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Running spectral index from large-field inflation with modulations revisited

Michael Czerny [a], Takeshi Kobayashi [b,c,*], Fuminobu Takahashi [a,d]

[a] Department of Physics, Tohoku University, Sendai 980-8578, Japan
[b] Canadian Institute for Theoretical Astrophysics, University of Toronto, 60 St. George Street, Toronto, Ontario M5S 3H8, Canada
[c] Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, Ontario N2L 2Y5, Canada
[d] Kavli IPMU, TODIAS, University of Tokyo, Kashiwa 277-8583, Japan

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ABSTRACT

We revisit large field inflation models with modulations in light of the recent discovery of the primordial B-mode polarization by the BICEP2 experiment, which, when combined with the *Planck* + WP + highL data, gives a strong hint for additional suppression of the CMB temperature fluctuations at small scales. Such a suppression can be explained by a running spectral index. In fact, it was pointed out by two of the present authors (TK and FT) that the existence of both tensor mode perturbations and a sizable running of the spectral index is a natural outcome of large inflation models with modulations such as axion monodromy inflation. We find that this holds also in the recently proposed multi-natural inflation, in which the inflaton potential consists of multiple sinusoidal functions and therefore the modulations are a built-in feature.

1. Introduction

The BICEP2 experiment detected the primordial B-mode polarization of the cosmic microwave background (CMB) with very high significance [1], giving a very strong case for inflation [2,3]. The inflation scale is determined to be

$$H_{\rm inf} \simeq 1.0 \times 10^{14}\ {\rm GeV} \left(\frac{r}{0.16}\right)^{\frac{1}{2}}, \tag{1}$$

$$r = 0.20^{+0.07}_{-0.05}\ (68\%\ {\rm CL}), \tag{2}$$

where $H_{\rm inf}$ is the Hubble parameter during inflation, and r denotes the tensor-to-scalar ratio. The preferred range of r is modified to $r = 0.16^{+0.06}_{-0.05}$, after subtracting the best available estimate for foreground dust. The BICEP2 result strongly suggests large-field inflation occurred, and by far the simplest model is the quadratic chaotic inflation [4].[1] The discovery of the tensor mode perturbations is of significant importance not only for cosmology but also for particle physics, because the suggested inflation energy scale is close to the GUT scale. If the primordial B-mode polarization is measured with better accuracy by the Planck satellite and other ground-based experiments, it will pin down the underlying inflation model, providing invaluable information on the UV physics such as string theory.

The BICEP2 data, when combined with the *Planck* + WP + highL data, gives a strong hint for some additional suppression of the CMB temperature fluctuations at small scales [1]. This is because the large tensor mode perturbations also contribute to the CMB temperature fluctuations at large scales, which causes the tension on the relative size of scalar density perturbations at large and small scales. The suppression of the density perturbations at small scales can be realized by e.g. (negative) running of the spectral index, hot dark matter, etc.[2] In this letter we focus on the running spectral index as a solution to this tension.

The spectral index of the curvature power spectrum $\mathcal{P}_\mathcal{R}$ is defined by

$$n_s(k) - 1 = \frac{d \ln \mathcal{P}_\mathcal{R}(k)}{d \ln k}, \tag{3}$$

and the running of the spectral index is obtained as the differentiation of n_s with respect to $\ln k$. The preferred range of the

* Corresponding author.
E-mail addresses: mczerny@tuhep.phys.tohoku.ac.jp (M. Czerny), takeshi@cita.utoronto.ca (T. Kobayashi), fumi@tuhep.phys.tohoku.ac.jp (F. Takahashi).

[1] For various large-field inflation models and their concrete realization in the standard model as well as supergravity and superstring theory, see e.g. [5–22].

[2] Before the BICEP2 results, there was a hint for the presence of hot dark matter, such as sterile neutrinos [23–25]. Non-thermally produced axions are also an interesting candidate [26,27].

