles do not run significantly. Therefore, if the effects of physics beyond the Standard Model are small at scales $\mu \lesssim m_t$, the analysis of texture zeros will not be affected by the exact choice of μ.

[5] Since in the quark sector all ten observables have been measured and their relative errors are not exceedingly small, we employ here a uniform predictivity measure for all observables. This was not possible in the lepton sector [7].

[6] In case of asymmetric error intervals, σ_j is replaced by σ_j^{left} and σ_j^{right} for $P_j(x) < \overline{\mathcal{O}}_j$ and $P_j(x) \ge \overline{\mathcal{O}}_j$, respectively.

Table 2
The 27 maximally restrictive textures in general quark mass matrices. A matrix entry 0 denotes a texture zero, and entries 1 and 2 stand for real positive and complex parameters, respectively. None of these textures is predictive with respect to any observable.

	M_d	M_u		M_d	M_u		M_d	M_u
G_1	$\begin{pmatrix}0&1&1\\0&1&2\\1&0&1\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$	G_2	$\begin{pmatrix}0&0&1\\0&1&0\\1&1&2\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&1\end{pmatrix}$	G_3	$\begin{pmatrix}0&0&1\\0&1&0\\1&1&2\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&1\\1&0&0\end{pmatrix}$
G_4	$\begin{pmatrix}0&0&1\\0&1&0\\1&2&1\end{pmatrix}$	$\begin{pmatrix}0&0&1\\1&1&0\\0&1&0\end{pmatrix}$	G_5	$\begin{pmatrix}0&0&1\\0&1&1\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&1\end{pmatrix}$	G_6	$\begin{pmatrix}0&0&1\\0&1&1\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&0\\1&1&0\end{pmatrix}$
G_7	$\begin{pmatrix}0&0&1\\0&1&2\\1&0&1\end{pmatrix}$	$\begin{pmatrix}0&1&1\\0&0&1\\1&0&0\end{pmatrix}$	G_8	$\begin{pmatrix}0&0&1\\0&1&1\\1&2&0\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&1\end{pmatrix}$	G_9	$\begin{pmatrix}0&0&1\\0&1&1\\1&2&0\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&0\\1&1&0\end{pmatrix}$
G_{10}	$\begin{pmatrix}0&0&1\\0&1&2\\1&1&0\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&1\\1&0&0\end{pmatrix}$	G_{11}	$\begin{pmatrix}0&0&1\\0&1&1\\1&2&0\end{pmatrix}$	$\begin{pmatrix}0&0&1\\1&1&0\\0&1&0\end{pmatrix}$	G_{12}	$\begin{pmatrix}0&0&1\\0&1&2\\1&1&0\end{pmatrix}$	$\begin{pmatrix}0&1&1\\0&0&1\\1&0&0\end{pmatrix}$
G_{13}	$\begin{pmatrix}0&0&1\\0&1&1\\1&2&0\end{pmatrix}$	$\begin{pmatrix}0&1&1\\1&0&0\\0&0&1\end{pmatrix}$	G_{14}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&0\\1&1&1\end{pmatrix}$	G_{15}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&1\\1&0&1\end{pmatrix}$
G_{16}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&1\\1&1&0\end{pmatrix}$	G_{17}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&0&1\\1&1&0\\0&1&1\end{pmatrix}$	G_{18}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&0&1\\1&1&1\\0&1&0\end{pmatrix}$
G_{19}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&1&1\\0&0&1\\1&0&1\end{pmatrix}$	G_{20}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&1&1\\0&0&1\\1&1&0\end{pmatrix}$	G_{21}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&1&1\\1&0&0\\1&0&1\end{pmatrix}$
G_{22}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&1&1\\1&0&1\\0&0&1\end{pmatrix}$	G_{23}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&1&1\\1&0&1\\0&1&0\end{pmatrix}$	G_{24}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&1&1\\1&0&1\\1&0&0\end{pmatrix}$
G_{25}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}1&1&1\\0&0&1\\0&1&0\end{pmatrix}$	G_{26}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&1\\1&1&1\\0&1&2\end{pmatrix}$	G_{27}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$	$\begin{pmatrix}0&1&1\\0&1&2\\1&0&1\end{pmatrix}$

2. With B_i we formulate the predictivity measure for $\mathcal{O}_i$ as

$$\Delta(\mathcal{O}_i) = \max_{x \in B_i} \chi_i^2(x). \tag{8}$$

Note that for $x \in B_i$ the $P_j(x)$ with $j \neq i$ are within the 5σ region of their experimental mean values $\overline{\mathcal{O}}_j$, *i.e.* they are "close" to $\overline{\mathcal{O}}_j$ in the sense of our compatibility criterion of a texture with the data. In Eq. (7), a non-zero $\delta\chi^2$ accelerates the convergence of the numerical maximization of $\Delta(\mathcal{O}_i)$ [7]. In the present paper we have set $\delta\chi^2 = 0.1$.

