Trajectory optimization and applications using high performance solar sails

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Abstract The high performance solar sail can enable fast missions to the outer solar system and produce exotic non-Keplerian orbits. As there is no fuel consumption, mission trajectories for solar sail spacecraft are typically optimized with respect to flight time. Several investigations focused on interstellar probe missions have been made, including optimal methods and new objective functions. Two modes of interstellar mission trajectories, namely “direct flyby” and “angular momentum reversal trajectory”, are compared and discussed. As a foundation, a 3D non-dimensional dynamic model for an ideal plane solar sail is introduced as well as an optimal control framework. A newly found periodic double angular momentum reversal trajectory is presented, and some properties and potential applications of this kind of inverse orbits are illustrated. The method how to achieve the minimum periodic inverse orbit is also briefly elucidated. © 2011 The Chinese Society of Theoretical and Applied Mechanics. [doi:10.1063/2.1103301]

Keywords solar sail, non-Keplerian orbit, angular momentum reversal, time optimal control

I. INTRODUCTION

With the second booming of deep space exploration, the solar sail has been considered as one of the most potential propulsion system without fuel consumptions. Different from conventional propulsion systems for spacecraft, solar sails are continuously accelerated to escape the solar system or spiral inwards to the inner solar system. The fundamental idea of using solar radiation pressure for spacecraft propulsion might date back to the early 1920s, proposed by the Soviet pioneers of astronautics, Tsiolkovsky and Tsander. However, there was little technical advancement until a feasibility study of a comet Halley rendezvous mission proposed by NASA/JPL during 1970s. In real mission designs, solar sails have to be physically large to improve the payload size. The deployment of a large sail is still highly challenging to date. Coupled attitude-orbit control is also a problem as well as the flexible solar sail surface. Despite so many difficulties of solar sail technology, practical experiments on sail deployment and mission design have never ceased since 1990s. In 2010, Japan’s IKAROS spacecraft became the first vehicle successfully accelerated only by the solar radiation pressure. Such an effort undoubtedly enhanced the confidence on solar sail development.

In solar sail mission designs, an important parameter is the sail lightness number β. The value β = 1 corresponds to the sail acceleration 5.93 mm/s² at 1 Astronomical Unit (1 AU=1.496×10¹¹ m) away from the Sun. Since it has great advantages in the interplanetary missions, solar sailing has been investigated in various areas, such as attitude control, trajectory optimization and mission analysis. It is well known that Zhukov first studied the time-optimal Earth-Mars transfer by using a solar sail. Then with different sail accelerations, Jayaraman restudied the mission and obtained a result with totally different control history of sail orientation compared with Zhukov’s. Actually, the time-optimal result for such a rendezvous mission should not be different by preconception. Shortly after, Wood made a comment on Ref. 26 and pointed out the incorrect application of the transversality condition of variational calculus in reference. For such missions, the Geocentric escape phase, which can be completed by various ways, are always neglected by assuming C₂=0 km²/s². Generally, demonstrative applications in low performance solar sails were mostly concentrated on interplanetary transfers or planet rendezvous missions. However, high performance solar sails (in an acceleration greater than 3 mm/s²) can enable exotic highly non-Keplerian orbits, such as linear escape trajectories, displaced orbits and interstellar missions. As there is an assumption of cruise phase (after 5 AU away from the Sun) about the interstellar mission, it should be different from the rendezvous problem in the final constraints and objective functions. So the previous results should be considered as near time optimal. The truly time optimal escape trajectory to a prescribed distance is still an open question, waiting for further investigations.

