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Time-varying Sliding Mode Controls in Rigid Spacecraft Attitude Tracking

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Abstract

To solve the problem of attitude tracking of a rigid spacecraft with an either known or measurable desired attitude trajectory, three types of time-varying sliding mode controls are introduced under consideration of control input constraints. The sliding surfaces of the three types initially pass arbitrary initial values of the system, and then shift or rotate to reach predetermined ones. This way, the system trajectories are always on the sliding surfaces, and the system work is guaranteed to have robustness against parameter uncertainty and external disturbances all the time. The controller parameters are optimized by means of genetic algorithm to minimize the index consisting of the weighted index of squared error (ISE) of the system and the weighted penalty term of violation of control input constraint. The stability is verified with Lyapunov method. Compared with the conventional sliding mode control, simulation results show the proposed algorithm having better robustness against inertia matrix uncertainty and external disturbance torques.

Keywords: attitude tracking control; time-varying sliding mode; control input constraint; genetic algorithm

1 Introduction

The rapid developments in space technology, have led to a series of formidable challenges to space-crafts, such as fly around, formation flying, and deep space exploration[1-2]. In some cases, the spacecraft is required to track an either known or measurable attitude trajectory known as attitude tracking. As a typical example, in the fly around the deputy satellite is required to maneuver from any initial attitude to become oriented towards the chief satellite. This could be regarded as the body frame of the deputy satellite to track a desired attitude trajectory in the inertia axis frame.

During attitude tracking, the spacecraft is most likely subjected to a variety of environmental disturbance torques mainly inclusive of torque of gravity gradient, torque of solar radiation, geomagnetic torque, and aerodynamic torque[3-4]. Changes of mass and inertia matrix and shifts of mass center due to fuel consumption and/or liquid sloshing constitute the problem of parameter uncertainty in attitude tracking. It is known that the torques used in the attitude control system mainly come from propulsion system, solar radiation pressure, momentum changing devices, and magnetic torque-rods[5]. These kinds of control torque output are all limited. Besides, both attitude kinematics and dynamic equation of a spacecraft are of marked nonlinear form. In short, the attitude tracking control is a nonlinear problem with control input constraints, parameter uncertainties, and external disturbances.

In an attempt to solve the problem, the sliding mode control is advantageous in being insensitive to parameter uncertainty and robustness against external disturbances[6-7], which has made it widely applicable to attitude control[8-10]. The sliding mode

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control law formed in Ref.[8] effectively curbs influences of the low-speed friction of the reaction wheels on the attitude thereby enhancing small satellites' attitude pointing accuracy and stability. In the fuzzy variable structure control method proposed in Ref.[9], the fuzzy system is used to approximate the system uncertain function. The coupled torque between the flexible appendage and the rigid center was considered as an internal reaction torque in Ref.[10], and a term relating to the internal reaction torque to the design of sliding surface was added. This is preferable because by preserving the structure of output feedback control, it avoids the modeling error that stems from the finite dimensional approximation in controller design. Till date, all these methods have used the predetermined sliding surfaces without taking into account the initial values of the system. The motion of the control system in association with the sliding mode control can be divided into two phases: the reaching phase and the sliding phase. The former means that the system trajectory moves to reach the sliding surface. The latter implies that the system trajectory slides along the sliding surface to the origin. The drawback of a predetermined sliding surface is that the robustness can not be ensured in the reaching phase. To solve this problem, a stepwise time-varying sliding mode control of the fast and robust track for an uncertain second-order system was put forward in Ref.[11]. This method achieves a faster track by significantly shortening the reaching phase. On the basis of Ref.[11], a continuous time-varying sliding mode control for the simple second-order system, which eliminates the reaching phase, and enables the system work the robustness all the time was presented in Ref.[12]. Three types of time-varying sliding mode controls—constant acceleration sliding mode control, constant velocity sliding mode control, and terminal slider control—for the second-order single-input single-output (SISO) system with control input constraints were proposed in Ref.[13]. They eliminate the reaching phase and guarantee the robustness all the time. Two parameters of the controller are determined with an analytical method based on certain assumed approximations. The optimal values of the parameters are destined to minimize the integral of the abstract value of the error. However, in a complicated nonlinear system, this analytical method is no more valid. A new approach is thus required to solve the nonlinear optimization problem.

In this article, the authors try to introduce the application of first two cited time-varying sliding mode control laws into the application of attitude tracking control, which is a multiple-input multiple-output (MIMO) system, and present a constant velocity slope-varying sliding mode control. The time-varying sliding surfaces of the three types pass the initial values of system at the initial moment. Then they shift or rotate to a certain scheme until the predetermined sliding surfaces are reached. With the system trajectory lying on the sliding surface all the time the whole reaching phase is eliminated. Hence, the system is always insensitive to parameter uncertainty and external disturbances. The parameters of the controller are optimized with the genetic algorithm in place of an analytical method. The optimization index includes two parts. Of them, one is the weighted index of squared error (ISE) of the system and the other the weighted penalty term in case the constraint on control torque is violated. Results from the simulation and their comparison with those of the conventional sliding mode control show that the proposed methods have an edge over the conventional one by greatly shortening the setting time.

2 Problem Description

In the problem of attitude tracking, roll, pitch, and yaw angles between the body frame and the desired axis frame might be very large. Consequently, the modified Rodrigues parameter (MRP) with the largest nonsingular range and without redundancy is used to represent the attitude. Denote the attitude MRP and the attitude angular velocity of the rigid spacecraft body frame with respect to the inertia axis frame as $\boldsymbol{\sigma}(t)\in\mathbf{R}^3$ and $\boldsymbol{\omega}(t)\in\mathbf{R}^3$, respectively. Similarly, with the attitude trajectory of desired axis frame with respect to the inertia axis frame, the respective denotations are $\boldsymbol{\sigma}_{\rm d}(t)\in\mathbf{R}^3$ and $\boldsymbol{\omega}_{\rm d}(t)\in\mathbf{R}^3$. The attitude tracking problem is meant to find a control law to transfer the state of spacecraft $(\boldsymbol{\sigma}(t),\boldsymbol{\omega}(t))$ to the desired state $(\boldsymbol{\sigma}_{\rm d}(t),\boldsymbol{\omega}_{\rm d}(t))$. Here, the desired axis frame, denoted by $\boldsymbol{\sigma}_{\rm d}(t)$, $\boldsymbol{\omega}_{\rm d}(t)$ and $\dot{\boldsymbol{\omega}}_{\rm d}(t)$, is assumed to be either already known or measurable.

