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## Heavy quark and gluino potentials to two loops

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## ABSTRACT

The static potentials for systems of a heavy quark and anti-quark, two gluinos and a quark and a gluino are computed for all possible color configurations of a  $SU(N_c)$  gauge group.

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The potential energy between two heavy quarks is one of the fundamental quantities of the strong interaction and has been in the focus of the theoretical investigations already in the early days of QCD [1]. The potential arises in a natural way when considering the non-relativistic limit of a heavy quark and anti-quark system as an ingredient of the resulting Schrödinger-like equations (see Ref. [2] for a review). Thus, the potential constitutes a crucial input whenever the production of heavy particles is considered at threshold or bound state properties are calculated. Examples of Standard Model processes are the production of top quark pairs in electron positron collisions for a center-of-mass energy in the vicinity of twice the mass or the invariant-mass distribution of  $t\bar{t}$  pairs at hadron colliders. Furthermore, one should also mention the evaluation of the energy levels and corrections to the wave function for heavy quark bound states like the  $\Upsilon$  or  $\Psi$  systems.

As far as processes beyond the Standard Model are concerned there have been recent publications where bound states of two gluinos, the massive super partners of the gluons, have been examined. Again, the corresponding potential, which has been used to two-loop order, plays a crucial role [3,4]. Similarly, in Ref. [5] the threshold production of a gluino–squark pair is considered. The required potential can be obtained from the quark–gluino potential which is discussed below.

In this Letter we systematically compute the potentials of all color configurations of a quark–anti-quark, gluino–gluino and quark–gluino bound state. To be precise, we consider the heavy-particle systems given in Table 1 and compute the potentials for the corresponding color decomposition.

Note that in our framework both the heavy quark  $q$  and gluino  $\tilde{g}$  are treated as external static color sources added to the (massless) dynamical degrees of freedom of QCD. Thus except for color

**Table 1**

Heavy-particle systems and their color decomposition into irreducible representations. The subscripts “S” and “A” distinguish the symmetric and anti-symmetric octet representations.

Bound state	Color representation	Irreducible representations
$q\bar{q}$	$3 \otimes \bar{3}$	$1 \oplus 8$
$\tilde{g}\tilde{g}$	$8 \otimes 8$	$1 \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27 \oplus R_7$
$q\tilde{g}$	$3 \otimes 8$	$3 \oplus \bar{6} \oplus 15$

there is no difference in the treatment of the gluino and the quark. As a consequence the potential of an anti-quark and a gluino is identical to the  $q\tilde{g}$  potential.

One comment concerning the color decomposition of the  $\tilde{g}\tilde{g}$  potential is in order: As it is common practice we consider only the combination of  $10 \oplus \bar{10}$ . Furthermore, the color structure  $R_7$  is only non-vanishing for  $N_c \neq 3$  and thus it is not relevant for QCD [6–10].

We define the various potentials introduced above as follows

$$V_{ij}^{[c]}(\mu^2 = \bar{q}^2) = -C^{[c]} \frac{4\pi \alpha_s(\bar{q}^2)}{\bar{q}^2} \left[ 1 + \frac{\alpha_s(\bar{q}^2)}{4\pi} a_1 + \left( \frac{\alpha_s(\bar{q}^2)}{4\pi} \right)^2 (a_2 + \delta a_{2,ij}^{[c]}) \right], \quad (1)$$