As mentioned above, the angular momentum reversal trajectory (abbreviated as “reversal trajectory” throughout this paper) is an alternative for Interstellar missions compared to the direct flyby. It was first presented by Vulpetti in 1992 from numerical view point in fixed sail cone angle. As a result, one of its advantages is that the attitude control profile of solar sail is simple which will be beneficial to operation. The 2D (two-dimensional) and 3D reversal trajectories...
were also investigated in the following decades. A new type of periodic trajectory in fixed sail cone angle (referred to as “solar inverse orbit” throughout this paper) is found and then extended to a minimum periodic trajectory by Lab of Aerospace Dynamics at Tsinghua University (THU LAD). The trajectory is in the ecliptic plane by one side of the Sun and symmetrical with respect to the Sun-perihelion line. Mengali also obtained such trajectories (named as “H^2 RT”) with six assumptions made for 2D dynamic equations. However, one of the assumptions about the perihelion velocity given by Mengali is feasible but not appropriate. The problem will be discussed in detail as well as potential applications of the solar inverse orbit.

II. INTERSTELLAR PROBE TRAJECTORY OPTIMIZATION

Much of the recent interest in high performance solar sails is due to the consensus that a solar sail spacecraft can complete interstellar probe missions within 10–20 years. The concept of an interstellar mission is described by Jaffe, more than three decades ago. Compared to the velocity of 3 AU/a (1a=1year) for the Voyager spacecraft, a cruise speed of more than 10 AU/a for relatively mid or high performance solar sails is made attractive for scientists. The precursor Interstellar Probe missions were investigated in the direct flyby mode by Sauer within a single Solar Photonic Assist to 100 AU, 250 AU and 1 000 AU. Dachwald and Leipold completed solar system escape with multiple flybys for both ideal and non-ideal sails. Not surprisingly, the results of non-ideal sails are similar to the ideal ones but with a little increase of mission time. The maximum objective function to minimize the mission time adopted by previous studies is similar or equals to

\[ J = -\lambda_0 \int_0^{t_f} dt = -\lambda_0 t_f, \tag{1} \]

where \( t_f \) is the time at jettison point which is the ending time of the cruise phase, \( \lambda_0 \) is a positive constant. It is really difficult and challenging to calculate the trajectory to 100 AU in an optimal control framework, let alone an even further distance. Moreover, the effectiveness of solar sails in reducing the fight time is minimal after 5–10 AU away from the Sun. It is currently assumed that the sail is jettisoned at 5 AU. Within such a reasonable assumption, the result of Sauer’s single Solar Photonic Assist and Dachwald’s multiple Solar Photonic Assists are shown in Figs. 1 and 2, respectively. Recently, the interstellar probe optimal trajectories are reinvestigated by THU LAD with a new objective function

\[ \tilde{J} = \lambda_0 (1 - \varepsilon) \| V_f \| - \lambda_0 \varepsilon \int_0^{t_f} dt, \tag{2} \]

where \( \| V_f \| \) is the magnitude of the sail final speed at jettison point, and the scale parameter \( \varepsilon \) is located within interval \([0, 1]\). The new function \( \tilde{J} \) takes into account both the solar approach and the pseudo-cruise phase. Better results have been achieved compared to Sauer with respect to flight time and sample trajectories, as shown in Fig. 3 and Table 1. The problem of sail deployment point is discussed and the differences between the direct flyby and the angular momentum reversal trajectory are also investigated within the same optimal control framework.

III. ANGULAR MOMENTUM REVERSAL TRAJECTORY

The angular momentum reversal trajectory is a novel approach to escape the solar system. Until now, the reversal trajectory has been occasionally mentioned by Sauer and McInnes. In 2008, Wokes classified the 2D heliocentric trajectories in fixed-sail-cone-angle and found the reversal trajectory via hodograph method. THU LAD extended Woke’s result and found
The feasible region of the reversal trajectory for a sail of fixed-cone-angle. As a new potential mission application for the reversal trajectories, the periodic double angular momentum reversal trajectory (solar in-verse orbit) will be introduced in this part. As a ba-

Table 1. Trajectory parameter variations in response to $\varepsilon$ corresponding to Fig. 3.