The attitude dynamic equation for a rigid spacecraft is[14]

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}+[\boldsymbol{\omega}\times]\boldsymbol{J}\boldsymbol{\omega}=\boldsymbol{T}_{\rm c}+\boldsymbol{T}_{\rm d} \tag{1}$$

where $\boldsymbol{J}\in\mathbf{R}^{3\times3}$ is the inertia matrix of a spacecraft, and $\boldsymbol{J}=\boldsymbol{J}_0+\Delta\boldsymbol{J}$, where $\boldsymbol{J}_0$ is the nominal value and $\Delta\boldsymbol{J}$ the bounded uncertainty, $\boldsymbol{T}_{\rm d}\in\mathbf{R}^3$ is the bounded external disturbance torque, and $\boldsymbol{T}_{\rm c}\in\mathbf{R}^3$ the control torque that satisfies the constraint:

$$\|\boldsymbol{T}_{\rm c}\|_\infty\le T_{\max} \tag{2}$$

where $\|\boldsymbol{T}_{\rm c}\|_\infty=\sup(|T_{\rm c1}|,|T_{\rm c2}|,|T_{\rm c3}|)$, $T_{{\rm c}i}(i=1,2,3)$ is element of $\boldsymbol{T}_{\rm c}$.

The error attitude MRP $\boldsymbol{\sigma}_{\rm e}(t)$ and the error attitude angular velocity $\boldsymbol{\omega}_{\rm e}(t)$ of the body frame with respect to the desired axis frame are defined as[15]

$$\boldsymbol{\sigma}_{\rm e}(t)=\boldsymbol{\sigma}(t)\otimes\boldsymbol{\sigma}_{\rm d}^{-1}(t) \tag{3}$$

$$\boldsymbol{\omega}_{\rm e}(t)=\boldsymbol{\omega}(t)-\boldsymbol{\omega}_{\rm d}^{\rm b}(t) \tag{4}$$

where $\boldsymbol{\omega}_{\rm d}^{\rm b}(t)=\boldsymbol{R}_{\rm bd}\boldsymbol{\omega}_{\rm d}(t)$ is the expression of $\boldsymbol{\omega}_{\rm d}(t)$ in the body frame, $\boldsymbol{R}_{\rm bd}$ the transformation matrix from the desired axis frame to the body frame, $\otimes$ denotes the MRP multiplication which could be defined by

$$\boldsymbol{\sigma}_1(t)\otimes\boldsymbol{\sigma}_2^{-1}(t)=\frac{(1-\boldsymbol{\sigma}_2^{\rm T}\boldsymbol{\sigma}_2)\boldsymbol{\sigma}_1+(\boldsymbol{\sigma}_1^{\rm T}\boldsymbol{\sigma}_1-1)\boldsymbol{\sigma}_2-2\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_1}{1+(\boldsymbol{\sigma}_2^{\rm T}\boldsymbol{\sigma}_2)(\boldsymbol{\sigma}_1^{\rm T}\boldsymbol{\sigma}_1)+2\boldsymbol{\sigma}_2^{\rm T}\boldsymbol{\sigma}_1}$$

Next, assuming that

(1) $\boldsymbol{\sigma}_{\rm e}(0)$ and $\boldsymbol{\omega}_{\rm e}(0)$ are exactly known, and

(2) $\boldsymbol{\omega}_{\rm e}(0)=0$ and

by substituting Eq.(4) into Eq.(1), the error attitude dynamic equation becomes

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}_{\rm e}+\boldsymbol{J}\dot{\boldsymbol{\omega}}_{\rm d}^{\rm b}+[\boldsymbol{\omega}\times]\boldsymbol{J}\boldsymbol{\omega}=\boldsymbol{T}_{\rm c}+\boldsymbol{T}_{\rm d} \tag{5}$$

The attitude kinematics equation is[16]

$$\dot{\boldsymbol{\sigma}}_{\rm e}(t)=\boldsymbol{M}(\boldsymbol{\sigma}_{\rm e})\boldsymbol{\omega}_{\rm e} \tag{6}$$

where

$$\boldsymbol{M}(\boldsymbol{\sigma}_{\rm e})\boldsymbol{\omega}_{\rm e}=\{(1-\boldsymbol{\sigma}_{\rm e}^{\rm T}\boldsymbol{\sigma}_{\rm e})\boldsymbol{I}+2[\boldsymbol{\sigma}_{\rm e}\times]+2\boldsymbol{\sigma}_{\rm e}\boldsymbol{\sigma}_{\rm e}^{\rm T}\}/4 \tag{7}$$

Then, for simplicity, by replacing $\boldsymbol{M}(\boldsymbol{\sigma}_{\rm e})$ with $\boldsymbol{M}$, Eq.(7) implies $\boldsymbol{M}\boldsymbol{M}^{\rm T}=q\boldsymbol{I}_{3\times3}$, where

$$q=(1+\boldsymbol{\sigma}_{\rm e}^{\rm T}\boldsymbol{\sigma}_{\rm e})/16$$

The attitude tracking problem can be reidentified to find a proper control law $\boldsymbol{T}_{\rm c}$ to transfer the error state $(\boldsymbol{\sigma}_{\rm e}(t),\boldsymbol{\omega}_{\rm e}(t))$ into $(\mathbf{0},\mathbf{0})$ in the presence of bounded parameter uncertainty $\Delta\boldsymbol{J}$ and bounded external disturbance torque $\boldsymbol{T}_{\rm d}$ without violating the control input constraint $\|\boldsymbol{T}_{\rm c}\|_\infty\le T_{\max}$.

