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Regge spectrum from holographic models inspired by OPE

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ABSTRACT

The problem of obtaining the Regge-like behaviour for meson mass spectrum in the hard-wall AdS/QCD models is addressed. We show that the problem can be solved by a simple incorporation of the effects of local vacuum condensates into such models. The slope of trajectories turns out to be determined by the local condensate of dimension 2 that is absent in the standard Operator Product Expansion. This pitfall, however, can be escaped by means of physically plausible modification of boundary conditions for the holographic fields corresponding to the usual gluon condensate, the latter then determines the slope.

1. Introduction

Recently the bottom-up holographic models have experienced noticeable success in description of non-perturbative QCD (see, e.g., [1–4]). In particular, many important aspects of chiral dynamics can be reproduced within the simplest hard-wall model [1]. However, the spectrum of highly excited mesons in the hard-wall models [1,3,4] behaves like $m_n \sim n$ that does not agree with the Regge-type spectrum, $m_n^2 \sim n$, expected in QCD. To solve this problem one resorted to an additional input from the 5d side — the quadratic dilaton background (the so-called soft-wall model [2]). Such an *ad hoc* solution, however, lacks for a simple description of the chiral symmetry breaking (see, e.g., the recent discussions in [5]). In addition, the physical meaning of the dilaton background is completely obscure. The slope of trajectories determines the physical masses of hadrons, hence, the slope itself is determined by the confinement. The authors of Ref. [2] suggested that the quadratic dilaton could reflect the closed string tachyon condensation since the latter process is often believed to be dual to the confinement in the gauge theories. It was further speculatively assumed in [6] that a closed string tachyon could be associated holographically to a dimension 2 condensate whose relation to various aspects of non-perturbative physics had been widely discussed in the literature (see, e.g., [7]). This conjecture opens the doors to a more general idea that, instead of "playing" with metric, some important aspects of the confinement can be incorporated into the AdS/QCD models through the account for the local condensates. An essential feature in this program is that the local condensates must be taken into consideration dynamically, this is opposite to the geometric approach of [8] where the condensates were included through a modification of metric.

In the present Letter, we propose a very simple and straightforward way of incorporation of local condensates into the AdS/QCD models. We make the case of the vector mesons within the hard-wall holographic model on the simplest Randall–Sundrum background (a somewhat similar but much more complicated analysis was performed in [6] for the glueball spectrum). It is shown that with this theoretical setup the spectrum becomes Regge-like at not very high energies, with the slope of trajectories being determined by the dim2 condensate. The latter circumstance is troublesome because such a gauge-invariant local condensate is absent in the standard Operator Product Expansion (OPE) [9]. If we regard this condensate as non-local then we will be in conflict with the AdS/CFT correspondence principle: The holographic fields must be associated to local gauge-invariant operators in the field theory. We have found a simple and intuitively plausible solution of this problem — the slope can be related to the usual dim4 gluon condensate by a certain modification of boundary conditions for the corresponding dual holographic field, therefore there is no need for introducing the dim2 condensate.

The Letter is organized as follows. The general scheme is presented in Section 2. In Section 3, we consider a concrete model demonstrating the underlying idea. Section 4 is devoted to discussions where it is shown how the problem with dim2 condensate can be avoided. Finally we provide concluding remarks in Section 5.

2. General scheme

We will demonstrate the idea for the case of isoscalar vector mesons (ω-mesons), this case can be straightforwardly generalized to other kinds of mesons. To select the operators that are important in the formation of masses of resonances we will be guided by

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the OPE for the two-point correlators of quark currents at large Euclidean momentum Q. It has the following structure [9] (we omit the Lorentz indices and irrelevant factors)

$$\Pi\left(Q^2\right) = C_0 \log \frac{\Lambda^2}{Q^2} + \sum_{k=1}^{\infty} C_k \frac{\langle \mathcal{O}_{2k} \rangle}{Q^{2k}}, \tag{1}$$

here $\mathcal{O}_{2k}$ are local gauge-invariant operators of canonical dimension $2k$ (several operators can correspond to the same k) and C_k are constants that can be calculated by means of the perturbation theory. The vacuum expectation values (v.e.v.) of the operators in the r.h.s. of Eq. (1) determine the masses of mesons, this is especially evident in the large-N_c limit of QCD, where the l.h.s. of Eq. (1) represents a sum over meson poles. As is usual in calculation of masses within the QCD sum rules [9] we should retain only the first several contributions to the r.h.s. of Eq. (1).