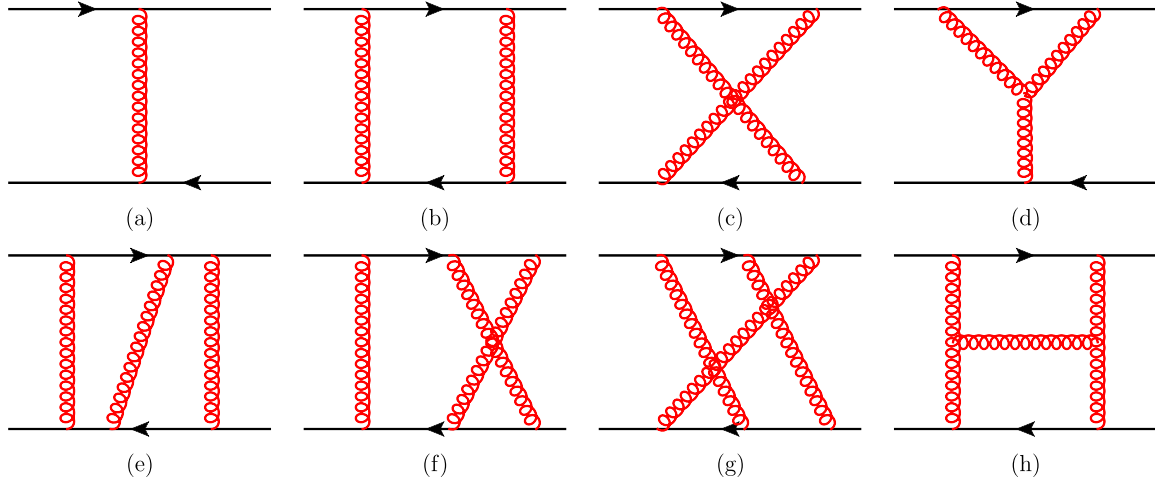
where  $ij \in \{q\bar{q}, \tilde{g}\tilde{g}, q\tilde{g}\}$  and  $c$  defines the color state as given in Table 1. The renormalization scale is set to  $\mu^2 = \bar{q}^2$  to suppress the trivial renormalization group terms on the r.h.s. of Eq. (1). For the potential  $V_{\tilde{g}\tilde{g}}^{[10]}$  we have to modify Eq. (1) slightly since there is no tree and one-loop contribution. Thus we write

$$V_{\tilde{g}\tilde{g}}^{[10]}(\mu^2 = \bar{q}^2) = -\frac{4\pi \alpha_s(\bar{q}^2)}{\bar{q}^2} \left( \frac{\alpha_s(\bar{q}^2)}{4\pi} \right)^2 \delta a_{2,\tilde{g}\tilde{g}}^{[10]}. \quad (2)$$

In Eq. (1) the coefficients  $a_1$  and  $a_2$  are the one- and two-loop corrections which are already present in the singlet contribution

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**Fig. 1.** Sample diagrams contributing to  $V_{ij}^{[c]}$  at tree-level (a), one-loop (b)–(d) and two-loop order (e)–(h). The straight lines correspond to quarks or gluinos, respectively, and the curly lines represent gluons.

of the  $q\bar{q}$  potential. They have been computed in Refs. [11–15] and can be found in Ref. [16] including higher order terms in  $(d-4)$  (where  $d$  is the space–time dimension). The three-loop coefficient  $a_3$  has been computed in Refs. [16–18]. In less than four dimensions the static potential has recently been studied in Ref. [19], see also [20], and the  $N=4$  supersymmetric Yang–Mill theory has been considered in Ref. [21].

At tree-level and at one-loop order the only difference among the various potentials is due to the overall color factor. At two-loop order we have introduced the quantity  $\delta a_{2,ij}^{[c]}$  which parametrizes the difference to the singlet result. It is currently only known for  $V_{q\bar{q}}^{[8]}$  [22]. Furthermore, also for  $V_{\tilde{g}\tilde{g}}^{[1]}$  the two-loop corrections have been computed [4,23] with the result  $\delta a_{2,\tilde{g}\tilde{g}}^{[c]} = 0$ . (In Ref. [23] also the three-loop term of  $V_{\tilde{g}\tilde{g}}^{[1]}$  has been evaluated.) In this Letter we present the two-loop results for all remaining potentials listed in Table 1.

For the calculation we have employed standard techniques which include the automatic generation of the diagrams (see Fig. 1), the classification into different families of integrals, the application of projectors [6–10] and the reduction to master integrals using the Laporta algorithm [24–26]. The latter have been taken over from Ref. [27]. The color factors have been computed with the help of the program `color` [28]. We have performed the calculation for general gauge parameter and have checked that it drops out in the final result.