3 Time-varying Sliding Mode Control Laws

This section will present the design method of three types of time-varying sliding mode controls. Of them, the first two shift the sliding surfaces by changing their intercepts, whereas the third rotates the sliding surface by changing its slope.

3.1 Constant acceleration intercept-varying sliding mode control

Define the constant acceleration intercept-varying sliding surface as[13]

$$\boldsymbol{S}=\dot{\boldsymbol{\sigma}}_{\rm e}(t)+k\boldsymbol{\sigma}_{\rm e}(t)+\begin{cases}\boldsymbol{A}_1t^2+\boldsymbol{B}_1t+\boldsymbol{C}_1,\ t\le T\\ \mathbf{0},\ t>T\end{cases} \tag{8}$$

where $T>0$ is the switching time, $k>0$ the slope of the predetermined sliding surface, and both are submitted to constants. Let $\boldsymbol{A}_1\in\mathbf{R}^3$, $\boldsymbol{B}_1\in\mathbf{R}^3$, and $\boldsymbol{C}_1\in\mathbf{R}^3$ be constant vectors, such that

(1) the initial value of the system belongs to the sliding surface defined by Eq.(8) at $t=0$

$$\dot{\boldsymbol{\sigma}}_{\rm e}(0)+k\boldsymbol{\sigma}_{\rm e}(0)+\boldsymbol{C}_1=\mathbf{0} \tag{9}$$

(2) the right side of Eq.(8) is continuous with its derivative at $t=T$

$$\left.\begin{aligned}\boldsymbol{A}_1T^2+\boldsymbol{B}_1T+\boldsymbol{C}_1=\mathbf{0}\\ 2\boldsymbol{A}_1T+\boldsymbol{B}_1=\mathbf{0}\end{aligned}\right\} \tag{10}$$

By combining Eq.(9) and Eq.(10), can be obtained

$$\left.\begin{aligned}\boldsymbol{A}_1&=-[\dot{\boldsymbol{\sigma}}_{\rm e}(0)+k\boldsymbol{\sigma}_{\rm e}(0)]/T^2\\ \boldsymbol{B}_1&=2[\dot{\boldsymbol{\sigma}}_{\rm e}(0)+k\boldsymbol{\sigma}_{\rm e}(0)]/T\\ \boldsymbol{C}_1&=-\dot{\boldsymbol{\sigma}}_{\rm e}(0)-k\boldsymbol{\sigma}_{\rm e}(0)\end{aligned}\right\} \tag{11}$$

By choosing

$$V=\boldsymbol{S}^{\rm T}\boldsymbol{M}\boldsymbol{J}\boldsymbol{M}^{\rm T}\boldsymbol{S}/2 \tag{12}$$

as the Lyapunov function, the time derivative of V

is

$$\dot{V} = \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}\dot{\boldsymbol{S}} + \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJ\dot{M}}^{\mathrm{T}}\boldsymbol{S} = \begin{cases} \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}(\boldsymbol{H} + \boldsymbol{M}\dot{\boldsymbol{\omega}}_{\mathrm{e}} + 2\boldsymbol{A}_1 t + \boldsymbol{B}_1) + \\ \quad \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJ\dot{M}}^{\mathrm{T}}\boldsymbol{S},\ t \le T \\ \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}(\boldsymbol{H} + \boldsymbol{M}\dot{\boldsymbol{\omega}}_{\mathrm{e}}) + \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJ\dot{M}}^{\mathrm{T}}\boldsymbol{S},\ t > T \end{cases} \tag{13}$$

where $\boldsymbol{H} = \dot{\boldsymbol{M}}\boldsymbol{\omega}_{\mathrm{e}} + k\dot{\boldsymbol{\sigma}}_{\mathrm{e}}$.

Using Eq.(5), can be obtained

$$\dot{V} = \begin{cases} \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}\boldsymbol{H} + q\boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{T}_{\mathrm{c}} - [\boldsymbol{\omega}\times]\boldsymbol{J\omega} - \boldsymbol{J}\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}} + \boldsymbol{T}_{\mathrm{d}}) + \\ \quad \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}(2\boldsymbol{A}_1 t + \boldsymbol{B}_1) + \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}\,\boldsymbol{J\dot{M}}^{\mathrm{T}}\boldsymbol{S},\ t \le T \\ \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}\boldsymbol{H} + q\boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{T}_{\mathrm{c}} - [\boldsymbol{\omega}\times]\boldsymbol{J\omega} - \boldsymbol{J}\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}} + \boldsymbol{T}_{\mathrm{d}}) + \\ \quad \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJ\dot{M}}^{\mathrm{T}}\boldsymbol{S},\ t > T \end{cases} \tag{14}$$

Suppose that the control law is

$$\boldsymbol{T}_{\mathrm{c}} = \begin{cases} [\boldsymbol{\omega}\times]\boldsymbol{J}_0\boldsymbol{\omega} + \boldsymbol{J}_0\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}} - [\boldsymbol{J}_0\boldsymbol{M}^{\mathrm{T}}\boldsymbol{H} + \boldsymbol{J}_0\boldsymbol{M}^{\mathrm{T}}(2\boldsymbol{A}_1 t + \\ \quad \boldsymbol{B}_1) + \boldsymbol{J}_0\dot{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{S}]/q - \boldsymbol{M}^{\mathrm{T}}\boldsymbol{\eta}\,\mathrm{sgn}(\boldsymbol{S})/q^2,\ t \le T \\ [\boldsymbol{\omega}\times]\boldsymbol{J}_0\boldsymbol{\omega} + \boldsymbol{J}_0\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}} - (\boldsymbol{J}_0\boldsymbol{M}^{\mathrm{T}}\boldsymbol{H} + \boldsymbol{J}_0\dot{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{S})/q - \\ \quad \boldsymbol{M}^{\mathrm{T}}\boldsymbol{\eta}\,\mathrm{sgn}(\boldsymbol{S})/q^2,\ t > T \end{cases} \tag{15}$$

