Nonlinear Dynamic Study on Effects of Flywheel Eccentricity in a Turbine Generator with a Squeeze Film Damper

Wei-Chun Hsu\textsuperscript{a*}, Chuen-Ren Wang\textsuperscript{b}, Ting-Nung Shiau\textsuperscript{b}, De-Shin Liu\textsuperscript{b}, Tyau-Her Young\textsuperscript{c}

\textsuperscript{a}Department of Mechanical Engineering, WuFeng University. 621 Chia Yi, Taiwan, R.O.C
\textsuperscript{b}Department of Mechanical Engineering, National Chung Cheng University. 621 Chia Yi, Taiwan, R.O.C
\textsuperscript{c}Department of Mechanical Engineering, National Taiwan University of Science and Technology. 106 Taipei, Taiwan, R.O.C

Abstract

This study presents the nonlinear dynamics of a turbine generator subjected to an unbalance force caused by mass eccentricity of the flywheel. The proposed turbine generator is equipped with a squeeze film damper under the effect of rub-impact in the oil film rupture. This system consists of a turbine rotor, generator, flywheel with mass eccentricity, and a squeezed film damper. The turbine and generator are connected by a coupling which is regarded as rigid. System equations of motion are formulated by the Global Assumed Mode Method (GAMM) and the Lagrange's approach. This study investigates the nonlinear behavior of the system, including the trajectory of the rotor in the time domain, frequency spectrum, Poincaré map, and bifurcation diagram, by solving the system's equation of motion with the Runge-Kutta method. Results show that the system displays period-one motion when the rotor speed ratio is very small under the effects of flywheel eccentricity. However, the periodic motion is suddenly transformed into aperiodic motion without any transition. The squeeze film damper fails to support the rotor if the speed ratio is in the interval between 0.93 and 1.065 when flywheel eccentricity is 1e-4m. And the interval of speed ratio shifted to between 0.5 and 1.06 and when flywheel eccentricity increases to 5e-4 m.

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* Corresponding author. Tel.: +886-5-226-7125*21931; fax: +886-5-2263723.
E-mail address: ingmar.hsu@wfu.edu.tw
1. Introduction

The turbine generator is widely used to generate electric power in the power industries. A turbine generator is generally equipped with a flywheel energy storage system (FESS), in which the flywheel can be used to store rotational inertia. However, the system experiences problematic vibration if the flywheel exhibits mass eccentricity or if there is friction in the bearings. This study presents the flywheel eccentricity effect on the nonlinear dynamic behavior of a turbine generator equipped with a squeeze film damper.

Over the last five decades, the squeeze film damper (SFD) has been widely used on industrial rigs, and studies on the subject are abundant. For example, Vance and Kirton [1,2] performed experiments showing the pressure of
hydrodynamic oil film excited by a squeeze film damper. Their experimental results are in agreement with the oil film model based on the long and short bearing theory of the Reynolds’ Equation, respectively. Gunter [3] investigated the steady-state and transient response of a squeeze film bearing. This analysis includes the effects of unbalance, cavitation, and retainer springs. Humes and Holmes [4] conducted experiments on the performance of a squeeze film bearing as a load-carrying member, showing that a theoretical model based on a squeeze film allowing a limited negative gauge pressure can reliably predict the vibration orbit based on the non-dimensional static, unbalanced coefficient, and oil film coefficient. Qinchang et al. [5] considered the effects of fluid inertia on fluid velocity profiles and provided an axial inertia velocity profile for short squeeze film dampers. Their experimental results confirm that the new fluid force models are better than traditional short squeeze film damper theory. Chu and Holmes [6] showed that a change in the rotor unbalance, SFD static eccentricity ratio, and SFD supply pressure can cause significant movement of system resonance and delay the onset of instability because of excessive damping. However, excessive damping may lock up any retainer spring in the SFD position, even causing severe vibration at the position of any overhung disk. Inayat-Hussain et al. [7] used the continuation method to discuss the nonlinear response of a rigid rotor equipped with a squeeze film damper without centering springs. They considered variables such as bearing parameter (B), unbalance force (U), and gravity (W). Their results show that the appropriate combination of parameters U and B can avoid non-synchronous vibration and the jump phenomenon. Della and Adiletta [8,9] investigated the contribution of SFD in many studies over the last four decades, and these studies can help the reader understand SFD. Ping et al. [10] showed that the solution of a discrete model that can deflect the continuum model because the system model neglects the effect of mass distribution and other inner nonlinear factors. Therefore, the analysis of a continuum model combined with the direct integration method and mode superposition method successfully shows a typical oil whip phenomenon. Flowers and Wu [11] developed a simplified model to investigate the lateral vibration of a shaft-disk system with bearing clearance nonlinearity. An analysis of the numerical simulation and limit cycle reveals a super-harmonic vibration and aperiodic behaviour. Wang et al. [12] discussed the coupled dynamic behaviour of a flexible rotor-bearing system with interaction between the blades and the rotor. They used bifurcation diagrams, 3D spectral plots, and Poincaré maps to analyze the dynamic behaviour of this system, and showed that the blades vibration is significantly affected by the rotational speeds.