Clearly, the smaller $\Delta(\mathcal{O}_i)$ is, the better it is determined by the other observables. By choosing a bound b^2, we can define a predictivity criterion: for $\Delta(\mathcal{O}_i) \leq b^2$ we say the model is capable to predict the observable $\mathcal{O}_i$; in this case, its value deviates from its mean value by at most $b\sigma$. The choice of b is rather arbitrary. We follow Ref. [7] and take $b = 10$. Thus our predictivity criterion is

$$\Delta(\mathcal{O}_i) \leq 100. \tag{9}$$

The results of our analysis are presented in two tables, Table 2 for general mass matrices and Table 3 for symmetric mass matrices. In the general case, removing those pairs of mass matrices (M_d, M_u) whose texture zeros can be generated by weak-basis transformations and those with at least one singular matrix, we find 243 inequivalent classes with representatives $(M_d^{(i)}, M_u^{(i)})$. Of these, 214 classes are compatible with the data and among them there are 27 maximally restrictive classes whose representatives $(M_d^{(i)}, M_u^{(i)})$ are listed in Table 2. None of these mass matrices are predictive in the sense discussed above.

For the symmetric mass matrices, we cannot apply general weak-basis transformations which generate texture zeros, without destroying the symmetry of the mass matrices. Thus we discard only those pairs (M_d, M_u) which have at least one singular matrix and arrive in this way at 230 classes with representatives $(M_d^{(i)}, M_u^{(i)})$. Out of these, 79 classes survive the χ^2-test. Finally, these classes contain 15 maximally restrictive classes which are displayed in Table 3. According to our predictivity criterion, 11 of these 15 textures are predictive with respect to one or more of the three light quark masses m_u, m_d and m_s; the masses of the heavy quarks and the Wolfenstein parameters cannot be predicted.

At first sight, the last sentence seems to exclude approximate relations of the form of Eq. (1) or, more generally, relations of the form[7]

$$f\left(\frac{m_1}{m_2}, W\right) = 0, \tag{10}$$

where f denotes a function specified by the particular form of the texture, m_1 and m_2 are quark masses and W is one of the Wolfenstein parameters. Clearly, if a relation of the type of Eq. (10) follows from a texture, then W is predicted by m_1/m_2, but mathematically we can turn this conclusion around and say that m_1 is predicted by m_2 and W or m_2 is predicted by m_1 and W. However, *numerically* this will in general not be the case because of the different relative errors $\sigma_j/\overline{\mathcal{O}}_j$ of the observables. From Table 1 one finds that the relative errors of the light quark masses m_u, m_d and m_s are between 5% and 40%, the errors of the heavy quark masses are between 1% and 3% and the errors of the Wolfenstein parameters range from 0.3% for λ to 15% for $\bar{\rho}$. For instance, for the famous relation (1), varying m_d and m_s around their experimental mean values within ranges of the order of magnitude of their respective experimental errors, one may well obtain values for $\lambda \equiv \sin\theta_c$ which, due to the small relative error $\sigma_\lambda/\lambda \sim 0.3\%$, lie more than 10 sigmas off its mean value, *i.e.* $\Delta(\lambda) > 100$. Conversely, fixing λ and one of the masses gives a prediction for the second mass which, since the relative errors of m_d and m_s are much larger, may very well be within ten sigmas of the experimental value, *i.e.* $\Delta(m_q) < 100$ for $q = d, s$. To summarize, even if there is a relation of the form (10), the predictivity analysis will in general not detect all involved observables. This has to be kept in mind in the assessment of the results displayed in Table 3.

[7] For the sake of simplicity, in this paragraph we confine ourselves to three observables.

Table 3
The 15 maximally restrictive textures in symmetric quark mass matrices. A matrix entry 0 denotes a texture zero, and entries 1 and 2 stand for real positive and complex parameters, respectively. Several of these textures are predictive with respect to some of the light quark masses.