where $\boldsymbol{\eta} = \mathrm{diag}(\eta_1, \eta_2, \eta_3)$ is the positive definite gain of the discontinuous control, by substituting Eq.(15) into Eq.(14), can be acquired

$$\dot{V} = -\boldsymbol{S}^{\mathrm{T}}\boldsymbol{\eta}\,\mathrm{sgn}(\boldsymbol{S}) + \boldsymbol{S}^{\mathrm{T}}\boldsymbol{f}_1(\Delta\boldsymbol{J}, \boldsymbol{T}_{\mathrm{d}}) \tag{16}$$

where

$$\boldsymbol{f}_1(\Delta\boldsymbol{J}, \boldsymbol{T}_{\mathrm{d}}) = \begin{cases} q\boldsymbol{MT}_{\mathrm{d}} - q\boldsymbol{M}([\boldsymbol{\omega}\times]\Delta\boldsymbol{J\omega} + \Delta\boldsymbol{J}\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}}) + \boldsymbol{M}\Delta\boldsymbol{JM}^{\mathrm{T}}\boldsymbol{H} + \\ \quad \boldsymbol{M}\Delta\boldsymbol{JM}^{\mathrm{T}}(2\boldsymbol{A}_1 t + \boldsymbol{B}_1) + \boldsymbol{M}\Delta\boldsymbol{J\dot{M}}^{\mathrm{T}}\boldsymbol{S},\ t \le T \\ q\boldsymbol{MT}_{\mathrm{d}} - q\boldsymbol{M}([\boldsymbol{\omega}\times]\Delta\boldsymbol{J\omega} + \Delta\boldsymbol{J}\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}}) + \boldsymbol{M}\Delta\boldsymbol{JM}^{\mathrm{T}}\boldsymbol{H} + \\ \quad \boldsymbol{M}\Delta\boldsymbol{J\dot{M}}^{\mathrm{T}}\boldsymbol{S},\ t > T \end{cases}$$

denotes the uncertainty due to $\Delta\boldsymbol{J}$ and $\boldsymbol{T}_{\mathrm{d}}$. The boundedness of both $\Delta\boldsymbol{J}$ and $\boldsymbol{T}_{\mathrm{d}}$ also implies the boundedness of $\boldsymbol{f}_1(\Delta\boldsymbol{J}, \boldsymbol{T}_{\mathrm{d}})$. Assuming that $\Delta_{1\max} = \|\boldsymbol{f}_1(\Delta\boldsymbol{J}, \boldsymbol{T}_{\mathrm{d}})\|_\infty$, $\boldsymbol{f}_1(\Delta\boldsymbol{J}, \boldsymbol{T}_{\mathrm{d}}) = [\Delta_{11}\ \ \Delta_{12}\ \ \Delta_{13}]^{\mathrm{T}}$, and setting $\eta_i = \Delta_{1\max} + \lambda\ (i = 1,2,3)$, where λ is an arbitrary positive number, from Eq.(16), can be obtained

$$\dot{V} = \sum_{i=1}^{3} S_i[-(\Delta_{1\max} + \lambda)\mathrm{sgn}(S_i) + \Delta_{1i}] \le -\sum_{i=1}^{3}\lambda|S_i| \tag{17}$$

where $S_i (i = 1,2,3)$ is one of the elements of $\boldsymbol{S}$. From Eq.(17), it is believed that the asymptotically stable system meets the sliding condition.

3.2 Constant velocity intercept-varying sliding mode control

The constant velocity intercept-varying sliding surface is defined as[13]

$$\boldsymbol{S} = \dot{\boldsymbol{\sigma}}_{\mathrm{e}}(t) + k\boldsymbol{\sigma}_{\mathrm{e}}(t) + \begin{cases} \boldsymbol{A}_2 t + \boldsymbol{B}_2,\ t \le T \\ \boldsymbol{0},\ t > T \end{cases} \tag{18}$$

where $\boldsymbol{A}_2 \in \mathbf{R}^3$ and $\boldsymbol{B}_2 \in \mathbf{R}^3$ are constant vectors, such that

(1) the initial value of the system belongs to the sliding surface defined by Eq.(18) at $t = 0$

$$\dot{\boldsymbol{\sigma}}_{\mathrm{e}}(0) + k\boldsymbol{\sigma}_{\mathrm{e}}(0) + \boldsymbol{B}_2 = \boldsymbol{0} \tag{19}$$

(2) the right side of Eq.(18) is continuous at $t = T$

$$\boldsymbol{A}_2 T + \boldsymbol{B}_2 = \boldsymbol{0} \tag{20}$$

By combining Eq.(19) and Eq.(20), can be obtained

$$\left.\begin{aligned} \boldsymbol{A}_2 &= [\dot{\boldsymbol{\sigma}}_{\mathrm{e}}(0) + k\boldsymbol{\sigma}_{\mathrm{e}}(0)]/T \\ \boldsymbol{B}_2 &= -\dot{\boldsymbol{\sigma}}_{\mathrm{e}}(0) - k\boldsymbol{\sigma}_{\mathrm{e}}(0) \end{aligned}\right\} \tag{21}$$

In much the same way Eq.(12) has been derived, and considering Eq.(18), can acquire the time derivative of V as follows:

$$\dot{V} = \begin{cases} \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}(\boldsymbol{H} + \boldsymbol{M}\dot{\boldsymbol{\omega}}_{\mathrm{e}} + \boldsymbol{A}_2) + \\ \quad \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJ\dot{M}}^{\mathrm{T}}\boldsymbol{S},\ t \le T \\ \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}(\boldsymbol{H} + \boldsymbol{M}\dot{\boldsymbol{\omega}}_{\mathrm{e}}) + \\ \quad \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJ\dot{M}}^{\mathrm{T}}\boldsymbol{S},\ t > T \end{cases} \tag{22}$$