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running spectral index and its statistical significance are not given in [1]. Since the combination of the *Planck* + WP + highL data constrains the running as [28] $dn_s/d\ln k = -0.022 \pm 0.010$ (68% CL), we expect that, once the BICEP2 data is combined, non-zero values of $dn_s/d\ln k \approx -0.02 \sim -0.03$ will be suggested with strong significance. As a reference value, we will assume that the running is approximately given by $dn_s/d\ln k \sim -0.025$ over the observed cosmological scales, but the precise value is not relevant for our purpose.[3]

In a single-field slow-roll inflation model with a featureless potential, the running of the spectral index is of second order in the slow-roll parameters, and therefore of order 10^{-3}. Thus, it is a challenge to explain a running as large as $dn_s/d\ln k \sim -0.025$. For various proposals on this topic, see e.g. Refs. [30–37]. In particular, [37] pointed out that a large negative running that is more or less constant over the observed cosmological scales would quickly terminate inflation within $N \lesssim 30$ in terms of the e-folding number. However, it should be noted that such a discussion is based on the assumption that the inflaton potential is expanded in the Taylor series of the inflaton field with finite truncation. In fact, it is possible to realize the running spectral index in simple single-field inflation models. In Ref. [38], two of the present authors (TK and FT) showed that a sizable running spectral index can be realized without significant impact on the overall behavior of the inflaton if there are small modulations on the inflaton potential. (See also [39] for related work.) Here, in order for the inflaton dynamics to be locally affected by the modulations, the inflaton field excursion must be relatively large as in the large-field inflation. Therefore, both the tensor mode and the running spectral index are a natural outcome of the large field inflation with modulations. Examples such as monomial inflaton potentials ($V = \lambda\phi^n$) with superimposed periodic oscillations were studied in [38].

In this letter we revisit the large-field inflation with modulations in light of the recent discovery of the primordial B-mode polarization by the BICEP2. Along the lines of Ref. [38], we study the recently proposed multi-natural inflation [19,20,40] as an example. Interestingly, the existence of the periodic oscillations is a built-in feature of multi-natural inflation. We show that the negative running spectral index can be realized without significant impact on the overall inflation dynamics, similar to the case studied before. We will also show that the predicted values of (n_s, r) for quadratic chaotic inflation and natural inflation can also be realized in multi-natural inflation.

2. Implications of BICEP2 for inflation

Before proceeding to the analysis, let us here briefly discuss the implications of the BICEP2 results for inflation. First, the inflaton field excursion during the last 60 e-foldings exceeds the Planck scale, $M_p \simeq 2.4 \times 10^{18}$ GeV, in a large field inflation model suggested by the BICEP2 result (1). One plausible way for having a good control of the inflaton potential over super-Planckian field values is to introduce a shift symmetry, under which the inflaton ϕ transforms as

$$\phi \to \phi + \alpha, \tag{4}$$

[3] Note that both the spectral index and its running are usually evaluated at a pivot scale, and the running is assumed to be scale-independent in the MCMC analysis of the CMB data [28]. On the other hand, there is no firm ground to assume that they are completely scale-independent, and in fact, they do depend on scales in many scenarios. Therefore, the comparison between theory and observation must be done carefully, and a dedicated analysis to each theoretical model would be necessary to deduce some definite conclusions. We also note that the joint analyses of the *Planck* and BICEP2 datasets have been performed in recent works such as [29], see Eq. (22).

where α is a real transformation parameter. The shift symmetry needs to be explicitly broken in order to generate the inflaton potential. That is to say, the global continuous shift symmetry can be explicitly broken down to a discrete one. For instance, the symmetry breaking could manifest itself as sinusoidal functions in the inflaton potential. If a single sinusoidal function dominates the inflaton potential, it is the natural inflation [5]. On the other hand, if there are many sources for the explicit breaking, the inflaton potential may consist of multiple sinusoidal functions with different height and periodicity. Such a case was investigated in [19, 20], which we refer to as multi-natural inflation. In this sense, the existence of small periodic oscillations is a built-in feature of the multi-natural inflation. As we shall see below, a sizable negative running spectral index can be generated in multi-natural inflation.

3. Running spectral index from inflation with modulations

3.1. Basic idea

Let us first review our basic idea from [38] on generating a sizable running spectral index. The point is that substructures in the inflaton potential can affect the tilt and/or the running spectral index in a non-negligible way, while not changing the overall behavior of the inflaton dynamics.

We consider an inflaton potential $V(\phi)$ with modulations, which can be decomposed as

$$V(\phi) = V_0(\phi) + V_{mod}(\phi), \tag{5}$$

where the second term represents the modulations. We assume that the modulations are so small that the slow-roll approximations

$$3H\dot{\phi} \simeq -V'(\phi), \tag{6}$$

$$3H^2 M_p^2 \simeq V(\phi), \tag{7}$$

remain valid. (Here an overdot denotes a time derivative.)