Previous studies have introduced the dynamic behavior of a gas-turbine generator and a FESS, and Bolund et al. [13] provided an overview of flywheel technology and a 200 kW flywheel with high-voltage technology. They illustrated the configuration and model of the flywheel energy storage system. This system achieves excellent performance when operated at low-surrounding air pressure. Han et al. [14] also illustrated the general design methodology of a flywheel by analyzing these influences, and presented a practical method for determining the geometric parameters. Bachschmid et al. [15] analyzed the steam-whirl instability using a model of a 425 MW steam turbo generator. This model includes the effects of seals, steam-whirl exciting force coefficients, and oil film bearing coefficients. Their model can observe the occurrence of steam-whirl instability phenomena in response to the instability factor. Ricci et al. [16] proposed updating the torsional model of a steam turbo generator with a modification factor based on experimental Eigen-frequencies. Shiau et al [17,18] proposed the new method of the Generalized Polynomial Expansion Method to simulate the deflections of the flexible shaft, and discussed the stability of system with the hybrid method which combines the merits of the harmonic balance and collocation method. The numerical results illustrated that a large side load may cause unstable synchronous equilibrium response when the journal is rotating at speed twice the synchronous resonant speed. Their results show that the updating formulation achieves excellent agreement between theory and experiment.

Squeeze film dampers have been widely used in aircraft, gas-turbine generators, and turbo machinery, and FESSs often serve as supplementary uninterruptible power supply (UPS) storage in several power industries. However, few publications have considered the dynamics of the rotor system between the squeeze film bearing and FESS. Consequently, this study investigates the nonlinear dynamic response of a gas-turbine generator with a squeeze film damper when the equipped flywheel is subjected to an unbalance force.

2. Formulation of Dynamic Model

Most turbine generators consist of a turbine rotor and generator connected by a coupling. This study assumes that this coupling is rigid, and the turbine and generator are regarded as components of the same shaft. The squeeze film damper is equipped near the flywheel, and the equation of motion is expressed as follows.
2.1. Squeeze film force

The pressure distribution of the oil film in a hydrodynamic bearing as shown in figure 1 is employed by the Reynolds’ equation, and this expression can be described as

$$\frac{\partial}{\partial \eta} \left( h^3 \frac{\partial P}{\partial \eta} \right) + \frac{\partial}{\partial x} \left( h^3 \frac{\partial P}{\partial x} \right) = 6\mu \left( \frac{\partial h}{\partial x} + 2\frac{\partial h}{\partial t} \right) + P \left( R^2 \frac{\partial P}{\partial h} \right) \frac{\partial h}{\partial x}$$

(1)

Previous scholars have cited the mathematic formulation of pressure distribution for oil film based on short bearing theory [1,2], The pressure distribution is described as

$$P(\theta, r) = \frac{3\mu}{c^2 (1 + \varepsilon \cos \theta)^3} \left[ -2\phi \varepsilon \sin \theta - 2\dot{\varepsilon} \cos \theta \right] \left( x^2 - L^2 \right)$$

(2)

where $\varepsilon = e/c$. The oil film force can then be obtained as

$$\begin{bmatrix} F_r \\ F_\theta \end{bmatrix} = -R \int_0^{2\pi} \frac{\partial}{\partial \theta} P(\theta, r) \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \, d\theta$$

(3)

After substituting equation (2) into equation (3), the resulting oil film forces, as shown in figure 1, are obtained by using the cavitated (t-film) short bearing theory in a rotating reference frame and expressed as following.