Also, by using Eq.(5), can be obtained

$$\dot{V} = \begin{cases} \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}\boldsymbol{H} + q\boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{T}_{\mathrm{c}} - [\boldsymbol{\omega}\times]\boldsymbol{J\omega} - \boldsymbol{J}\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}} + \\ \quad \boldsymbol{T}_{\mathrm{d}}) + \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}\boldsymbol{A}_2 + \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}\,\boldsymbol{J\dot{M}}^{\mathrm{T}}\boldsymbol{S},\ t \le T \\ \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}\boldsymbol{H} + q\boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{T}_{\mathrm{c}} - [\boldsymbol{\omega}\times]\boldsymbol{J\omega} - \boldsymbol{J}\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}} + \\ \quad \boldsymbol{T}_{\mathrm{d}}) + \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJ\dot{M}}^{\mathrm{T}}\boldsymbol{S},\ t > T \end{cases} \tag{23}$$

Here, by setting the control law as

$$\boldsymbol{T}_{\mathrm{c}} = \begin{cases} [\boldsymbol{\omega}\times]\boldsymbol{J}_0\boldsymbol{\omega} + \boldsymbol{J}_0\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}} - (\boldsymbol{J}_0\boldsymbol{M}^{\mathrm{T}}\boldsymbol{H} + \boldsymbol{J}_0\boldsymbol{M}^{\mathrm{T}}\boldsymbol{A}_2 + \\ \quad \boldsymbol{J}_0\dot{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{S})/q - \boldsymbol{M}^{\mathrm{T}}\boldsymbol{\eta}\,\mathrm{sgn}(\boldsymbol{S})/q^2,\ t \le T \\ [\boldsymbol{\omega}\times]\boldsymbol{J}_0\boldsymbol{\omega} + \boldsymbol{J}_0\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}} - (\boldsymbol{J}_0\boldsymbol{M}^{\mathrm{T}}\boldsymbol{H} + \boldsymbol{J}_0\dot{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{S})/q - \\ \quad \boldsymbol{M}^{\mathrm{T}}\boldsymbol{\eta}\,\mathrm{sgn}(\boldsymbol{S})/q^2,\ t > T \end{cases} \tag{24}$$

and by substituting Eq.(24) into Eq.(23), yields

$$\dot{V} = -\boldsymbol{S}^{\mathrm{T}}\boldsymbol{\eta}\,\mathrm{sgn}(\boldsymbol{S}) + \boldsymbol{S}^{\mathrm{T}}\boldsymbol{f}_2(\Delta\boldsymbol{J}, \boldsymbol{T}_{\mathrm{d}}) \tag{25}$$

where

$$\boldsymbol{f}_2(\Delta\boldsymbol{J}, \boldsymbol{T}_{\mathrm{d}}) =$$

$$\begin{cases} q\boldsymbol{M}\boldsymbol{T}_{\mathrm{d}}-q\boldsymbol{M}([\boldsymbol{\omega}\times]\Delta\boldsymbol{J}\boldsymbol{\omega}+\Delta\boldsymbol{J}\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}})+\boldsymbol{M}\Delta\boldsymbol{J}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{H}+ \\ \quad \boldsymbol{M}\Delta\boldsymbol{J}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{A}_2+\boldsymbol{M}\Delta\boldsymbol{J}\dot{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{S},\ t\le T \\ q\boldsymbol{M}\boldsymbol{T}_{\mathrm{d}}-q\boldsymbol{M}([\boldsymbol{\omega}\times]\Delta\boldsymbol{J}\boldsymbol{\omega}+\Delta\boldsymbol{J}\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}})+\boldsymbol{M}\Delta\boldsymbol{J}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{H}+ \\ \quad \boldsymbol{M}\Delta\boldsymbol{J}\dot{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{S},\ t>T \end{cases}$$

denotes the uncertainty due to $\Delta\boldsymbol{J}$ and $\boldsymbol{T}_{\mathrm{d}}$. Similar to $\boldsymbol{f}_1(\Delta\boldsymbol{J},\boldsymbol{T}_{\mathrm{d}})$, $\boldsymbol{f}_2(\Delta\boldsymbol{J},\boldsymbol{T}_{\mathrm{d}})$ is also bounded.

Supposing that $\Delta_{2\max}=\|\boldsymbol{f}_2(\Delta\boldsymbol{J},\boldsymbol{T}_{\mathrm{d}})\|_\infty$, and $\boldsymbol{f}_2(\Delta\boldsymbol{J},\boldsymbol{T}_{\mathrm{d}})=[\Delta_{21}\quad \Delta_{22}\quad \Delta_{23}]^{\mathrm{T}}$, and setting $\eta_i=\Delta_{2\max}+\lambda\ (i=1,2,3)$, then

$$\dot V=\sum_{i=1}^{3}S_i[-(\Delta_{2\max}+\lambda)\mathrm{sgn}(S_i)+\Delta_{2i}]\le-\sum_{i=1}^{3}\lambda|S_i| \quad (26)$$

holds true.

Obviously, the asymptotically stable system meets the sliding condition.

3.3 Constant velocity slope-varying sliding mode control

The constant velocity slope-varying sliding surface is defined as

$$\boldsymbol{S}=\begin{cases}\dot{\boldsymbol{\sigma}}_{\mathrm{e}}(t)+(\boldsymbol{A}_3t+\boldsymbol{B}_3)\boldsymbol{\sigma}_{\mathrm{e}}(t),\ t\le T\\ \dot{\boldsymbol{\sigma}}_{\mathrm{e}}(t)+k\boldsymbol{\sigma}_{\mathrm{e}}(t),\ t>T\end{cases} \quad (27)$$

where $\boldsymbol{A}_3=\mathrm{diag}(a_{31},a_{32},a_{33})$ and $\boldsymbol{B}_3=\mathrm{diag}(b_{31},b_{32},b_{33})$ are constant diagonal matrixes, such that