The running in the spectral index is expressed in terms of the slow-roll parameters as

$$\frac{dn_s}{d\ln k} \simeq -24\epsilon^2 + 16\epsilon\eta - 2\xi, \tag{8}$$

where

$$\epsilon \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta \equiv M_p^2 \frac{V''}{V}, \qquad \xi \equiv M_p^4 \frac{V'V'''}{V^2}, \tag{9}$$

and primes denote derivatives with respect to the inflaton field ϕ. Thus, in order to generate a sizable running, the third derivative of the inflaton potential must be large. At the same time, we consider the modulations to minimally affect the overall behavior of the inflaton dynamics. To this end we require the following conditions for most of the inflaton field values:

$$\left|V_0(\phi)\right| \gg \left|V_{mod}(\phi)\right|, \tag{10}$$

$$\left|V_0'(\phi)\right| \gtrsim \left|V_{mod}'(\phi)\right|, \tag{11}$$

$$\left|V_0''(\phi)\right| \lesssim \left|V_{mod}''(\phi)\right|, \tag{12}$$

$$\left|V_0'''(\phi)\right| \ll \left|V_{mod}'''(\phi)\right|. \tag{13}$$

We assume that the effect of the modulations V_{mod} (and its derivatives) on the inflaton dynamics should be negligibly small when averaged over a sufficiently long time or large field space. One example for such modulations is a sinusoidal function. If both V_0 and V_{mod} are given by sinusoidal functions as in multi-natural inflation,

