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Unsupervised construction of fuzzy measures through self-organizing feature maps and its application in color image segmentation ☆

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Abstract

The paper presents a framework for the segmentation of multi-dimensional images, e.g., color, satellite, multi-sensory images, based on the employment of the fuzzy integral, which undertakes the classification of the input features. The framework makes use of a self-organizing feature map, whereby the coefficients of the fuzzy measure are determined. This process is unsupervised and therefore constitutes one of the main contributions of the paper.

The performance of the framework is shown by successfully realizing the segmentation of color images in two different applications. First, the features of the framework and its parameterization are analyzed by segmenting different images used as benchmark in image processing. Finally, the framework is applied in the segmentation of different images taken under difficult illumination conditions. The images serve the development of an automated cashier system, where the weak segmentation constitutes the first step for the identification of different market items. The presented framework succeeds in the segmentation of all these color images.

Keywords: Fuzzy integral; Fuzzy measures; Self-organizing feature map; Hybrid system; Color image segmentation; Multi-dimensional image processing; Image processing; Multi-sensory fusion

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1. Introduction

The fuzzy integral [1] generalizes most of fuzzy fusion operators presented hitherto [2,3]. This operator has been successfully applied in numerous application fields, e.g., computer vision [3–5]. This fact succeeds in spite of the lack of methodologies for the automated parameterization of the fuzzy integral through the fuzzy measures [6], which is one of the shortcomings of its application. The paper presented herein tackles this problem by presenting a new methodology for the construction of the fuzzy measures. Furthermore, the methodology is applied in the implementation of a framework for color image segmentation.

Image segmentation, whose goal is the partition of the image domain by taking perceptual homogeneity into consideration, is a long standing research subject in computer vision [7–11]. The resulting image subdomains, which can be denoted as image segments, satisfy some condition of homogeneity with respect to a particular perceptual feature, e.g., present the same color or some kind of texture. Image segmentation plays a principal role in the realization of computer vision applications, as a previous stage for the recognition of different image elements or objects. In this context some authors [12] differentiate between weak and strong segmentation, where this last one attains the object-driven segmentation. Although multiple methodologies for the segmentation of images in the gray value domain have been developed [7,8,11], it is still an open research question how to segment images of larger dimensionality, e.g., color, satellite, multi-sensory images. In this context it is worth mentioning that in spite of the numerous approaches for color segmentation presented heretofore [8–10], this question is challenged by the variability of the artificial reproduction of color when the illumination conditions change.

This paper presents a procedure for the fuzzy segmentation of multi-dimensional images. A defuzzification stage can be added to the basic methodology. Nevertheless, the presented procedure can be exploited at its best when delivering a fuzzy result. Hence, the loss of image information in a vision system should occur in the last stages of the system by taking the *principle of least commitment* [13] into account. Being the segmentation of images a low-level processing stage, a fuzzy segmentation result presents more information and thus can be better exploited by the embedding system than its crisp counterpart. The segmentation of multi-dimensional images is achieved in the here presented framework through a classification stage based on the fuzzy integral [1], whereby the previously extracted features are fused and transformed into membership degrees of different segment classes. Moreover the utilization of the fuzzy integral for color image processing can improve the robustness of the resulting systems with respect to the illumination [3].

The fuzzy integral is operated with respect to so-called fuzzy measures [1], which are used for weighting the data. The fuzzy integral has been successfully used in problems of image segmentation [3,14–16]. In most of these frameworks however, the fuzzy measures are heuristically defined [3,14,16]. The remaining framework [15] makes use of a supervised strategy based on random search with simulated annealing. Moreover, the determination of the fuzzy measure coefficients is attained in other application fields through supervised procedures as well, i.e., numerical optimization [17–20], probability analysis [21], neural networks [22,23], and genetic algorithms [6,24,25]. In contrast to these methodologies, the construction of the fuzzy measures is achieved herein by an unsupervised procedure which was introduced in [26].

Since the automated determination of the fuzzy measure coefficients still constitutes a requisite in order for the fuzzy integral to be extensively employed in real applications [6], the presented methodology supposes an advance in the application of the fuzzy integral not only restricted to the field of computer vision. Thus, just another unsupervised approach for the construction of fuzzy measures has been presented [27]. This approach was applied in the performance evaluation of students by aggregating subject qualifications. It is based on the application of entropy functionals for the determination of the attribute importance.

The presented framework combines the fuzzy integral with a projection on a two-dimensional grid, which is implemented through a self-organizing feature map (SOFM) [28]. This last stage serves the automated construction of the fuzzy measures. Furthermore, the coefficients of the fuzzy measure are assessed by clustering the achieved projection. This hybrid system differs from a similar one, where a fuzzy integral and a SOFM are applied for handwriting recognition [23], on its unsupervised nature. The framework, which has been already applied in the classification of benchmark data sets [26], is extensively analyzed herein.

The paper is organized as follows. First the theoretical background of the different methodologies used in the framework for multi-dimensional image segmentation are presented in Section 2. Thence, Section 3 details the advances on the construction of fuzzy measures achieved herein. The framework for image segmentation is described in Section 4. Finally, the results of the framework's application (Section 5) and some conclusions (Section 6) are given.

2. Theoretical background and related works

The fuzzy integral employs the fuzzy measures in order to establish the a priori importance of the information sources that are fused. There is a lack of procedures attaining the automated construction of fuzzy measures [6]. The here presented approach employs a SOFM, which has been already used in image processing for analyzing feature saliency [29,30], in order to determine the aforementioned importance. Thus the projection of the input data on an output map by applying a SOFM is first undertaken in the methodology presented herein. The projection result is thence partitioned through the application of mathematical morphology. The theoretical background on the mentioned subjects, namely the fuzzy integral, SOFMs, and mathematical morphology are detailed in the following sections.

2.1. Information fusion through the fuzzy integral and the construction of fuzzy measures

There are several types of fuzzy integral [2]. Among them the so-called Choquet Fuzzy Integral showed hitherto the best performance in the resolution of classification problems [26]. This integral presents the following mathematical expression:

$$\mathscr{C}_\mu(\mathbf{x}) = \sum_{i=1}^{n} \left[h_{(i)}(x_i) - h_{(i+1)}(x_{i+1})\right] \cdot \mu(A_{(i)}). \tag{1}$$

The notation used in this expression is explained in the following paragraph. The fuzzy integral aggregates fuzzy information. Thus the data from the different information sources, $\mathbf{x} = x_1, \ldots, x_n$, are fuzzified, where h_i represent the fuzzifying functions. The fuzzified data are operated against the quantification of the a priori importance on the information sources. This quantification is realized through the fuzzy measure coefficients, $\mu(A_i)$, where A_i, $i = 0, \ldots, 2^{n-1}$, stands for the different subsets that can be established over the set of information sources. The parentheses in the subindices indicate a sorting operation previous to the aggregation, up to which the fuzzy measure coefficients are selected. The interested reader is referred to [2] for a detailed description on this fuzzy fusion operator and to [4] for a description of different aspects of its application in computer vision.