$$F_r = \mu R L \frac{2\phi}{C} \left[ \frac{\varepsilon^2}{(1 - \varepsilon^2)^2} + \frac{\pi}{4} \left( 1 - 2\varepsilon \right) \varepsilon \frac{e}{(1 - \varepsilon^2)^{1/2}} \right]$$

$$F_\theta = \mu R L \frac{2\phi}{C} \left[ -2\dot{\varepsilon} \frac{\pi e}{4(1 - \varepsilon^2)^{3/2}} + \frac{2\dot{\varepsilon}}{1 - \varepsilon^2} \right]$$

(4)

The oil film forces in fixed reference frame can be obtained by using the transformation matrix.

$$\begin{bmatrix} F_r^{\text{fixed}} \\ F_\theta^{\text{fixed}} \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} F_r \\ F_\theta \end{bmatrix}$$

(5)

2.2. Dynamic model

The configuration of a simple rotor-bearing system is shown in figure 2(a). The system consists of a rotating flexible shaft, rigid disks, and bearings. Two reference frames are used to describe the system motion. One is a fixed reference frame X-Y-Z, and the other is a rotating reference x-y-z. The rotating frame rotates around the X-axis at a whirl speed of $\Omega$. The term $\Omega$ denotes the rotating speed of the shaft around the X-axis. The deflections of the cross-section for the shaft include two translations ($V, W$) and two rotations ($B, \Gamma$). They are functions of positions $x$ along the rotating axis and time $t$ according to the global assumed mode method (GAMM). Therefore, they can be expressed as:

$$V = V(x, t), \quad W = W(x, t)$$

$$B = B(x, t), \quad \Gamma = \Gamma(x, t)$$

(6)

The rotations $B, \Gamma$ are related to the translations $V, W$ by the following equations:

$$B(x, t) = -\frac{\partial W(x, t)}{\partial x}, \quad \Gamma(x, t) = \frac{\partial V(x, t)}{\partial x}$$

(7)

Based on the mode superposition technique (GAMM) proposed by Shiau and Hwang [17], the displacement solutions are approximated by following functions.
\[ V(x,t) = \sum_{n=1}^{N_y} a_n(t)x^{n-1}, \quad W(x,t) = \sum_{n=1}^{N_y} b_n(t)x^{n-1} \]

\[ B(x,t) = -\frac{\partial W(x,t)}{\partial x}, \quad \Gamma(x,t) = \frac{\partial V(x,t)}{\partial x} \]

This study uses the Lagrange's approach to derive the system equation of motion. The kinetic energy \( T \) and the potential energy \( U \) of the rotating shaft are given by

\[ U = U_s + U_b \]
\[ T = T_s + T_d + T_e \]

where \( U_s \) and \( U_b \) are the potential energy of the shaft and bearing, respectively. The terms \( T_s \) and \( T_d \) are the kinetic energy of the shaft and disk, respectively. The term \( T_e \) is the kinetic energy related to mass eccentricity. These variables can be defined as follows:

\[ U_s = \frac{1}{2} \int_0^l EI \left\{ (V')^2 + (W')^2 \right\} dx \]
\[ U_b = \sum_{j=1}^{N_y} \left\{ \frac{1}{2} k_{mm} (V_j')^2 + \frac{1}{2} k_{pp} (W_j')^2 + k_{mp} V_j' W_j' \right\} + \left\{ \frac{1}{2} k_{mm} (V')^2 + \frac{1}{2} k_{pp} (W')^2 \right\} \]

\[ T_s = \frac{1}{2} \int_0^l \rho A \left\{ (V')^2 + (W')^2 \right\} dx + \frac{1}{2} \int_0^l \left\{ (B')^2 + (\Gamma')^2 \right\} dx + \frac{1}{2} \int_0^l \left\{ (V - B)^2 + (W - \Gamma)^2 \right\} dx + \frac{1}{2} \int_0^l \left\{ \Omega^2 I_{yy} \right\} dx \]
\[ T_d = \frac{1}{2} \sum_{j=1}^{N_y} m_j \left\{ (V_j')^2 + (W_j')^2 \right\} + \frac{1}{2} \sum_{j=1}^{N_y} \left\{ \Omega^2 I_{yy} \right\}_{j=1} \]

\[ T_e = \int_0^l e(x) \rho(x) A(x) \Omega \left[ -V \sin(\Omega t + \phi) + W \cos(\Omega t + \phi) \right] dx + \int_0^l e(x) \rho(x) A(x) \Omega^2 dx + \sum_{j=1}^{N_y} m_j (e_j')^2 \Omega^2 \]