(1) the initial value of the system lies on the sliding surface defined by Eq.(27) at $t=0$

$$\dot{\boldsymbol{\sigma}}_{\mathrm{e}}(0)+\boldsymbol{B}_3\boldsymbol{\sigma}_{\mathrm{e}}(0)=\boldsymbol{0} \quad (28)$$

(2) the right side of Eq.(27) is continuous at $t=T$

$$\boldsymbol{A}_3T+\boldsymbol{B}_3=k\boldsymbol{I} \quad (29)$$

By combining Eq.(28) and Eq.(29), can be obtained

$$\left.\begin{aligned} a_{3i}&=[k+\dot\sigma_{\mathrm{e}i}(0)/\sigma_{\mathrm{e}i}(0)]/T\\ b_{3i}&=-\dot\sigma_{\mathrm{e}i}(0)/\sigma_{\mathrm{e}i}(0)\end{aligned}\right\} \quad (30)$$

where $\sigma_{\mathrm{e}i}(0),\ \dot\sigma_{\mathrm{e}i}(0)\ (i=1,2,3)$ are elements of $\boldsymbol{\sigma}_{\mathrm{e}}(0),\ \dot{\boldsymbol{\sigma}}_{\mathrm{e}}(0)$, respectively.

By choosing the same candidate of Lyapunov function as is in Eq.(12), its time derivative is

$$\dot V=\begin{cases}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}[\dot{\boldsymbol{M}}\boldsymbol{\omega}_{\mathrm{e}}+\boldsymbol{M}\dot{\boldsymbol{\omega}}_{\mathrm{e}}+\boldsymbol{A}_3\boldsymbol{\sigma}_{\mathrm{e}}+(\boldsymbol{A}_3t+\\ \quad \boldsymbol{B}_3)\dot{\boldsymbol{\sigma}}_{\mathrm{e}}]+\boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}\,\boldsymbol{J}\dot{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{S},\ t\le T\\ \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}(\dot{\boldsymbol{M}}\boldsymbol{\omega}_{\mathrm{e}}+\boldsymbol{M}\dot{\boldsymbol{\omega}}_{\mathrm{e}}+k\boldsymbol{I}\dot{\boldsymbol{\sigma}}_{\mathrm{e}})+\\ \quad \boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJ}\dot{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{S},\ t>T\end{cases} \quad (31)$$

By setting the control law as

$$\boldsymbol{T}_{\mathrm{c}}=$$
$$\begin{cases}[\boldsymbol{\omega}\times]\boldsymbol{J}_0\boldsymbol{\omega}+\boldsymbol{J}_0\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}}-\boldsymbol{J}_0\boldsymbol{M}^{\mathrm{T}}[\dot{\boldsymbol{M}}\boldsymbol{\omega}_{\mathrm{e}}+\boldsymbol{A}_3\boldsymbol{\sigma}_{\mathrm{e}}+(\boldsymbol{A}_3t+\\ \quad \boldsymbol{B}_3)\dot{\boldsymbol{\sigma}}_{\mathrm{e}}+\boldsymbol{J}_0\dot{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{S}]/q-\boldsymbol{M}^{\mathrm{T}}\boldsymbol{\eta}\,\mathrm{sgn}(\boldsymbol{S})/q^2,\ t\le T\\ [\boldsymbol{\omega}\times]\boldsymbol{J}_0\boldsymbol{\omega}+\boldsymbol{J}_0\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}}-[\boldsymbol{J}_0\boldsymbol{M}^{\mathrm{T}}(\dot{\boldsymbol{M}}\boldsymbol{\omega}_{\mathrm{e}}+k\boldsymbol{I}\dot{\boldsymbol{\sigma}}_{\mathrm{e}})+\\ \quad \boldsymbol{J}_0\dot{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{S}]/q-\boldsymbol{M}^{\mathrm{T}}\boldsymbol{\eta}\,\mathrm{sgn}(\boldsymbol{S})/q^2,\ t>T\end{cases} \quad (32)$$

and substituting Eq.(32) into Eq.(31), yields

$$\dot V=-\boldsymbol{S}^{\mathrm{T}}\boldsymbol{\eta}\,\mathrm{sgn}(\boldsymbol{S})+\boldsymbol{S}^{\mathrm{T}}\boldsymbol{f}_3(\Delta\boldsymbol{J},\boldsymbol{T}_{\mathrm{d}}) \quad (33)$$

where

$$\boldsymbol{f}_3(\Delta\boldsymbol{J},\boldsymbol{T}_{\mathrm{d}})=$$
$$\begin{cases}q\boldsymbol{M}\boldsymbol{T}_{\mathrm{d}}+\boldsymbol{M}\Delta\boldsymbol{J}\dot{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{S}-q\boldsymbol{M}([\boldsymbol{\omega}\times]\Delta\boldsymbol{J}\boldsymbol{\omega}+\Delta\boldsymbol{J}\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}})+\\ \quad \boldsymbol{M}\Delta\boldsymbol{J}\boldsymbol{M}^{\mathrm{T}}[\dot{\boldsymbol{M}}\boldsymbol{\omega}_{\mathrm{e}}+\boldsymbol{A}_3\boldsymbol{\sigma}_{\mathrm{e}}+(\boldsymbol{A}_3t+\boldsymbol{B}_3)\dot{\boldsymbol{\sigma}}_{\mathrm{e}}],\ t\le T\\ q\boldsymbol{M}\boldsymbol{T}_{\mathrm{d}}-q\boldsymbol{M}([\boldsymbol{\omega}\times]\Delta\boldsymbol{J}\boldsymbol{\omega}+\Delta\boldsymbol{J}\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{b}})+\boldsymbol{M}\Delta\boldsymbol{J}\boldsymbol{M}^{\mathrm{T}}(\dot{\boldsymbol{M}}\boldsymbol{\omega}_{\mathrm{e}}+\\ \quad k\boldsymbol{I}\dot{\boldsymbol{\sigma}}_{\mathrm{e}})+\boldsymbol{M}\Delta\boldsymbol{J}\dot{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{S},\ t>T\end{cases}$$