The fuzzy measures $\boldsymbol{\mu}$ are functions on fuzzy sets, $\boldsymbol{\mu} : \mathscr{P}(X) \to [0, 1]$, which satisfy the following axiomatic conditions in case of a finite number of information sources:

I. Limits condition: $\mu(\emptyset) = 0$; $\mu(X) = 1$.
II. Monotonicity condition: $A \subseteq B \to \mu(A) \leqslant \mu(B)$.

Being n the number of information sources to be aggregated, a fuzzy measure presents 2^n coefficients. Because of the limits condition just $2^n - 2$ coefficients have to be determined.

The n coefficients of the individual sources are denoted as fuzzy densities. There are different types of fuzzy measures, whose differentiating characteristic is the kind of relationship between the fuzzy densities and the coefficients of the other subsets. Therefore, the construction of the fuzzy measure reduces to the determination of the n fuzzy densities when using a particular type of fuzzy measure. In fuzzy λ-measures [2] the fuzzy densities are related to the remaining coefficients through a parameter λ, as stated by

$$\mu(\{x_i\} \cup \{x_j\}) = \mu_{ij} = \mu_i + \mu_j + \lambda\mu_i\mu_j. \tag{2}$$

The standard procedure for finding λ up to the values of the fuzzy densities can be found in [4]. Moreover, possibility fuzzy measures fulfill:

$$\mu(\{x_i\} \cup \{x_j\}) = \mu_{ij} = \vee(\mu_i, \mu_j), \tag{3}$$

where $\vee$ states for the maximum operator.

When using the fuzzy integral as classification function M fuzzy measures have to be constructed, where M is the number of classes. Moreover, M fuzzy integrals are computed with respect to the M different fuzzy measures, what results in a possibilistic classification [31] of the incoming data.

Among all strategies for the construction of fuzzy measures [6], data-driven ones offer the best alternative because they are not based on a previously defined measure. Genetic algorithms are in this context the most often used methodology in real applications [6,25,24]. However, the utilization of such supervised strategies requires the collection of a labeled data set, which is employed in the training phase. The supervised operation supposes a shortcoming in image processing applications, where the generation of the training set is not trivial, e.g., for image segmentation. In the here presented framework these measures are found without supervision by applying a SOFM on the input data.

2.2. Self-organizing feature maps and associated methodologies

SOFMs are neural networks used as tools for data visualization and knowledge engineering with a well-known good performance in high-dimensional feature spaces [32].

The SOFM paradigm seeks the projection of the incoming data set into a discrete grid of nodes. Two auxiliary matrices, which help in the visualization of the generated clusters, can be defined on the output map [33]. These matrices are denoted as the *U-matrix* and *hit-matrix*. If the projection grid is two-dimensional, the auxiliary matrices can be represented in the image space, i.e., as gray value images.

A SOFM is used in the here presented segmentation approach in order to avoid the clustering of high-dimensional feature spaces by taking into consideration the data projection achieved by the application of this neural paradigm. For this purpose, the SOFM is first trained with a subset of the input data. The neurons of the output map, which represent the vector prototypes denoted as $\mathbf{w}_{ij}$, are randomly initialized and iteratively adapted in the training phase. This is achieved by applying:

$$\mathbf{w}_{ij}(t+1) = \mathbf{w}_{ij}(t) + \alpha * h_c(t) * [\mathbf{x}(t) - \mathbf{w}_{ij}(t)],$$

where α is the learning rate, and $h_c(t)$ the so-called neighborhood function. More details on the SOFM features can be found in [28]. The projection that is achieved by completing the training phase is used for the computation of the *U-matrix* and *hit-matrix* [33].

The *U-matrix* represents the distances between the prototypes of the output map in a gray value image space. Once the SOFM has been trained the *U-matrix* is constructed as follows. For a map of dimensions $P \times Q$, the *U-matrix* is an image with $(2P-1) \times (2Q-1)$ pixels, whose grayvalues are denoted as u_{ij}. Three different types of pixels can be distinguished:

- Pixels whose position correspond to the prototype position, thus presenting odd subindices. The values of these pixels are computed as the average of their four-neighborhood:

$$u_{ij} = \frac{u_{(i-1)j} + u_{(i+1)j} + u_{i(j-1)} + u_{i(j+1)}}{4}, \tag{4}$$

  where $i = 1, 3, \ldots, 2P-1$, and $j = 1, 3, \ldots, 2Q-1$.
- Pixels between two prototypes, thus only one of the subindices is even. The values of these pixels are computed as the distance between the value in the output map of the adjacent prototypes $\mathbf{w}_{ij}$:

$$u_{ij} = \begin{cases} \|\mathbf{w}_{(i-1)j} - \mathbf{w}_{ij}\| : & \forall i = 2, 4, \ldots, 2P, \ \ j = 1, 3, \ldots, 2Q-1, \\ \|\mathbf{w}_{i(j-1)} - \mathbf{w}_{ij}\| : & \forall i = 1, 3, \ldots, 2P-1, \ \ j = 2, 4, \ldots, 2Q. \end{cases} \tag{5}$$

  Here any distance function can be used, but the Euclidean distance is customary taken. The value of these *distance* pixels are first computed.
- Pixels whose four-neighborhood is exclusively formed by the formerly mentioned *distance* pixels, thus with both subindices being even. The values of these pixels are computed again as the average of their four-neighborhood:

$$u_{ij} = \frac{u_{(i-1)j} + u_{(i+1)j} + u_{i(j-1)} + u_{i(j+1)}}{4}, \tag{6}$$

  where $i = 2, 4, \ldots, 2P$, and $j = 2, 4, \ldots, 2Q$.

The *U-matrix* is customary used in order to get a first idea of the cluster distribution [34]. Clusters are characterized in this image through an homogeneous area of low gray-values separated by edge-wise elongated areas of large grayvalues (see Fig. 1c).