2.3. Rub-Impact force

When the system is subject to a large unbalance force because of mass eccentricity, the oil film ruptures, and the rub-impacts occur intermittently between the squirrel cage supported structure of SFD and housing. In figure 1(b), the forces \( F_{r^{\text{rub}}} \) and \( F_{w^{\text{rub}}} \) of a fixed reference frame generated from the interaction between the squirrel cage supported structure of SFD and housing are nonlinear. These rubbing impacts occur intermittently and could last a short duration. Thus, this study considers the Coulomb-type frictional relationship at contacts. When rubbing occurs, as in figure 1(b), the radial impact force \( F_N \) and tangential rub force \( F_T \) can be expressed as:

\[ F_N = \begin{cases} 0, & \text{for } (c < \delta) \\ (c - \delta) K_s, & \text{for } (c \geq \delta) \end{cases} \quad \text{and} \quad F_T = \mu_\text{im} F_N \]

where \( \delta = \sqrt{V^2 + W^2} \) represents the radial displacement of the rotor, \( K_s \) is the housing stiffness, and \( \mu_\text{im} \) is the frictional coefficient. The forces \( F_{r} \) and \( F_{T} \) can be described as:

\[ F_{r^{\text{rub}}} = -F_N \left( \frac{V}{c} \right) - \psi_r F_T \left( \frac{W}{c} \right) \]
\[ F_{W^{\text{rub}}} = -F_N \left( \frac{W}{c} \right) + \psi_r F_T \left( \frac{V}{c} \right) \]

(15)
Alternatively, these forces can be written as:

\[
\begin{bmatrix}
F_{v}^{rub} \\
F_{w}^{rub}
\end{bmatrix} = \begin{bmatrix}
\left(c - \delta \right) K_f & 1 \\
-\psi_f H_{m} & 1
\end{bmatrix} \begin{bmatrix}
V \\
W
\end{bmatrix}
\]

(17)

where \(\psi_f\) is the function that determines the direction of frictional forces.

\[
\psi_f = \begin{cases}
-1, & \text{for } v_i > 0 \\
0, & \text{for } v_i = 0 \text{ and } v_i = \frac{\dot{W}(V)}{e} - \frac{V}{e} \\
1, & \text{for } v_i < 0
\end{cases}
\]

(18)

Fig. 1 Schematic diagram of (a) the squeeze film damper; (b) the rub-impact force between squirrel cage supported structure and housing

2.4. Equation of motion

The system equation of motion can be derived by using Lagrange's approach as shown below.

\[
\frac{d}{dt} \left[ \frac{\partial}{\partial q_i} (T - U) \right] - \frac{\partial}{\partial q_i} (T - U) = 0
\]

(19)

where \(q_i = \{a_1, a_2, \cdots a_{N_a}, b_1, b_2, \cdots b_{N_b} \} \).

After substituting equation (5), equations (10)-(14) and equation (17) into equation (19), the equation of motion can be expressed as follows:

\[
\begin{bmatrix}
M_T + M_R & 0 \\
0 & M_T + M_R
\end{bmatrix} \begin{bmatrix}
\dot{a} \\
\dot{b}
\end{bmatrix} + \begin{bmatrix}
G & \begin{bmatrix}
K_{yy} \\
K_{yz}
\end{bmatrix} & \begin{bmatrix}
K_{zy} \\
K_{zz}
\end{bmatrix}
\end{bmatrix} \begin{bmatrix}
\dot{a} \\
\dot{b}
\end{bmatrix} + \begin{bmatrix}
K_S & 0 \\
0 & K_S
\end{bmatrix} \begin{bmatrix}
a \\
b
\end{bmatrix} = \begin{bmatrix}
R_a \\
R_b
\end{bmatrix}
\]

(20)

where \(R_a\) and \(R_b\) are of the form of generalized force.

\[
R_a = \{R_{a_1}, R_{a_2}, \cdots R_{a_{N_a}} \}^T
\]

\[
R_b = \{R_{b_1}, R_{b_2}, \cdots R_{b_{N_b}} \}^T
\]