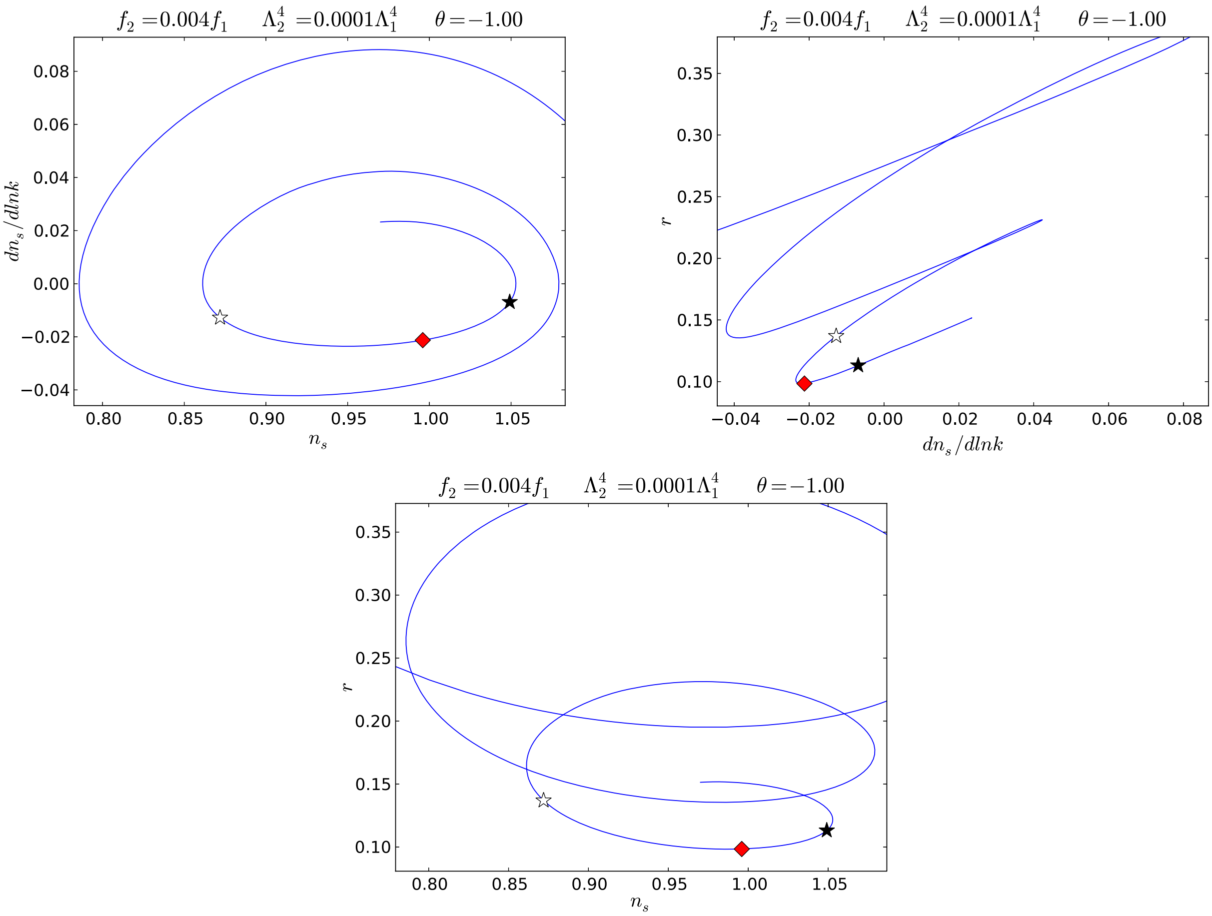

Fig. 1. Evolution of the spectral index n_s, its running $dn_s/d\ln k$, and the tensor-to-scalar ratio for $f = 100M_p$, $A = 4 \times 10^{-3}$, $B = 1 \times 10^{-4}$, and $\theta = -1$. The red diamond denotes $N = 60$ e-folds before the end of inflation, whereas the black (white) star corresponds to $N = 63$ (55).

V_{mod} should have a shorter period, i.e., smaller decay constant to satisfy the above conditions.

The curvature perturbation power spectrum can be computed as[4]

$$\mathcal{P}_{\mathcal{R}}(k) \simeq \left.\frac{H^4}{4\pi^2\dot{\phi}^2}\right|_{k=aH}. \tag{15}$$

Thus, the curvature power spectrum receives modulations due to $\dot{\phi}^2 \propto |V_0' + V_{mod}'|^2$. Note that the modulations of H due to V_{mod} is subdominant, as long as condition (10) is met. On the other hand, the spectral index as well as the running are strongly affected by the modulations if (12) and (13) are met.

3.2. Multi-natural inflation

Now let us consider multi-natural inflation, where the inflaton potential consists of multiple sinusoidal functions. For simplicity let us focus on the case of two sinusoidal functions with different heights and periodicities. The potential takes the form

$$V(\phi) = C - \Lambda_1^4 \cos(\phi/f_1) - \Lambda_2^4 \cos(\phi/f_2 + \theta), \tag{16}$$

where the decay constants f_1 and f_2 take different values. C is a constant that shifts the minimum of the potential to zero, and θ is a relative phase. The last term shifts the potential minimum from the origin to $\phi = \phi_{\rm min}$, and also modifies the potential shape. This model is reduced to the original natural inflation in the limit of either $\Lambda_2 \to 0$ or $f_2 \to \infty$. If we further take the limit of $\Lambda_1 \to \infty$ and $f_1 \to \infty$ while Λ_1^2/f_1 is kept constant, the model is reduced to the quadratic chaotic inflation.

To simplify the notation we set $f_1 = f$ and $\Lambda_1 = \Lambda$, and relate the parameters by,

$$f_2 = Af, \tag{17}$$

$$\Lambda_2^4 = B\Lambda^4, \tag{18}$$

where A and B are real and positive constants. Because we are interested in small modulations to the inflaton potential, we choose the parameters so that the conditions from (10) to (13) are met. That is to say, A and B satisfy

$$A^3 \ll A^2 \lesssim B \lesssim A \ll 1. \tag{19}$$

Figs. 1 and 2 show the running as a function of n_s and r for $f = 100M_p$ and $f = 10M_p$, respectively. In Fig. 1 we take $A = 4 \times 10^{-3}$, $B = 1 \times 10^{-4}$, and $\theta = -1$, whereas in Fig. 2 we take $A = 4.5 \times 10^{-2}$, $B = 1.24 \times 10^{-2}$ and $\theta = 2$. One can see that

[4] It should also be noted that the formulae obtained using the slow-roll approximations can break down when the higher order derivatives of the inflaton potential become too large. In particular, expressions such as (8) contain errors of order the approximate results operated by

$$\frac{1}{H}\frac{d}{dt} \simeq -M_p^2 \frac{V'}{V}\frac{d}{d\phi}. \tag{14}$$

This quantity is smaller than unity in the cases studied in this letter, as we consider examples where the oscillation amplitude of the spectral index is larger than that of the running. Hence we can invoke the slow-roll approximations.