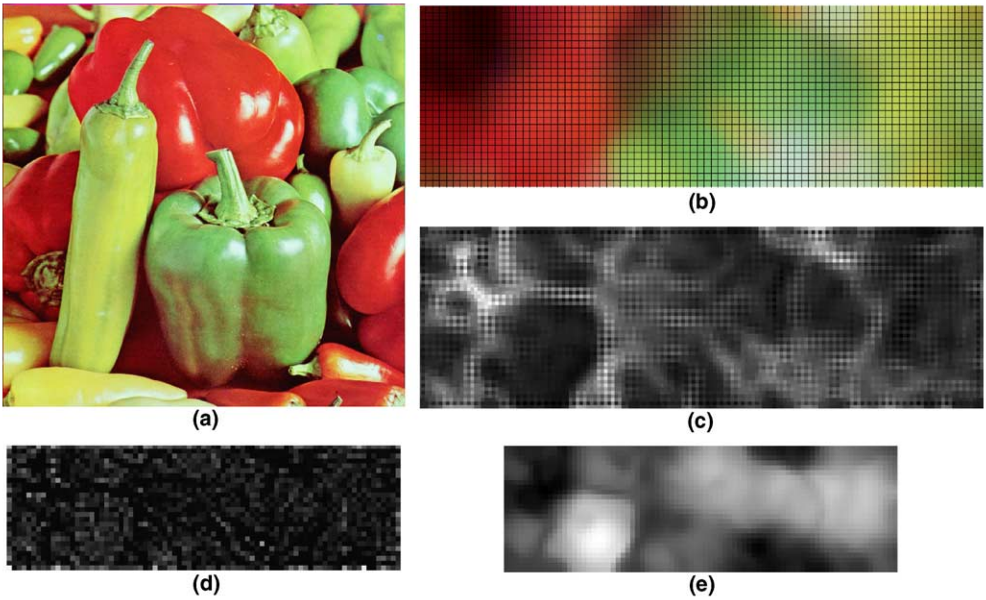

Fig. 1. Operations based on the SOFM neural paradigm on the "peppers" image. (a) Input image. (b) Projection on the output map of the SOFM network. (c) *U-matrix* of the SOFM projection. (d) *Hit-matrix* of the SOFM projection. (e) *Fuzzy hit-matrix* of the SOFM projection.

The *hit-matrix* is a two-dimensional histogram of $P \times Q$ positions, which correspond to each of the prototypes in the output map $\mathbf{w}_{ij}$. Once the output map has been trained, the data set is applied once again in order to obtain the winning prototype of each data point. This information is accumulated in the *hit-matrix* and represented in image form, where the gray value is proportional to the number of times a prototype wins (see Fig. 1d) [33]. Such prototypes with a larger number of hits are the most-frequent winning values, and thus can be considered as the most representative ones.

There is also a variant of the hit histogram [32], which receives the name of *fuzzy hit-matrix*. It presents a value FH_i for each point i in the two-dimensional output grid computable through the expression:

$$FH_i = \sum_{j=1}^{p} \frac{1}{1 + (\|\mathbf{x}_j - \mathbf{w}_i\| / \overline{Q})^2}, \tag{7}$$

where p is the number of points in the data set and $\overline{Q}$ is the average of the quantization error achieved in the data reduction (see Fig. 1e). Since the output map is clustered by applying a morphological procedure, an overview on mathematical morphology is given in the following section.

2.3. Mathematical morphology

Mathematical morphology is a theoretical framework used in image processing, whereby local operations seek the global transformation of the image being treated [35]. Thus the basic morphological operations dilation $\delta^{(1)}(f)$ and erosion $\epsilon^{(1)}(f)$ are applied on the

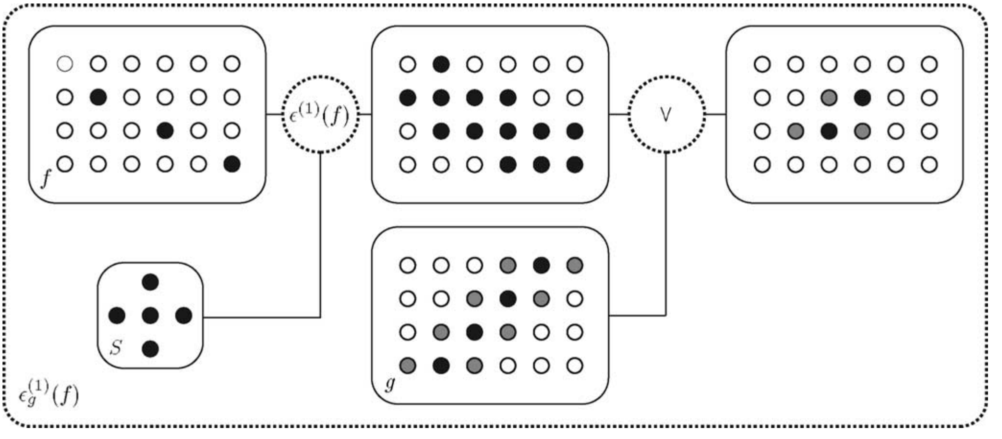

Fig. 2. Pictorial description of the geodesic erosion $\epsilon_g^{(1)}(f)$ with *marker set f* and *geodesic mask g*. *S*: structuring element, $\epsilon^{(1)}(f)$: erosion, $\vee$: maximum.

grayvalue input image f through a particular structuring element, which is the element determining the geometry of the local operation. Furthermore, the so-called geodesic dilation $\delta_g^{(1)}(f)$ and geodesic erosion $\epsilon_g^{(1)}(f)$ [35] take into consideration a third element g, which receives the name of *geodesic mask*. In these geodesic operators f is denoted as *marker set*. The *geodesic mask* limits the result of the basic morphological operation on the *marker set* to a particular image subdomain. The two geodesic operations are formally defined as

$$\delta_g^{(1)}(f) = \delta^{(1)}(f) \wedge g, \qquad \epsilon_g^{(1)}(f) = \epsilon^{(1)}(f) \vee g. \tag{8}$$

An exemplary pictorial description of the application of a geodesic erosion operator can be observed in Fig. 2.

The so-called morphological reconstruction results from the iterative repetition of a geodesic transformation until stability is achieved. Thus the reconstruction by dilation $R_g(f)$ and the reconstruction by erosion $R_g^\star(f)$ can be expressed as

$$\begin{aligned} R_g(f) &= \delta_g^{(i)}(f), \quad \forall i / \delta_g^{(i)}(f) = \delta_g^{(i+1)}(f), \\ R_g^\star(f) &= \epsilon_g^{(i)}(f), \quad \forall i / \epsilon_g^{(i)}(f) = \epsilon^{(i+1)}(f). \end{aligned} \tag{9}$$

It is worth mentioning that the morphological procedure used in the here presented framework is based on the application of the morphological reconstruction.