(21)

where \(a = \{a_1, a_2, \cdots a_{N_a} \}^T\) and \(b = \{b_1, b_2, \cdots b_{N_b} \}^T\) are generalized coordinates, \(R_a\) and \(R_b\) are generalized force, they are \(N_p \times 1\) matrices, \([M_T], [M_R], [G], [K_S], [K_{yy}], [K_{yz}], [K_{zy}], [K_{zz}]\) and \([K_{zz}]\) are \(N_p \times N_p\) real symmetric matrices. The detail information are expressed as Appendix A.
3. Formulation of Dynamic Model

Figures 2(a) and 2(b) show that the proposed system consists of a turbine rotor, generator, and flywheel with mass eccentricity, and a squeezed film damper with a squirrel cage supported structure. The turbine and generator are connected by a coupling, which are regarded as rigid. B1, B2, and B3 are ball bearings, and B4 is a squeeze film damper with a squirrel cage supported structure. The terms D1, D2, and D3 are turbine blades; D4 is the generator disk, and D5 is the flywheel. These variables are regarded as rigid disks. The data of the shaft of whole system, material properties and stiffness of ball bearings are given on Table 1, 2 and 3.

Table 1 The parameters of the turbine generator.

<table>
<thead>
<tr>
<th>Node no.</th>
<th>Node location(mm)</th>
<th>Element length(mm)</th>
<th>Bearing &amp; Disk</th>
<th>Out radius(mm)</th>
<th>Inner radius(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>19.2</td>
<td></td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>19.2</td>
<td>66.8</td>
<td>B1</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>86</td>
<td>48</td>
<td></td>
<td>75.5</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>134</td>
<td>61</td>
<td>D1</td>
<td>75.5</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>195</td>
<td>76</td>
<td>D2</td>
<td>75.5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>271</td>
<td>73</td>
<td>D3</td>
<td>75.5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>344</td>
<td>70</td>
<td></td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>414</td>
<td>68.8</td>
<td>B2</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>482.8</td>
<td>70.3</td>
<td>B3</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>10</td>
<td>553.1</td>
<td>27.5</td>
<td></td>
<td>55</td>
<td>6</td>
</tr>
<tr>
<td>11</td>
<td>580.6</td>
<td>36.45</td>
<td>D4</td>
<td>55</td>
<td>6</td>
</tr>
<tr>
<td>12</td>
<td>617.05</td>
<td>43.65</td>
<td>25</td>
<td>8.1</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>660.7</td>
<td>12.65</td>
<td>B4(SFD)</td>
<td>25</td>
<td>8.1</td>
</tr>
<tr>
<td>14</td>
<td>673.35</td>
<td>20.1</td>
<td></td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>693.1</td>
<td>-</td>
<td>D5</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2 The parameters of the location disk and support

<table>
<thead>
<tr>
<th>Location (cm)</th>
<th>Mass (Kg)</th>
<th>Polar inertia(Kg*m²)</th>
<th>Mass inertia(Kg*m²)</th>
<th>eccentricity (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>34 (D1)</td>
<td>1.341</td>
<td>0.0037</td>
<td>0.0069</td>
<td>1e-5</td>
</tr>
<tr>
<td>195 (D2)</td>
<td>1.71</td>
<td>0.0048</td>
<td>0.0091</td>
<td>1e-5</td>
</tr>
<tr>
<td>271 (D3)</td>
<td>1.863</td>
<td>0.0059</td>
<td>0.0115</td>
<td>1e-5</td>
</tr>
<tr>
<td>580.6 (D4)</td>
<td>5.587</td>
<td>0.020</td>
<td>0.040</td>
<td>1e-5</td>
</tr>
<tr>
<td>693.1 (D5)</td>
<td>15</td>
<td>0.06</td>
<td>0.12</td>
<td>5e-4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Location (cm)</th>
<th>K_yy=Kzz (10^7 N/m)</th>
<th>K_yz=K_zy (10^7 N/m)</th>
<th>C_yy=C_zy (10^3 N*s/m)</th>
<th>C_yz=C_zy (N*s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.2 (B1)</td>
<td>11.75</td>
<td>-0.8756</td>
<td>1.752</td>
<td>0</td>
</tr>
<tr>
<td>414 (B2)</td>
<td>11.75</td>
<td>-0.8756</td>
<td>1.752</td>
<td>0</td>
</tr>
<tr>
<td>482.8 (B3)</td>
<td>11.75</td>
<td>-0.8756</td>
<td>1.752</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3 Parameters of the shaft and SFD

