Demonstration on the indexes design of gravity satellite orbit parameters in the low-low satellite-to-satellite tracking mode

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Abstract: Combining with the exigent demand of the development of satellite gravimetry system in China, aiming at the determination of technical indexes of gravity satellite orbit parameters, on the basis of the numerical experiments and results analysis, the design indexes of gravity satellite orbit height, inter-satellite range and the orbit inclination are analyzed and calculated, and the issues towards twin gravity satellites such as coherence requirement of the orbit semi-major axes, control requirement of the pitch angle and time interval requirement to keep twin satellites formation in mobility are discussed. Results show that the satellite orbit height is 400 km to 500 km, the inter-satellite range is about 220 km, the satellite orbit inclination is between polar orbit and sun-synchronous orbit, the semi-major axes difference of twin satellites orbit is within ±70.146 m, the pitch angle of twin satellites is about 0.9 degree, and the time interval to keep twin satellites formation in mobility is 7 days to 15 days.

Key words: low-low satellite-to-satellite tracking (SST-ll); satellite orbit height; inter-satellite range; orbit inclination; index design

1 Introduction

Besides the combination with actual requirements of application, the technology difficulty and feasibility of realization should also be considered to develop satellite gravimetry technology in China. The sensitivity level of different satellite gravimetry modes and orbit parameters of the Earth’s gravity field are different, therefore, the different satellite gravimetry modes, such as the satellite-to-satellite tracking in the high-low mode (SST-hl), the satellite-to-satellite tracking in the low-low mode (SST-ll), the satellite gravity gradi-ent (SGG), and the orbit parameters should be tested, then, the satellite gravimetry system, which has good performance, a feasible technology and acceptable expense, can be chosen based on the comparison of test results

According to the actual development level of satellite gravimetry hardware facility in China and requirements of the Earth’s gravity field, the SST-II technology should be developed firstly, the precision of static and dynamic Earth’s gravity field model (EGM) obtained by the SST-II mode is at least ten times higher than that of the SST-II mode, and the detection precision of the SST-II mode of static and time variable Earth’s gravity field in intermediate and low frequency is rather high, of which the technology content is comparatively low and easy to be realized, and the successful experience of the whole system of Gravity Recovery and Climate
Experiment (GRACE) can also be used for reference. The demonstration on the indexes design of satellite orbit parameters includes the determination of satellite orbit height, inter-satellite range, satellite orbit inclination, coherence requirement of the orbit semi-major axes, control requirement of the pitch angle and time interval requirement to keep twin satellites formation in mobility. Based on the demonstration of the orbit parameters indexes design of satellite gravimetry system in China, the theoretical and technological support for analysis and demonstration of overall fighting technology indexes of satellite gravimetry system and technology indexes of the main effective loads are supplied, and some references for overall indexes of China’s own satellite gravimetry system and technology indexes of the effective loads can also be provided.

2 Optimum choice of satellite orbit parameters

2.1 Index design of satellite orbit height

The relationship between cumulative geoid height error and inter-satellite range rate error is

$$\delta N = R \sum \frac{1}{2^n} \frac{R}{fM} \left[ 1 - P_n(\cos \theta) \right] \delta R_{12}$$

(1)

where \(\delta N\) is the cumulative geoid height error, \(R\) is the Earth’s average radius, \(\delta R\) is the geocentric radius vector of a random point in the satellite orbit, \(fM\) is the gravitational constant, \(N_{\text{max}}\) is the highest degree of the EGM obtained by the Nyquest rule, \(P_n(\cos \theta)\) is the Legendre polynomial, \(\theta\) is the geocentric colatitude, \(\delta R_{12}\) is inter-satellite range rate error.

According to the equation (1), assuming that the inter-satellite range is 220 km, the sampling rate of the SST-II system is 1 s and the inter-satellite range rate error is \(10^{-6}\) m/s, when the satellite orbit height is 500 km, 400 km, 300 km and 200 km, respectively, the corresponding cumulative geoid height errors and the statistics results are listed in figure 1 and table 1.