3. Advances in the construction of fuzzy measures

In this section the two novel contributions concerning fuzzy measures presented herein are elucidated. The first one takes into consideration the utilization of a self-organizing feature map for the construction of the fuzzy measures, which are needed for the classification stage undertaken by the fuzzy integral. Therefore the here presented approach is first compared with the hybrid system for handwriting recognition presented in [23], which combines a fuzzy integral with this neural paradigm as well. Thence the procedure for the automated determination of the fuzzy densities based on the projection achieved by the SOFM is detailed. At the end of the section the procedure for the construction of the fuzzy measures up to the obtained fuzzy densities is described. This part is related to the used types of fuzzy measures, an aspect where some novelties are presented as well.

3.1. Comparison with other systems combining the fuzzy integral and self-organizing feature maps

SOFMs have been already used in order to construct fuzzy measures [23]. This hybrid system have been used for handwritten word recognition. The map's prototypes are clustered with a supervised procedure in the training phase. In the recall phase, the prototypes within a cluster are fused through a fuzzy integral in order to compute the fuzzy membership functions of the input image pattern to a particular character class. The computation of the fuzzy measures is based on the relative frequency of activation of the prototypes when a pattern corresponding to each particular character class is applied on the resulting map.

In contrast to the formerly described system, the construction of fuzzy measures in the *morphological clustering* of the SOFM, which is described in Section 3.2, results from an unsupervised procedure. Thus it just presents two parameters, namely γ and M, which characterize the achieved segmentation. Moreover, the computation of the coefficients is related to the winning frequency of the prototypes in both frameworks. In contrast to the hybrid system for handwritten recognition, the *morphological clustering* makes use of this concept through the utilization of the *fuzzy hit-matrix*.

3.2. Extraction of fuzzy densities from a SOFM output map through morphological clustering

A methodology based on mathematical morphology that takes into consideration the *fuzzy hit-matrix* and the *U-matrix* clusters the output map of a SOFM. The representation of these matrices in the image domain enables the application of mathematical morphology. The methodology is based on morphological reconstruction operators. The *morphological clustering*, whose algorithmical description is given in Algorithm 1, is applied on the *U-matrix* in order to obtain M different vectors of n components. The components of these vectors are used as the fuzzy densities of the fuzzy measures, whereby the fuzzy integral undertakes the classification. The goal of the algorithm is therefore the selection of the most outstanding prototype of M clusters in the *U-matrix*, where just one prototype in each cluster of the *U-matrix* is selected.

In the *morphological clustering* (see Algorithm 1) the *U-matrix* is used as *geodesic mask* g after undergoing a binarization through a threshold γ (line 1). The binarization of the *U-matrix* defines a set of the possible areas (PA), from which a prototype can be selected (line 1). After the initialization (lines 2 and 3), the clustering proceeds as described in the following paragraphs. The *fuzzy hit-matrix* is first binarized through a threshold corresponding to its maximal value (line 5). In this way, a set of candidate prototypes (CP) is defined (line 6). Thence a reconstruction by dilation is computed using each element (line 7) of the candidate set as the *marker set* and the set of possible areas as *geodesic mask* as described in Section 2.3 (line 8). The image resulting from the reconstruction, which defines a set of selected areas (SA), is used two-fold. On the one hand, the candidate prototypes falling within the selected areas SA are added to the set of selected prototypes SP (line 9). On the other hand, the set of possible areas PA is actualized by eliminating those areas with already selected prototypes (line 10). Hence, the selection of just one prototype per cluster is assured. The detailed operation is sequentially repeated by decreasing the threshold of the *fuzzy hit-matrix* (line 5) until M prototypes, which are embedded in the set SP, have been selected (line 4).

As a result of the application of the *morphological clustering*, M prototype vectors of the output map, which present therefore n components, are obtained. These prototypes can be formally denoted as

$$\boldsymbol{\rho}^k = \{\rho_1^k, \ldots, \rho_n^k\}, \quad \forall k \in [1, M].$$

Algorithm 1. Morphological clustering of the *U-matrix* (U) based on reconstruction by erosion ($R_g^{\star}(f)$). PA: Set of possible areas. γ: Threshold of the *U-matrix*, which determines the cluster structure. SP: Set of selected prototypes. M: Number of prototypes to be selected. FH: *Fuzzy hit-matrix*. CP: Set of candidate prototypes. sp: element of selected prototypes.

$PA \leftarrow \{U > \gamma\}$
$\theta \leftarrow 256$
$SP \leftarrow \{\emptyset\}$
while $|SP| < M$ **do**
5: $\quad \theta \leftarrow \theta - 1$
$\quad CP \leftarrow \{FH = \theta\} \cap PA$
$\quad$ **for** $sp \in CP$ **do**
$\quad\quad SA \leftarrow R_{PA}^{\star}(sp)$
$\quad\quad SP \leftarrow SP \cup \{sp\}$
10: $\quad\quad PA \leftarrow PA - SA$
$\quad$ **end for**
end while

3.3. Construction of the fuzzy measures and some related novelties

Once the winning prototypes for each segment class $\boldsymbol{\rho}^k$ have been determined, the fuzzy measures have to be constructed. For this purpose the expression (2) is applied, where the fuzzy densities μ_i^k take the value of the corresponding component of the selected vector prototypes:

$$\mu_i^k = \rho_i^k, \quad \forall k \in [1, M], \ \ i \in [1, n]. \tag{10}$$

Thus the M fuzzy λ-measures $\boldsymbol{\mu}^k$ are constructed and thence used in the classification. Nevertheless, the obtained fuzzy densities can previously undergo a normalization. The normalization of the fuzzy densities has been used in [36] in order for the fuzzy measure coefficients of the coalitions to gain on importance, therefore increasing the discrimination capability of the fuzzy integral. This normalization succeeds by applying a factor T to the fuzzy densities as expressed by

$$\mu_i^k = \frac{\rho_i^k}{T}, \quad \forall k \in [1, M], \; i \in [1, n]. \tag{11}$$

As it can be observed this normalization factor is the same for all classes.

As a novelty, another strategy can be used in order to improve the discrimination capability of the fuzzy integral in the classification. Hence, a new type of fuzzy measures can be defined. A so-called mix type of fuzzy measure, which constitute an alternative to the construction of fuzzy-λ measures herein, is tested in the here presented framework as well. A *mix fuzzy-λ-possibility measure* fulfill the expression of a fuzzy λ-measure just in the canonical region of the prototype vector $\boldsymbol{\rho}^k$. In the remaining regions of the feature hyper-cube, this mix fuzzy measure behaves as a possibility measure, thus fulfilling the expression (3). The fuzzy measures constructed in this fashion increase the result of fuzzy integrals for these input vectors that present the same canonical region as the selected prototypes.