<table>
<thead>
<tr>
<th></th>
<th>Oil viscosity (N m²/s)</th>
<th>Clearance of SFD (m)</th>
<th>Eccentricity of disk (m)</th>
<th>Density (Kg/m³)</th>
<th>Young modulus (N/m²)</th>
<th>Frictional coefficient of rub (µ)</th>
<th>Contact stiffness of stator (N/m)</th>
<th>Stiffness of squirrel cage support (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.66e-3</td>
<td>10e-5</td>
<td>5e-4 &amp; 1e-4</td>
<td>7900</td>
<td>2.0x10^11</td>
<td>0.02</td>
<td>1.4e9</td>
<td>3.503e7</td>
</tr>
</tbody>
</table>
3.1. Assumption and convergence testing

This study assumes that the oil film in squeeze film damper is incompressible and regardless of its weight. The inertia of the flow is neglected, and the coefficient of friction and radial pressure are constant. The thickness of the oil film is less than the radius of the bearing, which means that the oil film is very thin. Thus, the pressure does not vary across the thickness of the oil film, and any curvature of the oil film can be neglected. There is no slippery surface between the oil film and the outer surface of the squirrel cage supported structure. This means that the squirrel cage supported structure is non-rotary. The dimensions of $V(t)$ and $W(t)$ are meters. The dimensions of $dV(t)/dt$ and $dW(t)/dt$ are m/s. And $V(T)$ and $W(T)$ are non-dimensional units. The non-dimensional speed ratio of $S$ represents $\Omega/\omega_1$.

In order to ensure the accuracy of the global assumed mode method (GAMM) in this study, a comparison of first third of natural frequencies are obtained by global assumed mode method within number of polynomial $N_p=8$ to $N_p=12$. Table 4 shows the first third natural frequencies for GAMM. The results indicate the relative error decreases as the $N_p$ increases. The percentage differences defined by $|\omega_{n,S} - \omega_{n,S}^*/\omega_{n,S}^*|$ are smaller than 5% for first third natural frequencies when $N_p$ equals to 11. Consequently, the number of polynomial $N_p$ is 11 in the following study.

Table 4 Convergence tests w.r.t $N_p$ for the values of a whole system

<table>
<thead>
<tr>
<th>Number of polynomial $(N_p)$</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>1554.9</td>
<td>1464.8</td>
<td>1420.7</td>
<td>1318.5</td>
<td>1317.3</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>2605.0</td>
<td>2583.6</td>
<td>2540.5</td>
<td>2526.6</td>
<td>2511.2</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>9107.5</td>
<td>8994.7</td>
<td>8962.4</td>
<td>8759.2</td>
<td>8657.5</td>
</tr>
</tbody>
</table>

3.2. Numerical results

In this study, a squeeze film damper is located at station 13 near the flywheel, and the nonlinear dynamic behavior is investigated when the rotor speed ratio $S$ increases from 0 to 1.215. Figures 3(a) and 3(b) show a period-one motion, which indicates a synchronous response when the rotor speed ratio is small. As the rotor speed ratio $S$ exceeds 0.15 when flywheel eccentricity is $1e-4$ m, the system undergoes an irregular period bifurcation without any transition and results in an aperiodic motion as shown in the bifurcation diagrams. This irregular period motion occurs in the range of rotor speeds ratio $S$ between 0.15 and 1.215. This region, which is called the sensitive interval of the rotor speed, can exhibit nonlinear dynamic responses such as the quasi-periodic motion or the aperiodic motion. Figure 3(b) shows that the system undergoes quasi-periodic motion and then transforms into aperiodic motion as rotor speed ratio $S$ exceeds 0.039 when flywheel eccentricity $e$ is $5e-4$ m. The severity of rub-impact phenomena becomes more significantly from $S=0.93$ to 1.065 at an eccentricity $e$ of $1e-4$ m. The rubbing region is in the range of speed ratio $S$ between 0.5 and 1.1 at an eccentricity $e$ of $5e-4$ m. Therefore, speed ratio $S$ will be smaller as flywheel eccentricity increases when rub-impact phenomena occurs.
The time waveform, frequency spectrum, phase plane plot, and Poincaré map are shown in figures 4(a) and 4(b) to illustrate the nonlinear dynamic behavior of a squeeze film damper station when rotor speeds ratio S is in the range between 0.9519 and 0.5725. The associated motions represents the quasi-periodic motion, aperiodic motion, and rub-impact phenomena. As shown in figure 4(a), the dynamic behavior is unlike the usual ways into chaos. This figure shows the quasi-periodic motion without any rub-impact effect. Figure 4(b) illustrates the trajectory of the squeeze film damper station is irregular with rub-impact effect. Consequently, response with rub-impact exhibits strongly non-periodic motion as compared to those without rub-impact effect.

