North-Holland

FOREWORD

Graph coloring has been a field of attraction for many years; a wide collection of papers has been dedicated to the study of chromatic properties of graphs.

Initially such problems were just a kind of game for pure mathematicians; it was in particular the case of the famous four color problem. However, as people were getting used to applying the tools of graph theory for solving real-life organizational problems, chromatic models appeared as a quite natural way of tackling many situations. Among these are timetabling problems, or more generally scheduling with disjunctive constraints (pairwise incompatibility between jobs), clustering in statistics, automatic classification, group technology in production (partitioning a collection of parts into families of parts which are as similar as possible in their production process), VLSI design, etc.

The theory of perfect graphs and particularly the perfect graph conjecture of Claude Berge provided a strong impetus for the development of the theory of coloring. Several papers in this volume are dealing with special classes of perfect graphs which are characterized by chromatic properties. A natural extension of coloring problems – motivated by a polyhedral formulation of optimization in perfect graphs – consists in expressing an integral vector in a polyhedron as a sum of integral vectors contained in a smaller polyhedron. This extension is considered in some contributions of the present volume. Besides this, color-critical graphs have been a focusing point in many research works; such graphs, having some inherent structure, can hopefully be characterized by more and more properties.

Many other variations and extensions of the basic node (or edge) coloring problem have been proposed: for instance a node or an edge may receive a set of "consecutive" colors, or a color chosen in a given set of admissible colors. A color class may be extended from a stable set of nodes to a union of disconnected cliques. Such types of variations are presented in the volume. Related optimization problems are also discussed; among those is the maximum q-coloring problem: find the largest number of nodes which can be colored with q colors in a given graph G (with chromatic number larger than q).

Edge coloring problems form a special case of node coloring; deciding if there exists an edge coloring using $\Delta(G)$ colors in a simple graph G with maximum degree $\Delta(G)$ is an NP-complete problem. However, many results giving bounds of the edge chromatic number based on the degrees of the nodes and on some properties of the graphs can be obtained. This volume contains a few papers in this direction.

Many algorithmic approaches have been developed and have provided large classes of graphs for which coloring problems can be solved in polynomial time. Such results will certainly be extremely useful for applications.

The papers presented in this text do not provide an exhaustive survey of the various fields of chromatic optimization; in particular they do not describe the numerous fields of applications of colorings. We hope however that they will bring to the reader a view of some facets of this active research area. We will have reached our aim if the contributions collected here will stimulate new investigations in the chromatic properties of graphs and hypergraphs.

In such a wide field, it is difficult to unify notations. We have nevertheless encouraged the authors to use the definitions of Claude Berge, *Graphs* (North-Holland, 1985). Most of them followed these lines and the terms used in different ways are usually defined in the papers or given in appropriate references.

Finally we would like to thank the authors who have been the active contributors to this volume. Our gratitude extends also to the many anonymous referees who have devoted much of their time to improve the quality and readability of the papers.

We wish to acknowledge the enthusiastic support and the encouragements of Peter L. Hammer, Editor-in-chief of *Discrete Mathematics*.

The help of North-Holland in preparing this volume is also gratefully acknowledged.

Many thanks are due to Mrs. A.-L. Choulat for having handled the manuscripts with great care.

Lausanne, June 15, 1988.

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