4. Application of a hybrid system for multi-dimensional image segmentation

The block diagram of the framework for the segmentation of multi-dimensional images is depicted in Fig. 3. The utilization of a defuzzification stage is optional. Hence, the framework delivers a fuzzy segmentation of the input image. This is an interesting property if the result has to be used in other stages of a more complex embedding system, e.g., a content-based image retrieval system. The defuzzification succeeds herein in order for the reader to get a better idea of the properties and the performance of the system. The description of the different modules of the framework succeeds in the following subsections.

4.1. Module SOFM

A SOFM is first trained with a subset of the pixels extracted from the image to be segmented. This subset of pixels is obtained by applying a subsampling strategy denoted as Linear Pixel Shuffling (LPS) [37]. Therefore the here presented framework is *extensible*

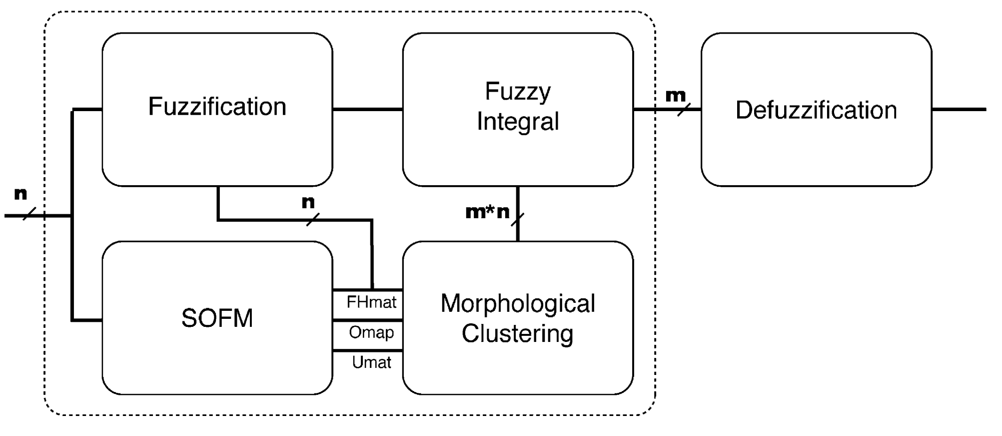

Fig. 3. Block diagram of the here presented framework for the segmentation of multi-dimensional images. Omap: Output map of the SOFM, Umat: *U-matrix*, FHmat: *Fuzzy hit-matrix*. The defuzzification stage can be optionally applied.

[38]. The extensibility of the algorithm allow it to be used with large data sets, i.e., large resolution images (see Section 5.4). However the sampling approach is undertaken herein in the spatial domain in contrast to other extensible methodologies [38], where the subsampling procedure is applied on the histogram.

Normally the SOFM is trained with training data set formed by feature vectors. Furthermore, one possible strategy for realizing this training in computer vision is the usage of the feature data set together with the corresponding coordinate of the features in the image domain. In case of using color features this procedure is denoted as working in a spatio-chromatic space [39].

As already mentioned the SOFM achieves a projection of the input data in a two-dimensional grid of prototypes that quantize it. Once the training phase has been completed, the *fuzzy hit-matrix* and *U-matrix* are computed (see Section 2.2). These three matrices are delivered to the module implementing the morphological clustering. Moreover, the *fuzzy hit-matrix* is used for the construction of fuzzy membership functions in the module *Fuzzification*.

4.2. Module morphological clustering

This module implements the algorithm described in Algorithm 1. Thus, the procedure fixes up the clusters on the *U-matrix* through the parameter γ. It selects thence the M larger local maxima of the *fuzzy hit-matrix* that fall within these clusters. Once these local maxima have been determined, it extracts the prototype vectors from the output map and delivers them to the module implementing the fuzzy integral.

4.3. Module fuzzification

In this module the *fuzzy hit-matrix* is transformed into one fuzzy membership function for each component of the input vector data. This operation is attained by taking the component planes of the output map and reorganizing them as a one-dimensional histogram. Hence, the abscise values are the values of the component planes, and the ordinate values, the value of the *fuzzy hit-matrix* for each neuron. Once being normalized, the resulting probability histogram is used as fuzzification function on the input data.

As an alternative, the one-dimensional histogram can be further transformed. On the one hand, the sum histogram is computed, what results in a monotonic increasing fuzzy membership function. On the other hand, Parzen windows [40] are applied on the one-dimensional histogram in order to smooth it.

4.4. Module fuzzy integral

In the application of the fuzzy integral for image segmentation an integral is computed for each pixel of the input image. This stage is realized in this module. The fuzzy integral is computed with respect to M fuzzy measures $\boldsymbol{\mu}^k$, whose fuzzy densities result from the application of the *morphological clustering* as stated by Eq. (10). The coefficients of the coalition subsets are computed by applying Eq. (2). Therefore a possibilistic classification [31] of the pixels on M segment classes (see Section 2.1) is obtained.

The two different strategies employed in order to improve the discrimination capability of the fuzzy integral, which are described in Section 3.3, are implemented in this module as well. First a factor T can be defined on each image in order to normalize the fuzzy densities as expressed by Eq. (11). This operation is undertaken previous to the construction of the fuzzy measures and therefore to the integration. Furthermore a *mix fuzzy-λ-possibility measure* (see Section 3.3) can be used. The application of these two alternatives is optional and determined by the user. Their effect on the segmentation results is analyzed in Section 5.1.4.

4.5. Module defuzzification

Following the possibilistic classification, the fuzzy integral results can be defuzzified. This is attained by finding out the argument, which delivers a maximum fuzzy integral result. The class of each pixel is finally assigned to this argument. As formerly mentioned, the application of this module is not compulsory.

5. Application results

The performance of the framework formerly presented is evaluated twofold. First the framework is applied for the segmentation of benchmark color images both for analyzing its different parameters and for comparing its performance with this of a well-known procedure [39]. Second, the framework is used in an application for market basket recognition. In this case, the framework attains the weak segmentation of the input images, which can lead to the identification of the depicted objects.

It is worth mentioning the kind of features employed in these applications. The segmentation is attained in a multi-dimensional feature space. The color features are the components of different color spaces simultaneously taken into account. Thus, the input images, which are represented in the *RGB* color space, are first transformed into different color spaces. The mathematical expressions of these transformations can be found in [41,42], where color spaces are reported to present invariance with respect to particular imaging conditions. Thus, each color space present different invariance properties, e.g., *HS* with respect to the illumination intensity. Therefore, the simultaneous employment of different color spaces is expected to improve the robustness of the here presented framework.

5.1. Benchmark images

Different aspects of the presented framework are analyzed on hand of benchmark images.[1] These results are described in the following sections. Since the result of a segmentation is subjectively analyzed at its best through a crisp image, the defuzzification stage has been applied herein.