	M_d	M_u	predicted obs.		M_d	M_u	predicted obs.
S_1	$\begin{pmatrix}0&1&1\\1&1&2\\1&2&2\end{pmatrix}$	$\begin{pmatrix}1&0&0\\0&0&1\\0&1&2\end{pmatrix}$	m_d	S_2	$\begin{pmatrix}0&1&1\\1&1&2\\1&2&2\end{pmatrix}$	$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	m_d
S_3	$\begin{pmatrix}0&1&1\\1&0&1\\1&1&2\end{pmatrix}$	$\begin{pmatrix}1&0&0\\0&1&2\\0&2&2\end{pmatrix}$	m_d	S_4	$\begin{pmatrix}0&0&1\\0&2&2\\1&2&2\end{pmatrix}$	$\begin{pmatrix}0&1&1\\1&0&1\\1&1&2\end{pmatrix}$	–
S_5	$\begin{pmatrix}0&0&1\\0&2&2\\1&2&2\end{pmatrix}$	$\begin{pmatrix}1&0&1\\0&0&1\\1&1&2\end{pmatrix}$	–	S_6	$\begin{pmatrix}0&0&1\\0&2&2\\1&2&2\end{pmatrix}$	$\begin{pmatrix}1&1&0\\1&2&1\\0&1&0\end{pmatrix}$	–
S_7	$\begin{pmatrix}0&0&1\\0&1&1\\1&1&2\end{pmatrix}$	$\begin{pmatrix}1&0&0\\0&1&2\\0&2&2\end{pmatrix}$	m_d	S_8	$\begin{pmatrix}0&0&1\\0&2&2\\1&2&2\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&1\\1&1&0\end{pmatrix}$	m_d
S_9	$\begin{pmatrix}0&0&1\\0&1&1\\1&1&2\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	m_d	S_{10}	$\begin{pmatrix}0&0&1\\0&1&1\\1&1&2\end{pmatrix}$	$\begin{pmatrix}0&1&0\\1&2&0\\0&0&1\end{pmatrix}$	m_u, m_d, m_s
S_{11}	$\begin{pmatrix}0&1&1\\1&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}1&0&0\\0&1&2\\0&2&2\end{pmatrix}$	m_d	S_{12}	$\begin{pmatrix}0&0&1\\0&2&2\\1&2&0\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&1\\1&1&2\end{pmatrix}$	m_d
S_{13}	$\begin{pmatrix}0&0&1\\0&2&2\\1&2&0\end{pmatrix}$	$\begin{pmatrix}0&1&0\\1&1&1\\0&1&2\end{pmatrix}$	m_d, m_s	S_{14}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}1&0&1\\0&1&2\\1&2&2\end{pmatrix}$	–
S_{15}	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&2\end{pmatrix}$	$\begin{pmatrix}0&1&0\\1&1&1\\0&1&2\end{pmatrix}$	m_d, m_s				

All of the 15 textures for symmetric quark mass matrices have four $(S_1, \ldots, S_7, S_{11}, S_{14})$ or five $(S_8, S_9, S_{10}, S_{12}, S_{13}, S_{15})$ independent texture zeros, the most predictive one being[8]

$$S_{10}: \quad M_d \sim \begin{pmatrix}0&0&1\\0&2&1\\1&1&1\end{pmatrix}, \quad M_u \sim \begin{pmatrix}0&1&0\\1&2&0\\0&0&1\end{pmatrix}, \tag{11}$$

which shows predictive power with respect to all of the three light quark masses. In the light of the discussion above, this does not mean that the texture S_{10} has three *independent* predictions. Indeed, in this case there are only two independent ones, which can be formulated as $\sin\theta_{12} \simeq \sqrt{m_d/m_s}$, *i.e.* Eq. (1), and $|U_{ub}| \equiv \sin\theta_{13} \simeq \sqrt{m_u/m_t}$.

We emphasize once more that, after the consideration of general quark mass matrices, we have investigated *symmetric* (but not *Hermitian*) mass matrices, in which context we have discovered six viable textures with five texture zeros—see Table 3. It is interesting to compare these six textures with the five viable *Hermitian* textures with five texture zeros discussed in the literature [18,19]. For this comparison we use the table in [19] where the five Hermitian patterns I–V are listed and check if there are corresponding patterns in our Table 3, with zeros in corresponding places after suitable weak-basis permutations. We find the correspondences II $\sim S_9$, III $\sim S_8$, IV $\sim S_{15}$ and V $\sim S_{10}$, while pattern I has no correspondence to viable texture zeros in symmetric quark mass matrices. This comparison reveals the fundamental difference between texture zeros in symmetric and Hermitian mass matrices.[9]

Summary: In this paper we have performed a systematic and complete analysis of texture zeros in general and symmetric quark mass matrices. Among all the possible texture zeros in general quark mass matrices, we identified the 27 maximally restrictive classes—see Table 2—which, however, do not show predictive power with respect to any of the quark masses and mixing parameters. This is very similar to the situation of Dirac neutrinos, where texture zeros are predictive at most with respect to the smallest neutrino mass and, in one case, also to the Dirac phase δ of the lepton mixing matrix [7]. In other words, pure Abelian flavor symmetries effective at the electroweak scale, *i.e.* texture zeros but no further restrictions on the quark mass matrices, do not seem to contribute to the solution of the mass and mixing problem in the quark sector.

The case of texture zeros in symmetric quark mass matrices is more promising, since there the majority of the 15 maximally restrictive textures has some predictive power—see Table 3. This may be compared to the case of Majorana neutrinos, where the neutrino mass matrix is symmetric. There the maximally restrictive textures are also more predictive than for the Dirac neutrino case [7].

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[8] Note that for the mass matrices of Eq. (11) we find numerically $|(M_d)_{22}| \simeq m_b$, $|(M_u)_{22}| \simeq m_t$. For achieving the usual ordering with large elements having higher indices, one has to apply a permutation $2 \leftrightarrow 3$ to the indices.

[9] Note that in [19] the Hermitian mass matrices I-V are studied at $\mu = M_Z$ just as in this paper. Hence, the results can directly be compared.

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