denotes the bounded uncertainty due to $\Delta\boldsymbol{J}$ and $\boldsymbol{T}_{\mathrm{d}}$. Obviously, $\boldsymbol{f}_3(\Delta\boldsymbol{J},\boldsymbol{T}_{\mathrm{d}})$ is bounded. Supposing $\Delta_{3\max}=\|\boldsymbol{f}_3(\Delta\boldsymbol{J},\boldsymbol{T}_{\mathrm{d}})\|_\infty$, and $\boldsymbol{f}_3(\Delta\boldsymbol{J},\boldsymbol{T}_{\mathrm{d}})=[\Delta_{31}\quad \Delta_{32}\quad \Delta_{33}]^{\mathrm{T}}$, and setting $\eta_i=\Delta_{3\max}+\lambda\ (i=1,2,3)$, then

$$\dot V=\sum_{i=1}^{3}S_i[-(\Delta_{3\max}+\lambda)\mathrm{sgn}(S_i)+\Delta_{3i}]\le-\sum_{i=1}^{3}\lambda|S_i| \quad (34)$$

holds true.

From Eq.(33), the asymptotically stable system meets the sliding condition.

Remarks

(1) Because of existence of the sign functions in Eq.(15), Eq.(24) and Eq.(32), there must be chattering in the control system. To solve the problem, the sign functions could be modified into saturation ones[17]:

$$\mathrm{sat}(S_i,\varepsilon)=\begin{cases}1,\ S_i>\varepsilon\\ S_i/\varepsilon,\ |S_i|\le\varepsilon\\ -1,\ S_i<-\varepsilon\end{cases} \quad (35)$$

where $i=1,2,3$, ε is a small positive number.

(2) From Eq.(17), Eq.(26), and Eq.(34), it can be seen that $\dot V\le 0$ and $\dot V=0$ hold true only when $\boldsymbol{S}=\boldsymbol{0}$. Because the time-varying sliding surface Eq.(8), Eq.(18), and Eq.(27) are designed under the consideration of system initial value, $\boldsymbol{S}(0)=\boldsymbol{0}$, which implies $V(0)=0$ and $\dot V(0)=0$. This means $V(t)$ and $\boldsymbol{S}$ are equal to zero all the time, that is, the system trajectory lies on the sliding surface at any time. Hence, the reaching phase is eliminated and

the system is robust against parameter uncertainty and external disturbances from the outset of the system work.

(3) In Sections 3.1-3.3, the sliding condition generalized squared norm $\boldsymbol{S}^{\mathrm{T}}\boldsymbol{MJM}^{\mathrm{T}}\boldsymbol{S}/2$, where $\boldsymbol{MJM}^{\mathrm{T}}$ is a positive definite, is used more than merely a $\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}$.

4 Parameter Optimization

In the methods of designing three types of time-varying sliding mode controls introduced in Section 3, there are two parameters—the slope of the predetermined sliding surface k and the switching time T—that are not determined as yet. Theoretical analysis shows that without any influence on the stability of the system, they have remarkable effects on its performances. It is thus expected to find out the optimal values of k and T, which can make the index of control performances optimum. The optimization of k and T is actually a problem of optimization with constraints involved because the optimal values should enable the system to work at the possibly highest speed and with the possibly highest accuracy, whereas the constraints on control input should always be satisfied. In an application of SISO second-order system in Ref.[13], this parameter optimization problem was solved in an analytical way. However, it does not work with attitude tracking control problem because of its complexity. Here, the genetic algorithm should be resorted to solve this constraint-submitted nonlinear parameter optimization problem. In this case, an index to evaluate the performances of the system is defined as

$$I = w_1\int_0^{\infty}\boldsymbol{\sigma}_{\mathrm{e}}^{\mathrm{T}}\boldsymbol{\sigma}_{\mathrm{e}}\mathrm{d}t + w_2\int_0^{\infty}h(\boldsymbol{T}_{\mathrm{c}})\mathrm{d}t \tag{36}$$

where the first term of the right side is the weighted ISE of system, and the second one the weighted penalty term when the constraint on the control input is violated. w_1 and w_2 are their weights, and the definition of $h(\boldsymbol{T}_{\mathrm{c}})$ is

$$h(\boldsymbol{T}_{\mathrm{c}}) = \begin{cases} 1, & \|\boldsymbol{T}_{\mathrm{c}}\|_{\infty} > T_{\max} \\ 0, & \|\boldsymbol{T}_{\mathrm{c}}\|_{\infty} \le T_{\max} \end{cases} \tag{37}$$

The optimal values of k and T are bound to be found to minimize the index. The advantage of using penalty term is to transform the constraint-submitted optimization problem into an non-constraint-submitted one. Nevertheless, an improper chosen penalty term would be possible to result in infeasible solutions[18]. Setting $w_2 \gg w_1$ and thereby greatly strengthening the penalty term might as well completely avoid infeasible solutions. The genetic algorithm is implemented in the following steps[19]:

(1) Choose search sets of both k and T, encode them as N bit binary strings, and generate the initial population with N_{p} individuals.

(2) Calculate the fitness value of each individual according to Eq.(36).

(3) Select from the current population the strings with a probability proportional to their fitness values.

(4) The single-point crossover operation is performed on the selected strings with a probability of P_{x}.

(5) Mutation is then applied to the new chromosomes with a set probability P_{m}. The new generation has now been completed.

(6) Evaluate the new generation by Eq.(36), and repeat the Steps (3)-(6) until the number of generations N_{g} is achieved.