5.1.1. Sampling and general methodology

The "peppers" image is first segmented with the here presented framework on a five-dimensional color feature space. Furthermore the color feature vectors are formed by the red, the green, and the blue components of the *RGB*-color space, and the hue, and the saturation of *HSI*-space. The SOFM is trained with 2% of the pixels in the input

[1] The images can be found in the USC-SIPI Image Database http://sipi.usc.edu/database/.

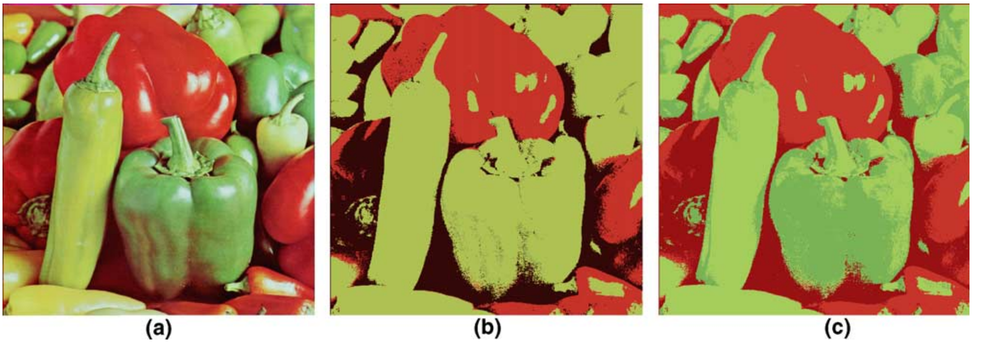

Fig. 4. Segmentation results of the "peppers" image (a) with the here presented framework. (b) Segmentation on a five-dimensional color space $RGB + HS$. (c) Segmentation on a seven-dimensional spatio-chromatic feature space, i.e., $RGB + HS + xy$.

image. The obtained results can be observed in Fig. 4b. The employment of the *fuzzy hit-matrix* instead of the *hit-matrix* improved the classification results (compare Fig. 1b and e). The segmentation was achieved by applying $\gamma = 40$ and $M = 6$ in the morphological clustering. Moreover the framework is tested on a seven-dimensional spatio-chromatic feature space (see Section 4.1), where the formerly used color space ($RGB + HS$) is taken into consideration together with the pixel coordinates (x, y). As it can be seen in Fig. 4c adding the spatial coordinates improves the results in this image.

The framework is tested on the "baboon" image as well, where again 2% of the pixels are used for training. The obtained results are depicted in Fig. 5. In this case, the parameters applied were $\gamma = 20$ and $M = 6$. Again the segmentation succeeds on the five-dimensional color space (Fig. 5b). The position of the extracted prototypes $\boldsymbol{\rho}^k$ on the *U-matrix* can be observed in Fig. 5c. As shown, the *morphological clustering* manages to select one prototype for each color cluster. Furthermore, some of the extracted prototypes are filtered out by the defuzzification stage.

5.1.2. Performance with increasing dimensionality

This section serves the analysis of the framework's performance by taking an increment in the dimensionality of the feature space into account. Hence, the results obtained with

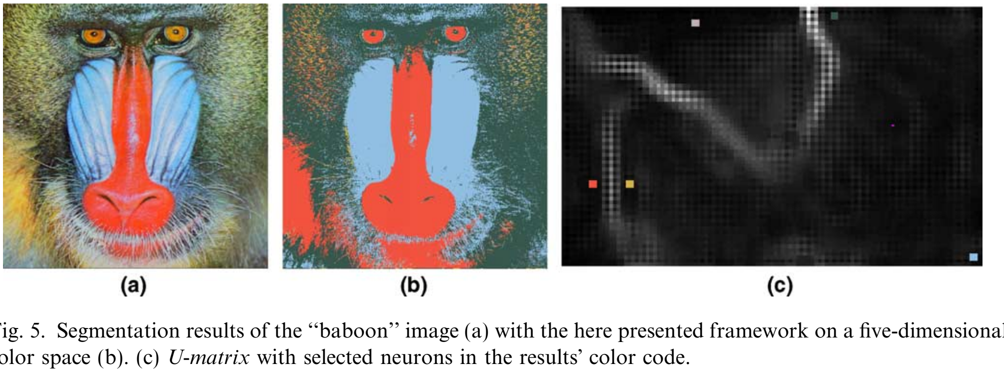

Fig. 5. Segmentation results of the "baboon" image (a) with the here presented framework on a five-dimensional color space (b). (c) *U-matrix* with selected neurons in the results' color code.

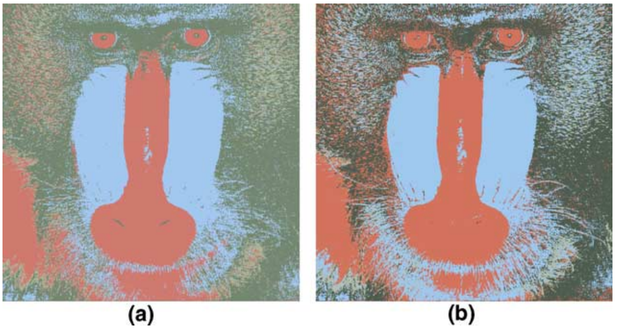

Fig. 6. Segmentation results of the "baboon" image (see Fig. 5a) with the here presented framework on a feature space of increasing dimensionality. See Fig. 5b for a comparison with the results on a five-dimensional color space $RGB + HS$. (a) Spatio-chromatic space of dimensionality 7, $RGB + HS + xy$. (b) *RGB* plus *HSI* plus *Gaussian* [42] color spaces, i.e., nine-dimensional color space.

the "baboon" image on spaces of dimensionality 7 and 9 are shown in Fig. 6, which can be compared with the result on a five-dimensional feature space (see Fig. 5b). As it can be observed, the framework performs analogously, showing a relative independence on the dimensionality. The question arises if the features selected are informative enough, but the quality of the feature selection is not analyzed herein. Furthermore the result on the nine-dimensional space can be improved as it is shown in Section 5.1.4.