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$r_{a}$/AU</th>
<th>$V_{t}$/AU/a</th>
<th>$t_{f}$/a</th>
<th>$t_{100}$AU/a</th>
<th>$t_{250}$AU/a</th>
<th>$t_{500}$AU/a</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>1.020</td>
<td>10.615</td>
<td>0.957</td>
<td>9.906</td>
<td>24.037</td>
<td>47.588</td>
</tr>
<tr>
<td>0.100</td>
<td>1.077</td>
<td>10.900</td>
<td>0.990</td>
<td>9.713</td>
<td>23.487</td>
<td>46.443</td>
</tr>
<tr>
<td>0.044</td>
<td>1.311</td>
<td>11.330</td>
<td>1.161</td>
<td>9.546</td>
<td>22.785</td>
<td>44.850</td>
</tr>
<tr>
<td>0.024</td>
<td>1.718</td>
<td>11.755</td>
<td>1.482</td>
<td>9.564</td>
<td>22.884</td>
<td>42.905</td>
</tr>
<tr>
<td>0.012</td>
<td>2.394</td>
<td>12.145</td>
<td>2.065</td>
<td>9.887</td>
<td>22.237</td>
<td>42.822</td>
</tr>
<tr>
<td>0.004</td>
<td>3.607</td>
<td>12.487</td>
<td>3.264</td>
<td>10.872</td>
<td>22.884</td>
<td>42.905</td>
</tr>
<tr>
<td>0.001</td>
<td>4.840</td>
<td>12.663</td>
<td>4.670</td>
<td>12.172</td>
<td>24.017</td>
<td>43.760</td>
</tr>
</tbody>
</table>

the reversal trajectory for a sail of fixed-cone-angle. As a new potential mission application for the reversal trajectories, the periodic double angular momentum reversal trajectory (solar inverse orbit) will be introduced in this part. As a basis of the solar inverse orbit, a time optimal control framework will be briefly presented in Sect. A. Some methods involved and solutions will be discussed in Sect. B.

A. Dynamic model and optimal control framework

In this study, a solar sail spacecraft starts from Earth orbit with the Geocentric escape phase neglected, which can be completed by conventional propulsion. All perturbation forces are not considered except for the solar gravitational force and the solar radiation pressure force exerting on the ideal plane sail. To benefit the calculation process, a set of 3D non-dimensional dynamic equations are employed in heliocentric inertial frame $xyz$. The distance unit is 1 AU while the time unit is so set as to make the solar gravitational constant equal to 1. The equations can thus be expressed as

$$\dot{R} = V,$$

$$V = -\frac{1}{R^3}R + \frac{1}{R^4}(R \cdot n)^2 n,$$

where $R$ and $V$ are the position and velocity vectors of the solar sail spacecraft, respectively. The vector $n$ is the sail orientation vector aligned with the direction of the sail acceleration normal to the sail surface. The second term $\beta(R \cdot n)^2 n/R^4$ is the sail acceleration.

Owing to symmetry of the inverse orbit, the problem can be optimized in only half period to reduce the simulation effort. According to the orbital features, its perihelion should be at the same side of the starting point with respect to the Sun. Therefore, the final constraint at perihelion is

$$\Phi(t_p, R_p) = [x(t_p) - x_p, y(t_p), z(t_p)] = 0,$$

where $t_p$ is the perihelion time and $R_p$ is the perihelion position vector. The minimum periodic inverse orbit obtained from the equations should be in the ecliptic plane guaranteed by zero values of perihelion position $y$ and $z$. To achieve the real optimal solution, the perihelion velocity should be free and can be written as

$$\Phi(t_p, V_p) = [V_x(t_p), V_y(t_p)] = 0.$$

In Eq. (5), $V_p$ is the perihelion velocity vector. Here, free perihelion velocity corresponds to the situation that there is no value constraint about $V_p$ along the $oy$ direction. Generally, there are two optimal indexes for the sail trajectory optimization problem. One is the minimization of time in a given sail lightness number. The other is the minimization of sail acceleration in required mission time. In many cases these two conditions are equivalent. For a relatively high performance solar sail the optimal objective function is defined as

$$J = -\lambda_0 t_p,$$

where $\lambda_0$ is a positive constant. The variational Hamiltonian for this problem is defined as

$$H = -\lambda_0 + \lambda_R(t) \cdot V + \lambda_V(t) \cdot \left[-\frac{1}{R^3}R + \frac{1}{R^4}(R \cdot n)^2 n\right].$$