![Figure 1: The cumulative geoid height errors recovered by satellite data in the different orbit height](image)

From the figure 1 and table 1, we can infer that the cumulative geoid error increases along with the increase of the model degree and the rise of the orbit height. When the orbit height is 500 km, the cumulative geoid error of the EGM in 120 degree is about 22.893 cm, it is close to the cumulative geoid error of the EGM recovered by the GRACE mission in 120 degree, which is about 18 cm. Therefore, according to the EGM precision recovered by satellite data in different orbit height, the orbit height of China’s future SST-II system should be 400 km to 500 km, and that is useful for the recovery of the EGM in 120 degree. The orbit height of the GRACE mission is 400 km to 500 km, so the analysis in this paper is correct.

### Table 1: The precision statistical results of the cumulative geoid height recovered by satellite data in the different orbit height (Unit: \(10^{-2}\) m)

<table>
<thead>
<tr>
<th>Satellite orbit height (km)</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.052316</td>
<td>0.058334</td>
<td>0.065996</td>
<td>0.080168</td>
<td>0.110124</td>
<td>0.175317</td>
</tr>
<tr>
<td>300</td>
<td>0.057961</td>
<td>0.071303</td>
<td>0.098632</td>
<td>0.168670</td>
<td>0.351424</td>
<td>0.833519</td>
</tr>
<tr>
<td>400</td>
<td>0.064909</td>
<td>0.093542</td>
<td>0.177450</td>
<td>0.446839</td>
<td>1.326432</td>
<td>4.341813</td>
</tr>
<tr>
<td>500</td>
<td>0.073584</td>
<td>0.132319</td>
<td>0.359179</td>
<td>1.275426</td>
<td>5.172612</td>
<td>22.892787</td>
</tr>
</tbody>
</table>
2.2 Index design of inter-satellite range

According to the equation (1), assuming that the orbit height is 500 km, the sampling rate of the SST-II system is 1s and the inter-satellite range rate error is $10^{-6}$ m/s, when the inter-satellite range is 110 km, 220 km, 330 km and 440 km, respectively, the corresponding cumulative geoid height errors and the statistics results are listed in figure 2 and table 2.

From table 2, we can infer that the cumulative geoid error is the least when the inter-satellite range is about 220 km, and that is useful for the recovery of the EGM in 120 degree. The inter-satellite range of the GRACE mission is changing between 170 km and 270 km, so the analysis in this paper is correct.

2.3 Index design of satellite orbit inclination

The Earth’s gravity field information which is distributed uniformly on the globe should be obtained as more as possible by satellite gravimetry technology. Therefore, the solution precision of the EGM is determined by many factors, such as data coverage on the globe, data uniformity and data precision, etc.

![Figure 2: The cumulative geoid height errors recovered by satellite data in the different inter-satellite range](image)

Table 2 The precision statistical results of the cumulative geoid height recovered by satellite data in the different inter-satellite range (Unit: $10^{-7}$ m)

<table>
<thead>
<tr>
<th>Inter-satellite range (km)</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>0.146676</td>
<td>0.258320</td>
<td>0.664326</td>
<td>2.177524</td>
<td>7.948811</td>
<td>30.796769</td>
</tr>
<tr>
<td>220</td>
<td>0.073584</td>
<td>0.132319</td>
<td>0.359179</td>
<td>1.275426</td>
<td>5.172612</td>
<td>22.892787</td>
</tr>
<tr>
<td>330</td>
<td>0.049533</td>
<td>0.091990</td>
<td>0.274563</td>
<td>1.124331</td>
<td>5.487751</td>
<td>29.238200</td>
</tr>
<tr>
<td>440</td>
<td>0.037299</td>
<td>0.073425</td>
<td>0.252383</td>
<td>1.264037</td>
<td>6.913751</td>
<td>29.443272</td>
</tr>
</tbody>
</table>

2.4 Index design of coherence requirement of orbit semi-major axes of twin satellites

In the entire flying period, the twin gravity satellites are always kept in the same orbit surface, however, because of the influences of G-force, atmosphere resistance, sun radiation and lunisolar gravitation, the orbit of twin satellites will change, and it will influence the original formation of twin satellites, therefore, the satellites must be kept in mobility in order to ensure that the inter-satellite range is 170 km to 270 km. In addition, the orbit eccentricities of twin satellites are not completely consistent, and these will influence the relative movement, so the maintenance of twin satellites...
formation is very important, and it is one of the key technologies for detection of the Earth's gravity field by the SST-II mission.