The standard techniques for the evaluation of the loop integrals appearing at one and two loops (see, e.g., Refs. [20,27]) can only be applied in a straightforward way to the singlet case since there, apart from light-fermion contributions, only the maximally non-Abelian parts contribute. In particular, no diagrams involving pinches occur, i.e. the integrals do not contain propagators of the form  $1/(k_0+i0) \times 1/(k_0-i0)$  where  $k$  is a loop momentum. Sample diagrams are shown in Figs. 1(c) and (d) at one-loop and Figs. 1(g) and (h) at two-loop order. However, for the non-trivial color configurations also contributions involving pinches (see, e.g., Figs. 1(b), (e) and (f)) have to be taken into account. The results of the corresponding diagrams are contained in the quantity  $\delta a_{2,ij}^{[c]}$ . We evaluate the integrals by either exploiting the exponentiation of the color singlet potential or by carefully evaluating the potential in coordinate space starting from the Wilson loop definition. Both methods are described in detail in Refs. [20,22]. In this Letter we have checked that they lead to the same result.

**Table 2**

Results for the  $C^{[c]}$  and  $\delta a_{2,ij}^{[c]}$  for the various color configurations. In the right column we set  $N_c = 3$  and evaluate  $\delta a_{2,ij}^{[c]}$  numerically. Note that  $c = 10$  refers to the combination  $10 \oplus \bar{10}$ . Furthermore,  $R_7$  has dimension zero for  $N_c = 3$ .

$ij$	$c$	$C^{[c]}$	$\delta a_{2,ij}^{[c]}$	$\delta a_{2,ij}^{[c]}(N_c = 3)$
$q\bar{q}$	1	$\frac{(N_c^2-1)}{2N_c}$	0	0
	8	$-\frac{1}{2N_c}$	$N_c^2(\pi^4 - 12\pi^2)$	-189.2
$\tilde{g}\tilde{g}$	1	$N_c$	0	0
	$8_S$	$\frac{N_c}{2}$	0	0
	$8_A$	$\frac{N_c}{2}$	$-6(\pi^4 - 12\pi^2)$	126.2
	10	-	$-\frac{3N_c}{2}(\pi^4 - 12\pi^2)$	94.62
	27	-1	$\frac{1}{2}(N_c+2)(N_c+1)(\pi^4 - 12\pi^2)$	-210.3
	$R_7$	1	$\frac{1}{2}(N_c-2)(N_c-1)(\pi^4 - 12\pi^2)$	-
$q\tilde{g}$	3	$\frac{N_c}{2}$	$-(\pi^4 - 12\pi^2)$	21.03
	$\bar{6}$	$\frac{1}{2}$	$\frac{1}{2}N_c(N_c-3)(\pi^4 - 12\pi^2)$	0
	15	$-\frac{1}{2}$	$\frac{1}{2}N_c(N_c+3)(\pi^4 - 12\pi^2)$	-189.2

In Table 2 we present our results for  $C^{[c]}$  and  $\delta a_{2,ij}^{[c]}$  for  $SU(N_c)$ . Note that  $\delta a_2$  is zero for the singlet contributions but also for gluino bound states in the symmetric octet configuration. One furthermore obtains a vanishing result for the representation  $\bar{6}$  ( $q\tilde{g}$ ) when specifying to QCD, i.e., setting  $N_c = 3$ . It is remarkable that all non-vanishing contributions are proportional to the same combination  $(\pi^4 - 12\pi^2)$  with a prefactor depending on the color state although the individual diagrams contributing to  $\delta a_{2,ij}^{[c]}$  do not show this proportionality and furthermore also contain terms without  $\pi^2$  or  $\pi^4$ .

As far as the numerical importance of  $\delta a_2$  is concerned one can compare the results in the last column of Table 2 with  $a_2$  for the bottom and top system given by

$$a_2(n_l = 4) \approx 211.1,$$

$$a_2(n_l = 5) \approx 155.8. \quad (3)$$

(The corresponding numbers for  $a_1$  are 5.889 and 4.778, respectively.) In some cases one observes a significant reduction of the two-loop coefficient (see, e.g., the 27 configuration for  $n_l = 4$  where  $a_2 + \delta a_{2,\tilde{g}\tilde{g}}^{[10]} \approx 0.8$ ) whereas in other cases the large value of  $a_2$  is even further increased.

To conclude, in this Letter the quark–anti-quark, gluino–gluino and quark–gluino potentials have been computed for all possible

color configurations up to two loops. In all cases it is possible to identify the two-loop coefficient  $a_2$  originating from the quark-anti-quark singlet potential. The additional contributions are given by a color factor times  $(\pi^4 - 12\pi^2)$ .

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