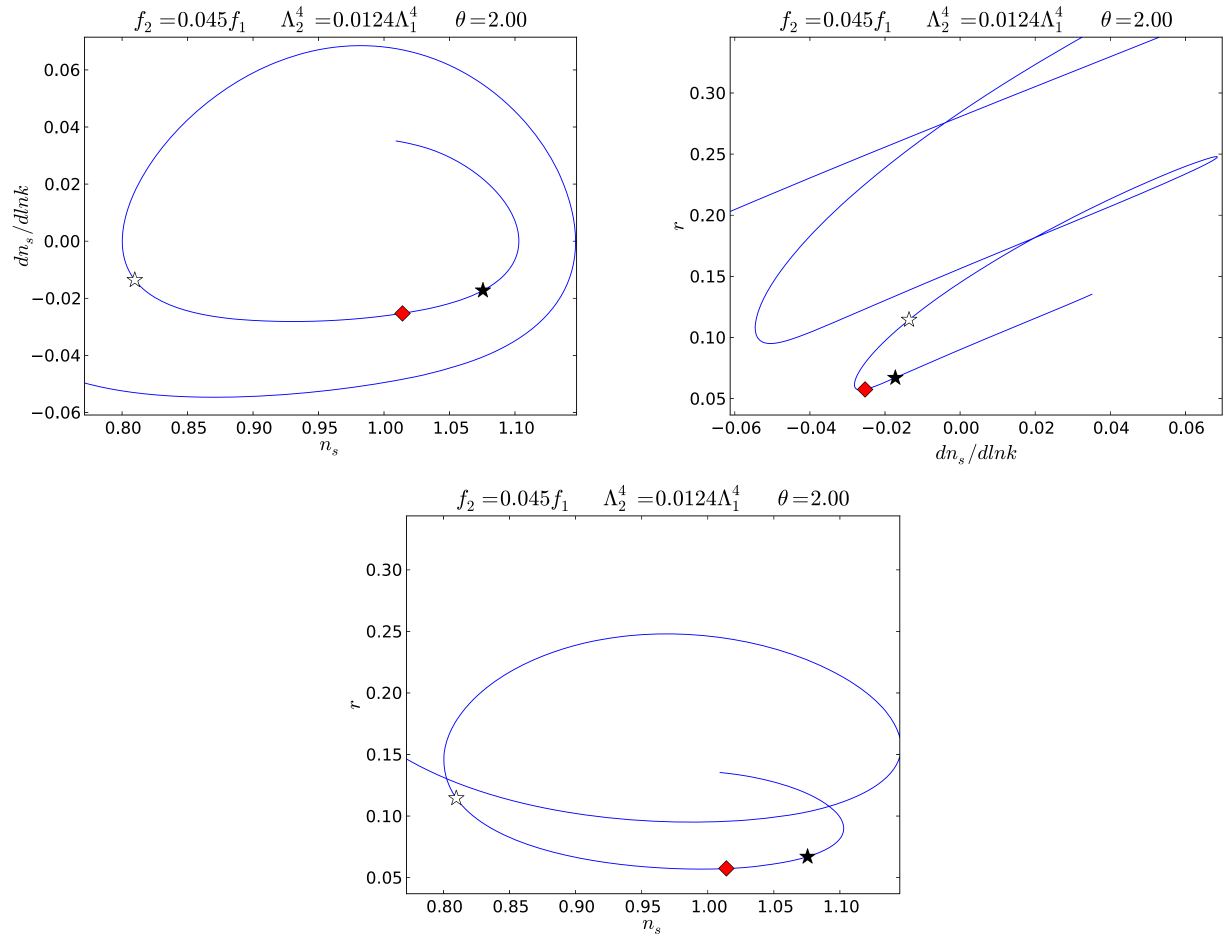

Fig. 2. Same as Fig. 1 but for $f = 10M_p$, $A = 4.5 \times 10^{-2}$, $B = 1.24 \times 10^{-2}$ and $\theta = 2$. The red diamond denotes $N = 60$ e-folds before the end of inflation, whereas the black (white) star corresponds to $N = 62$ (54).

the inequalities in (19) are satisfied for these choices of the parameters. The case of $f = 100M_p$ is approximately the same as the case of the quadratic potential with small modulations [38]. The potential height Λ is fixed by the Planck normalization on the primordial density perturbations [28]. Its typical value is approximately given by $\Lambda^2 \sim f \times 10^{13}$ GeV. The predicted values of n_s, r, and the running $dn_s/d\ln k$ evolve clockwise around the curves and the tail of the curves correspond to $N = 70$ e-folds before the end of inflation. In Fig. 1, the black (white) star corresponds to $N = 63$ (55). In Fig. 2, the black (white) star corresponds to $N = 62$ (54). In both cases, the red diamond is for $N = 60$. Therefore a negative running spectral index can be realized in between the $N = 50$ to 60 regime. Thus, by taking the second sinusoidal term to be modulations in the potential, multi-natural inflation readily accommodates a negative running of n_s.