The relationship between the relative inter-satellite range change $\Delta L$ and the average orbit semi-major axes difference $\Delta a$ of twin satellites is \cite{1,5,6}

$$\Delta L = -\frac{1}{2}n\Delta a\Delta t$$  \hspace{1cm} (2)$$

where $n$ is the average orbit angular rate, $\Delta t$ is the data sampling period.

The gravity data need to be sampled continuously during the SST-II mission, so the frequent orbit mobility is not appropriate, however, the relative inter-satellite range needs to be kept within a certain range, therefore, the SST-II mission has much higher requirement for twin satellites orbit. According to the GRACE mission, the gravity data should be sampled continuously in at least one month. According to the equation (2), the orbit semi-major axes difference of twin satellites should be controlled within $\pm 70.146$ m.

2.5 Index design of control requirement of pitch angle of twin satellites

The orbit height of twin satellites will descend from 500 km to 300 km in the whole flying period with the effect of non-conservative forces, such as atmosphere resistance. Although the twin satellites can be designed uniformly and symmetrically so as to keep the resistance they suffered consistent, the twin satellites should be maintained in a certain pitch angle in order to keep the inter-satellite link of K-band visible, it is shown in figure 6 as follows.

Assuming that the inter-satellite range is $L$, the satellite radius is $r$, the geocentric angle of twin satellites

\[ L = \frac{1}{2}n\Delta a\Delta t \]

where $n$ is the average orbit angular rate, $\Delta t$ is the data sampling period.

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Assuming that the inter-satellite range is $L$, the satellite radius is $r$, the geocentric angle of twin satellites

\[ L = \frac{1}{2}n\Delta a\Delta t \]
is $\theta$, in order to keep the inter-satellite link of K-band normal, the satellite pitch angle $\eta$ should fulfill the condition that $^{[1,5,6]}$

$$\eta = \frac{1}{2} \theta = \frac{360^\circ S}{4\pi L}$$  \hspace{1cm} (3)

The requirement of satellite pitch angle has to do with satellite orbit height and inter-satellite range. The satellite pitch angle and its statistics result corresponding to the different orbit height and inter-satellite range are listed in figure 7 and table 3.

From the figure 7 and table 3, we can infer that the satellite pitch angle is increasing along with the descending of orbit height and the increase of inter-satellite range, the satellite pitch angle is about 0.916°.

2.6 Index design of time interval requirement to keep twin satellites formation in mobility

The expression of the time interval $\Delta t$ to keep twin satellites formation in mobility is $^{[5-7]}$

$$\Delta t = \frac{2 \sqrt{\rho}}{a \pi} \frac{L_{\text{max}} - L_{\text{min}}}{\Delta B}$$  \hspace{1cm} (4)

where $a$ is the average orbit semi-major axes, $\Delta B$ is the trajectory coefficient difference of twin satellites, and $\rho$ is the atmosphere density in the different orbit height.

According to the atmosphere density in the different orbit height, which is listed in the second column in table 4, the time interval $\Delta t$ to keep twin satellites formation in mobility is calculated and its results are listed in table 4.

From the table 4, we can infer that the time interval to keep twin satellites formation in mobility is increasing along with the rise of orbit height. The actual time interval has to do with figure, mass, characteristic of surface material, attitude control of satellite and sun movement, etc, so it has some difference compared with the results in this paper.

3 Conclusion

In summary, the orbit parameters of future gravimetry satellite in the SST-II mode can be obtained,

1. The satellite orbit height is 400 km to 500 km.
2. The inter-satellite range is about 220 km.
3. The satellite orbit inclination is between polar orbit and sun-synchronous orbit.
4. The semi-major axes difference of twin satellites
orbit is within ±70.146 m.

(5) The pitch angle of twin satellites is about 0.9 degree.

(6) The time interval to keep twin satellites formation in mobility is 7 days to 15 days.

References