5.1.3. Fuzzification strategies

The presented fuzzification strategies (see Section 4.3) are compared herein. The segmentation of the "peppers" image is therefore attained in a color feature space. As it can be observed in Fig. 7 the employment of monotonic increasing fuzzy membership functions deliver similar results as those obtained without fuzzification but by taking a spatio-chromatic feature space into account (see Fig. 4c). Thus, the sum histogram can be used for the implementation of the fuzzification stage, whereas the other two

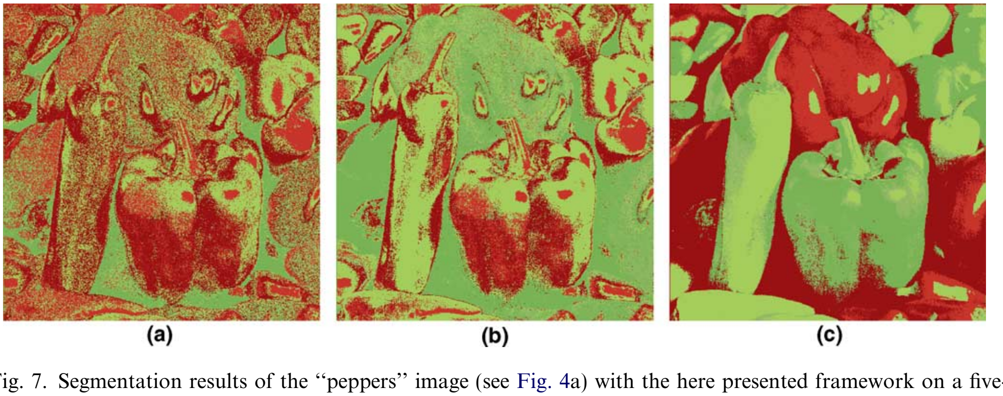

Fig. 7. Segmentation results of the "peppers" image (see Fig. 4a) with the here presented framework on a five-dimensional feature space ($RGB + HS$) for different fuzzification strategies. See Fig. 4b–c for a comparison with the results without fuzzification. (a) Probability histogram. (b) Probability histogram smoothed through Parzen windows [40]. (c) Sum histogram.

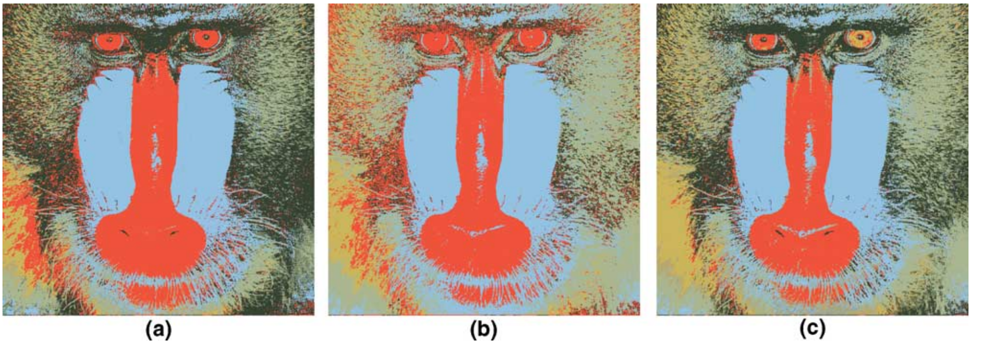

Fig. 8. Segmentation results of the "baboon" image (see Fig. 5a) with the here presented framework for different fuzzy measures. See Fig. 6b for a comparison with the results obtained by applying a fuzzy-λ measure on the same color space, namely *RGB* + *HSI* + *Gaussian*. (a) Fuzzy-λ measure with a normalization $T = 3$ on the fuzzy densities. (b) *Mix fuzzy-λ-possibility* measure. (c) *Mix fuzzy-λ-possibility* measure with a normalization $T = 3$ on the fuzzy densities.

strategies, namely the probability histograms and Parzen windows, do not perform well at all.

5.1.4. *Mix fuzzy-λ-possibility measures and normalization*

The two contributions related to the construction of the fuzzy measures are tested on the "baboon" image. The results are depicted in Fig. 8, where a segmentation in a nine-dimensional feature space is attained. As it can be observed, the significance of both strategies increases for spaces of such dimensionality.

5.2. *Comparison with the mean shift algorithm*

In the following section the framework presented herein is compared with the *mean shift* algorithm for image segmentation, which has been recently presented [39]. Both methodologies define the segmentation by taking a similar number of parameters into account, namely three. Having set up $M = 20$, the fuzzy integral framework leaves as free parameters: the utilization or not of a *mix fuzzy-λ-possibility* measure (β, boolean), the normalization factor of the fuzzy densities (T), and the threshold of the *U-matrix* (γ). It is worth mentioning that the results of the framework presented herein are obtained on some instance of the images[2] that present compression artifacts. Moreover, a multi-dimensional color space that is formed by the components of the *RGB*, the *Gaussian* [42], and the *c-space* [41] is used in the segmentation. Its results can be observed in Fig. 9.

It can be observed that the framework presented herein is pixel-based, what results in the presence of more details. Since the *mean shift* approach takes edge information and a post-processing stage into account, its results are more uniform. These facts sometimes generate some weird segment borders, e.g., the border of the two lake segments in the

[2] The input images and the *mean shift* results are available at http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html.