Let us discuss the relevant operators. The first term in the r.h.s. of Eq. (1) is perturbative, hence, it is not of interest for us. The local gauge invariant operator of dimension 2 is absent in the standard OPE [9], nevertheless, as is widely discussed in the literature (see, e.g., [7]), an effective formation of dim2 gluon condensate $\langle A_\mu^2 \rangle$ may turn out to be of high importance in the gluodynamics and lead to a rich phenomenology. In addition, such a quadratic correction is often associated with contributions of renormalons (see, e.g., discussions in [10]). For this reason we tentatively include this operator into our analysis. The v.e.v. $\langle \mathcal{O}_4 \rangle$ is contributed by $m_q \langle \bar{q}q \rangle$ and by the gluon condensate $\alpha_s \langle G_{\mu\nu}^2 \rangle$. The both have no anomalous dimension (the last in one loop). The relative contribution of $m_q \langle \bar{q}q \rangle$ is very small, we neglect it adopting the chiral limit $m_q = 0$. The physical meaning of higher terms in the asymptotic expansion (1) is not essential for our purposes.

According to the AdS/CFT correspondence, we must link each operator $\mathcal{O}(x)$ to a field $\varphi(x,z)$ in the 5d bulk theory, with the 5d masses of fields $\varphi(x,z)$ being determined via the relation [11,12]

$$m_5^2 = (\Delta - p)(\Delta + p - 4), \tag{2}$$

where Δ is the canonical dimension of the corresponding operator $\mathcal{O}(x)$ and p in our simple case is just the number of Lorentz indices.

Let us discuss the field content in our model. First of all, we have the vector meson that is interpolated by the current $\bar{q}\gamma_\mu q$. The corresponding 5d field will be denoted V_M. We have $p = 1$, $\Delta = 3$, hence, $(m_5)_V^2 = 0$. Second, we have an infinite set of operators $\langle \mathcal{O}_{2k} \rangle$ (for simplicity, we neglect the fact that, generally speaking, a finite number of different operators corresponds to each k). The corresponding 5d scalar fields X_{2k} have $p = 0$, $\Delta = 2k$, hence,

$$(m_5)_{2k}^2 = 4k(k-2). \tag{3}$$

The action of the theory in the bulk describing the vector mesons is

$$S = \int d^4x\, dz \sqrt{g}\, \mathrm{Tr}\left\{ \sum_k \left(|DX_{2k}|^2 - (m_5)_{2k}^2 |X_{2k}|^2 \right) - \frac{1}{4} F_{MN}^2 \right\}, \tag{4}$$

where

$$D_M X_{2k} = \partial_M X_{2k} - i g_5 V_M X_{2k}, \tag{5}$$

$$F_{MN} = \partial_M V_N - \partial_N V_M. \tag{6}$$

As usual, the holographic coordinate z corresponds to the inverse energy scale, $z \sim 1/Q$. The v.e.v. of the fields X are determined by the classical solutions satisfying the UV boundary conditions. Since in the limit of very high energies the vacuum in QCD is perturbative, i.e. there are no condensates, it is natural to impose the UV boundary condition

$$X_{2k}(x, z = 0) = 0. \tag{7}$$

At this stage we do not impose any IR boundary (to be discussed below) that determines a scale until which the running of the QCD gauge coupling is neglected. We believe that in dealing with renorminvariant (or almost renorminvariant) quantities, as we do, the problem of running coupling is irrelevant.