![Fig. 3 Bifurcation diagrams in the V and W direction of SFD station with the eccentricity of (a) e=1e-4 m and (b) e=5e-4 m.](image)

![Fig. 4 The nonlinear behaviour of the SFD station with eccentricity of (a) e=1e-4 m at S=0.9519 and (b) e=5e-4 m at S=0.5725.](image)

4. Conclusions

This study presents the nonlinear dynamics of a turbine generator under the consideration of flywheel eccentricity. The proposed turbine generator is equipped with a squeeze film damper under the effects of rub-impact in the oil film rupture. Results show that the system displays period-one motion when the rotor speed ratio is small. However, the periodic motion suddenly transforms into aperiodic motion without any transition. The squeeze film damper will fail to support the rotor in a certain range of rotor speed. And this range will be enlarged as the flywheel eccentricity increases.

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Appendix A. The matrix form of equation of motion

\[ M_{x}(m,n) = \int_{0}^{1} A \rho x^{n-2} dx + \sum_{i=1}^{N} m_{i}^{e}(x_{i}^{e})^{m_{i}^{e}} + m_{i}^{b}(x_{i}^{b})^{m_{i}^{b}} \]

\[ M_{x}(m,n) = (m-1)(n-1) \left( \int_{0}^{1} I_{x} x^{n-4} dx + \sum_{i=1}^{N} I_{i}^{x}(x_{i}^{x})^{m_{i}^{x}} \right) + (m-1)(n-1) I_{0}^{x}(x_{0}^{x})^{m_{0}^{x}} \]

\[ G(m,n) = (m-1)(n-1) \left( \int_{0}^{1} J_{x} x^{n-4} dx + \sum_{i=1}^{N} J_{i}^{x}(x_{i}^{x})^{m_{i}^{x}} \right) + (m-1)(n-1) J_{0}^{x}(x_{0}^{x})^{m_{0}^{x}} \]

\[ K_{s}(m,n) = (m-1)(n-1)(n-2) \int_{0}^{1} E x^{n-4} dx \]

\[ K_{s}(m,n) = \sum_{i=1}^{N} k_{i}^{x}(x_{i}^{x})^{m_{i}^{x}} + k_{0}^{x}(x_{0}^{x})^{m_{0}^{x}} \]

\[ K_{s}(m,n) = \sum_{i=1}^{N} k_{i}^{b}(x_{i}^{b})^{m_{i}^{b}} \]

\[ R_{s} = \int_{0}^{1} e_{s}(x) \rho(x) \sigma(x) \Omega^{2} \sin(\Omega t + \theta) x^{N-1} dx + \sum_{i=1}^{N} m_{i}^{e}(e_{i}^{e}) \Omega^{2} \sin(\Omega t + \theta)(x_{i}^{x})^{N-1} + \sum_{i=1}^{N} F_{i}^{e}(x_{i}^{x})^{N-1} + \sum_{i=1}^{N} F_{i}^{b}(x_{i}^{x})^{N-1} \]

\[ R_{b} = \int_{0}^{1} e_{b}(x) \rho(x) \sigma(x) \Omega^{2} \sin(\Omega t + \theta) x^{N-1} dx + \sum_{i=1}^{N} m_{i}^{b}(e_{i}^{b}) \Omega^{2} \sin(\Omega t + \theta)(x_{i}^{x})^{N-1} + \sum_{i=1}^{N} F_{i}^{e}(x_{i}^{x})^{N-1} + \sum_{i=1}^{N} F_{i}^{b}(x_{i}^{x})^{N-1} \]

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