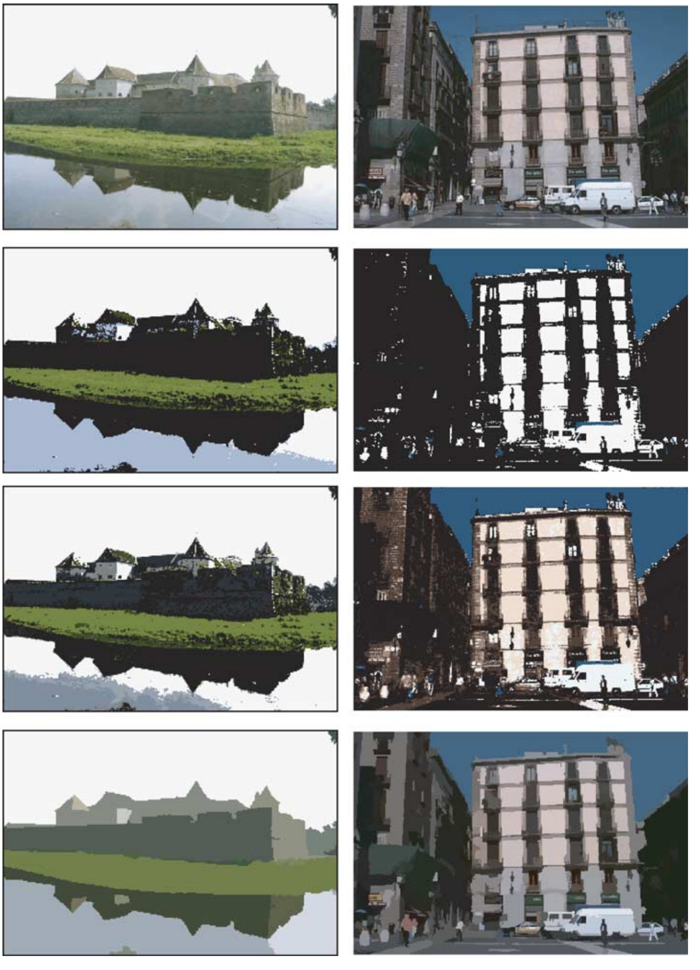

Fig. 9. Comparison of the framework presented herein with the *mean shift* [39] algorithm for image segmentation. The comparison is undertaken on the images *fagaras* (left) and *barcelona* (right), which are depicted at the top, after transforming them into a nine-dimensional color space, *RGB + Gaussian + C-space*. The segmentation of *fagaras* is undertaken (from top to the bottom) for ($\beta = 0$, $T = 8.0$, $\gamma = 45$), and ($\beta = 1$, $T = 9.5$, $\gamma = 50$). This of *barcelona*, for ($\beta = 1$, $T = 3.5$, $\gamma = 50$), and ($\beta = 1$, $T = 3.66$, $\gamma = 70$). The results of the *mean shift* plus post-processing are depicted at the bottom.

fagaras image. Furthermore, the segmentation based on the fuzzy integral uses less color codes. The convenience of this property is application dependent.

Both methodologies are driven by the free parameters, what can be exploited by the embedding systems as described in [39]. The intuitive worth of the parameters T and γ can be derived from the depicted results (see Fig. 9). Hence, T improves the discrimination capability and its value is image dependent. The value of γ controls the number of color codes used in the segmentation result. Furthermore, the framework described herein works somehow independent of the dimensionality of the input feature space. This fact is based on the utilization of the *U-matrix* projection in the clustering procedure. While "whenever the feature space has more than six dimensions, the analysis [with *mean shift*] should be approached carefully" [39], the fuzzy integral framework successfully operates in a nine-dimensional color space (see Fig. 9).

5.3. Segmentation for market basket recognition

The presented framework is used in a problem of market basket recognition, where different market items have to be recognized in order to implement an automated cahier system. In this context it is worth pointing out that the weak segmentation of images is attained for content-based image retrieval [12]. A weak segmentation attains the data-driven segmentation of image data in contrast to the object-driven one attained by a strong segmentation.

The weak segmentation of some beer can items is attained, which should lead to the resolution of a market basket recognition application. The images are taken in a scenario simulating uncontrolled conditions of illumination. Thus the items present some shadow areas and some highlights, which make more difficult the achievement of a good segmentation.

In order to cope with these illumination conditions the framework simultaneously represent the images in different color spaces as formerly detailed. The *Gaussian* [42], the *c-space*, and the *l-space* [41] are selected for attaining the segmentation. In this last space just, the second component is employed. Thus, the segmentation is realized on a seven-dimensional feature space. In this case, 5% of the image pixels are employed in order to train the SOFM.

The framework is applied by normalizing the extracted fuzzy densities and by constructing *mix fuzzy-λ-possibility* measures. The achieved results are depicted in Fig. 10.

5.4. Computational issues and framework properties

The framework presented herein is composed by three main computational modules. The first one undertakes the transformation among the different color spaces used, the subsampling of the resulting multi-dimensional color space and finally the training of the SOFM through this input data subset. The second one includes the morphological clustering of the SOFM map, whereby the fuzzy densities are extracted. The last module embeds the fuzzy integral operation for the M classes and the defuzzification of the result.

Although the assessment of the computational cost depends on software implementation, hardware, and other influencing factors, the reader can find in the following sentences some orienting CPU times determined on a computer with a Pentium M processor working at 1.3 GHz. The training of the SOFM takes approximately 3 s for a 20×20 map and 10,000 train iterations on a three-dimensional color space. This quantity depends on the mentioned parameters. The interested reader can use the SOM Toolbox [32] for a prototyping implementation of this module.[3] The clustering of the output map computes in the order of hundreds of milliseconds depending on the dimensions of the output map and the parameters of the procedure. Some software modules of mathematical morphology[4] can facilitate testing the *morphological clustering*. The fuzzy integration depends on the number of classes of the segmentation and the dimensions

[3] Free available at http://www.cis.hut.fi/projects/somtoolbox/.

[4] For example, free available at http://www.astro.rug.nl/~gipsy/pydoc/numarray.nd_image.morphology.html.

Fig. 10. Segmentation results of the here presented framework in an application for market basket recognition. It is worth mentioning the bad illumination conditions of the input images.

of the input image. A fuzzy integral of a three-dimensional image can be implemented in order to operate at 2 ms/pixel. It is worth mentioning that the utilization of look-up tables in the implementation of fuzzy measures can extremely speed up this operation.

The question arises if its feasible to use such a complex framework as the one described herein. In this context, it is worth pointing out some of the properties of the processing system. Hence, the training phase can be always computed off-line. This makes difficult its usage in real-time systems as video processing. However the *extensibility* of the framework can be exploited in this case. Moreover, some other application fields as content-based image retrieval can take advantage of the off-line training. In this context it is worth mentioning that the framework not only delivers the fuzzy segmentation of the image, which can be used to compute the similarity between two images, but it orders the colors present in the input image from more to less important. In the segmentation of color images the code of the different segments are delivered as the fuzzy densities, and thus form part of the system.

6. Conclusions

A framework based on the projection achieved by a SOFM and on the utilization of the fuzzy integral has been described. The fuzzy integral classifies each of the pixels in the input images and thus attains the segmentation of the image. The corresponding fuzzy measures are constructed through the application of a procedure for the morphological clustering of the *U-matrix*, which is computed in the two-dimensional data projection achieved by a SOFM. This approach constitutes a contribution to the field, since to the best of our knowledge just one other procedure has been presented hitherto for the unsupervised determination of the fuzzy measure coefficients.

The general methodology can be successfully applied for the segmentation of color images, as it has been shown on hand of benchmark images. The presented framework is characterized by two parameters (although the number of prototypes to be selected M can be coarsely determined). One of the main features of the here presented framework is that large dimensional feature spaces can be clustered somehow independently from the dimensionality of the feature space, i.e., just the response of the *U-matrix* to the high-dimensional space influences the result. The fuzzification stage is better attained through the application of monotone increasing membership functions. The normalization of the fuzzy densities, which adds a parameter to the system, and the utilization of the *mix fuzzy-λ-possibility* measures presented herein improve the results, especially in feature spaces of larger dimensionality.

Finally, the weak segmentation of color images for the resolution of a market basket recognition problem is realized. In this context, it is worth mentioning the robustness of the framework with respect to the illumination conditions. The obtained results demonstrate the capability of the here presented approach. Therefore, the segmentation of color images through texture analysis will be attained in the near future.

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