To obtain a concrete model one has to choose a metric, the classical solutions for $\langle X_{2k} \rangle$ form then a "potential" for the vector field V_M, fixing for the latter a gauge and boundary conditions one calculates the mass spectrum.

3. A model

We will consider the simplest metric exploited in the hard-wall model, the anti-de Sitter one [1],

$$ds^2 = \frac{1}{z^2}\left(dx_\mu\, dx^\mu - dz^2\right), \tag{8}$$

where for simplicity it is taken $R = 1$ for the radius of AdS space. Fixing the gauge $V_z = 0$, the normalized solutions v_n, $V_M^n(x,z) = V_\mu^n(x) v_n(z)$, of classical equation for the transverse components V_μ^T exist only for discrete values of 4d momentum $q^2 = m_n^2$,

$$\partial_z\left(\frac{1}{z}\partial_z v_n\right) + \frac{m_n^2 v_n}{z} = \frac{2g_5^2}{z^3} v_n \sum_k \langle X_{2k} \rangle^2, \tag{9}$$

where $\langle X_{2k} \rangle$ are solutions of

$$\frac{1}{z^3}\partial_\mu \partial^\mu X_{2k} - \partial_z\left(\frac{1}{z^3}\partial_z X_{2k}\right) = -\frac{1}{z^5}(m_5)_{2k}^2 X_{2k}, \quad k = 1, 2, \ldots. \tag{10}$$

We look for solutions for $\langle X_{2k} \rangle$ which are functions of z only. Making change of variables $v_n = \sqrt{z}\psi_n$ the system of equations (9), (10) takes the form

$$-\psi_n'' + \left(\frac{3}{4z^2} + \frac{2g_5^2}{z^2} \sum_k \langle X_{2k} \rangle^2 \right) \psi_n = m_n^2 \psi_n, \tag{11}$$

$$z^2 X_{2k}'' - 3z X_{2k}' - (m_5)_{2k}^2 X_{2k} = 0, \quad k = 1, 2, \ldots. \tag{12}$$

Eq. (11) is of Schrödinger type with the "potential" in brackets, this potential is determined by the solutions of Eqs. (12). Inserting the values of $(m_5)_{2k}^2$ from relation (3) into Eqs. (12) we obtain the following solutions satisfying the boundary condition (7),

$$\langle X_2 \rangle = c_2^{(1)} z^2 + c_2^{(2)} z^2 \log z, \tag{13}$$

$$\langle X_{2k} \rangle = c_{2k}^{(1)} z^{2k}, \quad k = 2, 3, \ldots, \tag{14}$$

where $c_{2k}^{(i)}$ are some dimensional constants.

In order to have the Regge-like spectrum, $m_n^2 \sim n$, the potential in Eq. (11) has to be of oscillator type, i.e. to behave as z^2 at large z. This is achieved if we set $c_2^{(2)} = 0$ and neglect the contributions of higher-dimensional operators, $c_{2k}^{(1)} = 0$ at $k > 1$. The spectrum will be

$$m_n^2 = 4\sqrt{2}\, g_5 \left| c_2^{(1)} \right| n + \text{const}. \tag{15}$$

In particular, choosing $c_2^{(1)} = (\sqrt{2}\, g_5)^{-1}$ in appropriate units of energy square we arrive at the spectrum obtained in the simplest soft-wall model [2]. We note also that since the slope $a =$

$4\sqrt{2}g_5|c_2^{(1)}|$ in Eq. (15) is proportional to the string tension σ, $a = 2\pi\sigma$, the quadratic correction in the OPE (1) turns out to be also proportional to σ, this is in a qualitative agreement with the results of holographic calculations performed in [13].

Thus, within the given holographic model, the first non-perturbative contribution to the two-point correlators, the so-called dim2 gluon condensate, is responsible for the Regge-like behaviour of meson spectrum, the higher non-perturbative contributions in the OPE (1) yield anharmonic corrections to the spectrum.