Here $\lambda_R(t)$ and $\lambda_V(t)$ are the adjoint variables to the position and velocity of the solar sail spacecraft, respectively. The rate of change of the adjoints are derived from the Hamiltonian as

$$\dot{\lambda}_R = -\frac{\partial H}{\partial R} = \frac{1}{R^3}\lambda_V - \frac{3}{R^5}(R \cdot \lambda_V) R - 2\beta \frac{1}{R^4}(R \cdot n)(n \cdot \lambda_V) \left[n \cdot \frac{2(R \cdot n) R}{R^2}\right],$$

$$\dot{\lambda}_V = -\frac{\partial H}{\partial V} = -\lambda_R.$$
In order to maximize $H$ at any time, the optimal sail orientation should satisfy the following equation

$$n(t) = \arg \max H(t, n, \lambda).$$

The corresponding final constraint of adjoint variables and stationary condition are

$$\lambda_{Vy}(t_p) = 0,$$
$$H(t_p) = 0.$$

As long as getting appropriate initial values of the adjoint variables, the optimal control problem is substantially transformed into an equivalent problem of solving a set of algebraic equations as

$$S = [\Phi(t_p, R_p), \Phi(t_p, V_p), \lambda_{Vy}(t_p), H(t_p)] = 0.$$

The minimum solar inverse orbit is obtained using an indirect method when the optimal solution exists. Equation (11) is eight dimensional while the required unknown variables are also eight, including $\lambda_R(t_0), \lambda_V(t_0), \lambda_0$ and $t_p$.

In the optimal control problem, the velocity adjoint $\lambda_V(t)$ is referred to as “primer vector” and defines the optimal direction for the solar radiation force vector. In order to maximize the Hamiltonian it is found that the solar radiation pressure force vector must lie in the plane defined by the position vector and primer vector such that

$$n = \frac{\sin (\alpha - \tilde{\alpha})}{\sin \hat{\alpha}} \frac{R}{\|R\|} + \frac{\sin \alpha}{\sin \hat{\alpha} \|\lambda_V\|} \lambda_V,$$

where $\alpha$ and $\tilde{\alpha}$ are cone angles of the sail normal vector $n$ and the primer vector, respectively.

**B. Periodic double angular momentum reversal trajectory**

After derivations of the optimal control framework, the solar inverse orbit will be numerically discussed. Without loss of generality, the sail lightness number adopted in this paper is 0.6 corresponding to 3.558 mm/s$^2$. The perihelion is selected at 0.6 AU away from the Sun by the same side as the starting point. It should be noted that the perihelion is determined by mission requirements and should match the maximum temperature allowed by the sail film. In most cases the perihelion is no less than 0.2 AU away from the Sun. For solar inverse orbits, the perihelion can be ranged from 0.2 AU to 2 AU or even far away. Three trajectories obtained from the time optimal control model are illustrated in Figs. 4 and 5. Orbit III in Fig. 4 is a type of non-Keplerian orbit without angular momentum reversal, named as “Doubly Periodic Orbit”, which was ever achieved by McInnes in a modulated radial sail acceleration. Although Doubly Periodic Orbit is not the required inverse orbit, it is the global time optimal solution. Orbit I in Fig. 5 is the required minimum periodic inverse orbit while Orbit II is the local optimal solution. In the solving process, a global
optimal method of particle swarm optimization\textsuperscript{62} can be employed to enhance the achievement of minimum periodic inverse orbit. The singular arcs can be eliminated by adding a constant flight time to the perihelion time $t_p$. As seen from Fig. 5, Orbits I and II have the same perihelion of 0.6 AU and are symmetric with respect to the Sun-Perihelion line. The aphelion points of these two trajectories are nearly located on the same circle. The propagations of the angular momentum for these two orbits are given in Fig. 6. In a period the orbital angular momentum changes twice. Some particular points are marked to benefit the understandings of angular momentum reversal. The concerned parameters of these trajectories are listed in Table 2.