In fact, there is a relation among the e-folding number during one period of modulations, ΔN, and the oscillation amplitudes of $dn_s/d\ln k$ and n_s [41], given by

$$A_{dn_s/d\ln k} \sim \frac{2\pi}{\Delta N} A_{n_s}. \tag{20}$$

This relation approximately holds in the above two examples.

3.3. *Comparison with observational constraints*

The combined *Planck* + WP + highL data constrains the running as

$$dn_s/d\ln k = -0.022 \pm 0.010 \text{ (68\%)}, \tag{21}$$

when allowing tensor fluctuations [28]. A recent work [29] have performed a joint analysis of the *Planck* and BICEP2 datasets, giving similar constraints,

$$dn_s/d\ln k = -0.024 \pm 0.010 \text{ (68\%)}. \tag{22}$$

The analyses assume a scale-independent running, while multi-natural inflation produces a running which itself oscillates with respect to the wave number. However, when the oscillation period of the modulations on the inflaton potential is large enough to incorporate of order 10 number of e-foldings, then the produced running is effectively constant over the observed CMB scales. This is the case for the example parameters chosen in the previous subsection; Figs. 1 and 2 show that over ∼10 e-folds around the pivot scale, the running varies by $\Delta(dn_s/d\ln k) \sim 0.01$, which is within the errors of the constraints. Hence the above running constraints should be valid for multi-natural inflation as well. It would also be interesting to investigate cases with smaller oscillation periods such that the running exhibits a strong scale-dependence within the observed scales. (Though in such cases one should also consider the applicability of the slow-roll approximations, see Footnote 4.) More precise data from upcoming experiments may allow detailed investigations of the oscillation period of the modulations in the inflaton potential.

4. Discussions

As we have pointed out in [38], large field inflation with substructures in the inflaton potential entails large tensor perturbations as well as a running spectral index. In this letter, we revisited large field models with modulations in the context of multi-natural inflation, where multiple effects breaking the shift symmetry give rise to a superposition of sinusoidal functions to the inflaton potential. We focused on the interesting case where a hierarchy exists among the periodicities of the sinusoidal oscillations, so that the model is a large field model with superimposed periodic oscillations. While the large field nature of the model produces large tensor mode perturbations, the oscillations on the potential source a running spectral index for the density perturbation spectrum. We have seen that multi-natural inflation possesses rich phenomenology, in particular, it produces a wide variety of values for $(r, n_s, dn_s/d\ln k)$ depending on the relative size of the sinusoidal functions.

We also remark that large field inflation with modulations not only sources the running spectral index for the density perturbations, but also for the tensor perturbation spectrum. The tensor spectral index and its running are given in terms of the slow-roll parameters as

$$n_T = \frac{d\ln \mathcal{P}_T}{d\ln k} \simeq -2\epsilon, \qquad \frac{dn_T}{d\ln k} \simeq -8\epsilon^2 + 4\epsilon\eta. \tag{23}$$

Unlike n_s, the tensor tilt depends only on ϵ and thus the tensor running is set by ϵ and η. Therefore, one sees from the conditions (10)–(13) that the running of the tensor tilt is smaller than that for the density perturbations. Nonetheless, it is worth noting that a non-negligible $dn_s/d\ln k$ entails some amount of $dn_T/d\ln k$ as well. This will become especially important when measuring the tensor tilt by combining tensor observations at different scales, such as when combining CMB experiments with direct observations of gravity waves. A simple extrapolation between widely different scales without considering the possibility of the tensor running could lead to a misinterpretation of the observational results; In particular, such a naive extrapolation would give rise to an apparent violation of the slow-roll consistency relation $r = -8n_T$ [42], which holds locally at each scale in our case.

Upcoming experimental data are expected to verify whether there actually are sizable tensor mode perturbations and a running of the spectral index. This will shed light on the substructure of the inflaton potential, which should be directly tied to the underlying microphysics.

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