4. Discussions

In a sense, the principle of AdS/CFT correspondence converts the asymptotic expansion in Q^{-2} in the OPE into asymptotic expansion in z^4 in the holographic potential of equation for mass spectrum (11),

$$U(z) = \frac{3}{4z^2} + 2g_5^2 \sum_{k=1}^{\infty} c_k z^{4k-2}. \tag{16}$$

In the QCD sum rules [9], only the lowest contributions in the OPE are essential for the determination of masses of ground states of mesons. Accepting the same principle in the model above we should also neglect the higher power-like contributions to the spectrum.

Until now we have nothing said about the IR boundary. An effective IR boundary z_{IR} should certainly exist in the presented model, but in contrast to the hard-wall models we do not impose any special boundary conditions at z_{IR} (the boundary conditions are determined by the requirement to have the Regge-like spectrum), rather z_{IR} shows the range of applicability of the model. For instance, by tuning z_{IR} one can achieve the dominance of the oscillator-type contribution z^2 in the potential of Eq. (11) in a certain range of large z. Following this way one obtains a Regge-like spectrum for a certain number of excited mesons which were observed experimentally, the description of higher states (not observed experimentally) are then beyond the validity of the model. More exactly, the shape of potential well is $\mathcal{O}(z^2)$ at $z_{\min} < z < z_{\mathrm{IR}}$, where $z_{\min}$ is the minimum of potential, and at $z = z_{\mathrm{IR}}$ one has a "hard" wall. As a consequence, the spectrum of normalizable modes is of oscillator type, $m_n^2 \sim n$, at small n and, after imposing the appropriate IR boundary condition, represents zeros of Bessel function, $m_n \sim n$, at large n, $m_n^2 > U(z_{\mathrm{IR}})$, where the model is supposed to be not applicable.

The theoretical status of quadratic correction in the OPE (1) is uncertain as long as the existence of dim2 gluon condensate (the effective "tachyonic" gluon mass) and the related phenomenology are somewhat speculative presently, let alone the problems with the implementation of AdS/CFT principle mentioned in Introduction. The question appears whether it is possible to modify the presented model such that the slope of meson trajectories were not determined by the dim2 gluon condensate? Intuitively it would be more natural to imagine that the slope is related to the dim4 gluon condensate, is it possible to implement this? The answer is positive. The dim4 gluon condensate is known to be contributed not only by the non-perturbative effects but also by the perturbation theory after summation over certain gluon exchanges. For this reason the UV boundary condition (7) may be just incorrect for the scalar field corresponding to the dim4 operator $\alpha_s G_{\mu\nu}^2$, it should be weakened to

$$X_4(x, z = 0) = \text{const}. \tag{17}$$

In this case, the solution of Eq. (12) for $k = 2$ is

$$\langle X_4 \rangle = c_4^{(1)} z^4 + c_4^{(2)}. \tag{18}$$

Substituting this solution in Eq. (11) we observe that $\langle X_4 \rangle$ yields contribution both to the UV (stemming from $c_4^{(2)}$) and to the IR (stemming from $c_4^{(1)}$) parts of potential, with the contribution of oscillator type representing an interplay of the both, $\mathcal{O}(c_4^{(1)} c_4^{(2)} z^2)$, i.e.

$$m_n^2 \sim c_4^{(1)} c_4^{(2)} n + \cdots. \tag{19}$$

This reflects holographically the fact that the gluon condensate encodes both perturbative and non-perturbative effects as we know it from the phenomenology. In addition, we see a direct realization of the AdS/CFT idea: The UV behaviour of the 5d dual theory determines the low-energy properties of the 4d theory on the boundary — the slope of discrete mass spectrum in the given case. Now we can remove the dim2 condensate basing our analysis on the standard OPE [9].