As seen from Table 2, the perihelion velocities of the inverse orbits are not local circular orbital velocity which should be 1.291 AU/a at 0.6 AU. Therefore, the final constraint about velocity magnitude should be free
Table 2. Comparison of three types of trajectories.

<table>
<thead>
<tr>
<th>Orbit type</th>
<th>Aphelion/AU</th>
<th>Perihelion/AU</th>
<th>$V_p$/AU⋅a$^{-1}$</th>
<th>Period/a</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>3.962</td>
<td>0.600</td>
<td>1.411</td>
<td>7.490</td>
</tr>
<tr>
<td>II</td>
<td>3.946</td>
<td>0.600</td>
<td>1.191</td>
<td>7.718</td>
</tr>
<tr>
<td>III</td>
<td>1.022</td>
<td>0.137</td>
<td>0.492</td>
<td>1.097</td>
</tr>
</tbody>
</table>

to obtain the real optimal solutions, instead of Mengali’s 5th assumption. Since the symmetry property of the inverse orbit is the basis of the research work, a theoretical proof for its existence is given by THU-LAD as well as three mission applications, i.e., space observation, heliocentric transfer trajectory and asteroid collision. It should be emphasized that the inverse orbit is suitable for space observation due to the quasi-helio-stationary property at its two symmetrical aphelia. Therefore, vast space not in the elliptic plane is waiting for observation via 3D inverse orbits. Such 3D orbits are given in Figs. 7 and 8. The orbit in Fig. 7 can be easily achieved by changing Eq. (4) into

$$\Phi(t_p, R_p) = \left[ x(t_p) - x_p, y(t_p), z(t_p) \pm z_p \right] = 0, \ (z_p \neq 0).$$

However, it has been found through simulations with final constraint of perihelion position $z_p$ at even 2.5 AU that the aphelia of such kind inverse orbits as shown in Fig. 7 are confined to the area of about $z_{\text{max}} = 0.4$ AU. With such a problem, a new kind of 3D inverse orbit is introduced by constraint of $z_{\text{max}}$. As seen from Fig. 8, there are four half parts of inverse orbits labeled from $T_p^1$ to $T_p^4$ as well as some feature points. If the evolution of sail orientations is symmetric with respect to plane $xoz$, the inverse orbit will be composed of $T_p^1$-$T_p^2$ or $T_p^4$-$T_p^3$. With feasible evolution of sail orientations this type of orbit can also be composed of $T_p^1$-$T_p^3$ or $T_p^4$-$T_p^2$. 

IV. CONCLUSION

For high performance solar sails in a characteristic acceleration of more than 3 mm/s$^2$, diversified applications have been investigated for future missions. Up to now, based on the optimal control model and the indirect method, Interstellar Probe mission has been investigated in a new optimal index. The mission duration has been further reduced and related problems were investigated. The difference between the direct flyby and the angular momentum reversal trajectory to complete the Interstellar mission was also discussed through numerical simulations. It is found that there is no need to reversal the orbital angular momentum for a sail within an inner flyby to achieve the minimum duration. Actually, an advantage of the reversal trajectory is that a solar sail can be in fixed cone angle to escape the solar system. As such, the periodic double angular momentum reversal trajectory is undoubtedly a new useful mission application for the reversal trajectories. A time optimal control framework for the inverse orbit is presented in this paper. Some potential applications are also suggested, especially for space observations with 3D inverse orbits. The properties and dynamics of 3D inverse orbit will be discussed in detail in THU-LAD’s publications. More properties of such inverse orbit may still deserve investigation in the further development of solar sails. Trajectory optimization and mission applications for high performance solar sails must remain an interesting and challenging research topic.

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34. C. R. McInnes, JGCD 21, 975 (1998).