It should be noted that the metric can be chosen such that the Regge slope is automatically determined by the dim4 operators in the OPE while the dim2 ones do not contribute to the slope even if they existed. We have found that this happens if the analysis above is formally performed in the flat metric, see Appendix A.

It seems that the description of Regge-like spectrum can be made fully compatible with the simultaneous description of the chiral symmetry breaking. If we neglect all contributions that lead to the anharmonic terms in the potential of Eq. (11) and take into account the contribution of the quark bilinear operator $\bar{q}q$ in the axial-vector channel following the procedure described in [1], this contribution will dominate in the IR-region, on the other hand, the UV asymptotics of the solutions will be unchanged. Since the description of chiral dynamics is based on these asymptotics, the corresponding results from [1] seem to be compatible with the simultaneous description of the Regge-like spectrum.

5. Concluding remarks

The experience of holographic models of QCD shows that it is really hard to achieve a satisfactory comprehensive description of non-perturbative QCD just by playing with the background metric and boundary conditions for the fields in the bulk, one should add some ingredients directly from QCD or low-energy effective theories of QCD. For instance, knowing that the quark condensate is the order parameter of the chiral symmetry breaking, one should introduce a 5d scalar field corresponding to the quark bilinear operator and organize a nontrivial solution that would correspond to the v.e.v. of the operator under consideration — the quark condensate. This was the first indispensable step for the correct description of chiral dynamics in [1]. In essence, we have proposed a simple and compact demonstration of the fact that in order to obtain the correct spectrum of meson excitations it is not necessary to complicate the bulk geometry, it is sufficient to add more QCD to the simplest model in a similar way. Thus, the question "Why does the simplest hard-wall model, being successful in description of the chiral dynamics, fail to reproduce the Regge-like spectrum?" has a simple and natural answer: Because the 5d field corresponding to the QCD operator that is crucial for chiral dynamics — the quark bilinear operator — was taken into account while the 5d fields corresponding to the QCD operators responsible for the masses of hadrons were not taken into consideration.

The problem that emerges along this line is the appearance of anharmonic corrections to the spectrum. As a result, the spectrum looks like (at least if these corrections are small)

$$m_n^2 \sim \sum_{i=1}^{\infty} c_i n^i. \tag{20}$$

The role of terms with $i > 1$ is an open question. On the one hand, one may try to elaborate some mechanism for their suppression (or, say, regard them as an artifact of asymptotic nature of OPE), on the other hand, the experiment [14] does not provide convincing indications that they must be suppressed. Within the holographic models, such anharmonic contributions were systematically analyzed in [15]. It is quite intriguing to observe that the found solution for the slope (19) shares with the analysis of [15] the following general feature: The slope is equally determined by the IR and UV sectors of the underlying theory.

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Appendix A

Consider the flat metric

$$ds^2 = dx_\mu \, dx^\mu - dz^2. \tag{A.1}$$

In this metric, the classical equations (11), (12) take the form

$$-\psi_n'' + \left(2g_5^2 \sum_k \langle X_{2k} \rangle^2\right)\psi_n = m_n^2 \psi_n, \tag{A.2}$$

$$X_{2k}'' - (m_5)_{2k}^2 X_{2k} = 0, \quad k = 1, 2, \ldots. \tag{A.3}$$

Making use of relation (3) and boundary condition (7), the solutions of Eqs. (A.3) are

$$\langle X_2 \rangle = c_2 \sin(2z), \tag{A.4}$$

$$\langle X_4 \rangle = c_4 z, \tag{A.5}$$

$$\langle X_{2k} \rangle = c_{2k} \sinh\left(2\sqrt{k(k-2)}\, z\right), \quad k = 3, 4, \ldots. \tag{A.6}$$

The harmonic contribution to the potential of Eq. (A.2) stems from the v.e.v. $\langle X_4 \rangle$, at least for large enough z. It seems that we should not use our logic for the operators with $k > 2$ as the solutions (A.6) do not look physical.

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