5 Simulation Results

Take as an example the following attitude tracking problem with parameters: $\boldsymbol{\omega}_{\mathrm{d}} = [0.57\ \ 0.57\ \ 0.57]^{\mathrm{T}}$ (°)/s, $T_{\max} = 4$, $\boldsymbol{\eta} = \mathrm{diag}(0.8, 0.8, 0.8)$, $\varepsilon = 0.001$, $w_1 = 1$, $w_2 = 100$. In the genetic algorithm, $N = 10$, $N_{\mathrm{p}} = 30$, $N_{\mathrm{g}} = 100$, $P_{\mathrm{x}} = 0.9$, $P_{\mathrm{m}} =$ 0.005. The search sets of k and T are $[0.01, 0.08]$ and $[50, 400]$, respectively. The initial values are $\boldsymbol{\omega}_{\mathrm{e}}(0) = [0\ \ 0\ \ 0]^{\mathrm{T}}$ (°)/s, $\boldsymbol{\sigma}_{\mathrm{e}}(0) = [-0.654\ \ 0.520\ \ 0.241]^{\mathrm{T}}$, whose corresponding 3-1-2 set of Euler angle is $[6.792\,3^{\circ}\ \ 144.74^{\circ}\ \ 105.18^{\circ}]$. Table 1 shows the optimal values of k and T of different time-varying sliding mode controls. Fig.1 shows the curve of I with respect to the number of generations N_{g}.

Table 1 Optimal values of three types of time-varying sliding mode controls

Type of sliding surface	k	T/s	I
Constant acceleration intercept-varying sliding surface	0.039 4	126.979 5	35.068 6
Constant velocity intercept-varying sliding surface	0.038 7	62.316 7	30.570 4
Constant velocity slope-varying sliding surface	0.079 5	128.348 0	27.317 3

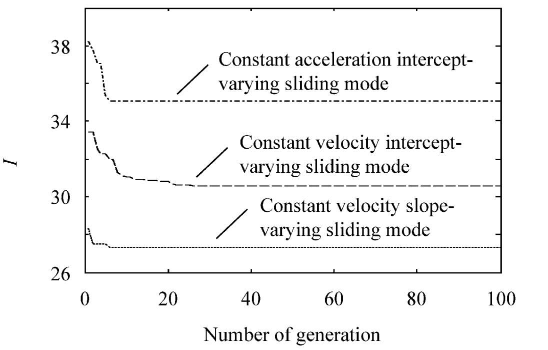

Fig.1　Value of I with respect to the number of generation.

Given $\boldsymbol{T}_{\mathrm{d}} = 0.5[\sin 0.1t \quad \sin 0.1t \quad \sin 0.1t]^{\mathrm{T}}$ N·m, $\Delta \boldsymbol{J} = 0.2\boldsymbol{J}$, the dynamic behavior of the attitude tracking control system operating according to above-cited three types of time-varying sliding mode controls is simulated and its control quality is compared to the conventional sliding mode. Figs.2-5 show the results of the simulation, Fig.2 shows the three-axis attitude point precision, Fig.3 the three-axis attitude stability, Fig.4 the control torque, Fig.5 the three phase portraits of the system.

From Figs.4-5, it can be seen that when the constraints on control torque are all time satisfied, all time-varying sliding mode controls could guarantee robustness against parameter uncertainty and external disturbances from the outset of the system work; whereas the conventional sliding mode control does not. From Figs.2-3, it can be seen that three time-varying sliding mode control laws also yield shorter setting time. Table 1 shows that the optimal value of index of constant velocity slope-varying sliding mode control is a little less than that of the other two kinds of time-varying sliding mode controls.

For comparison, the following criteria are specified:

(1) duration of the reaching phase, which basically gives the information on how long the system will sense undesired effects of disturbance and parameter variations;

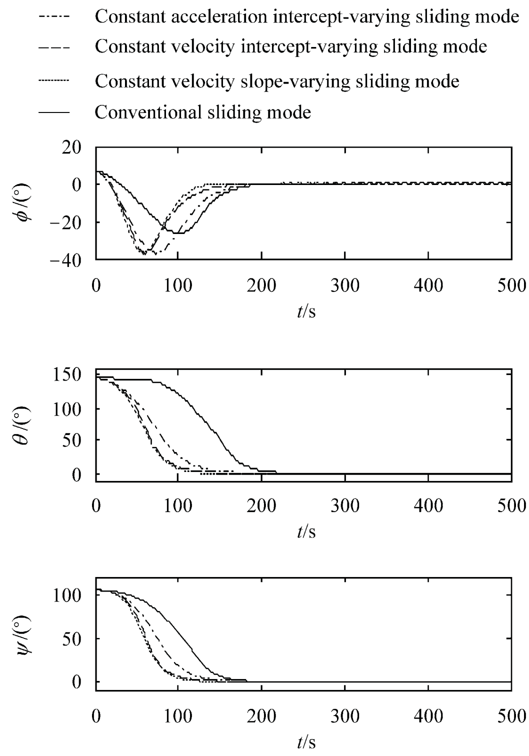

Fig.2　Three-axis attitude point precision.

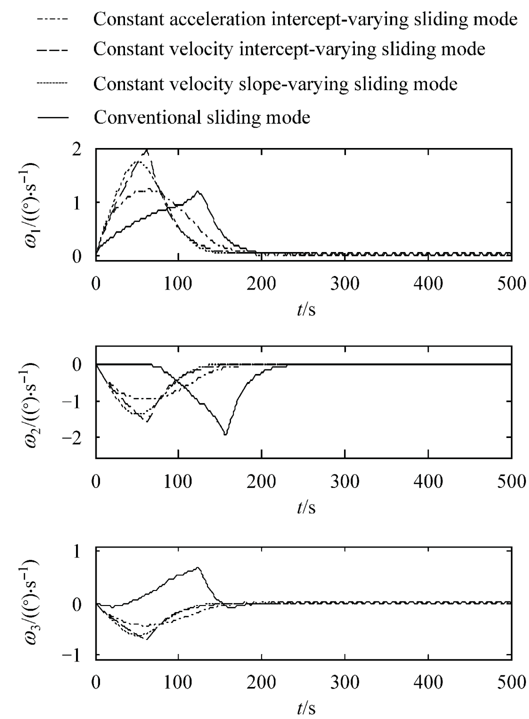

Fig.3　Three-axis attitude stability.

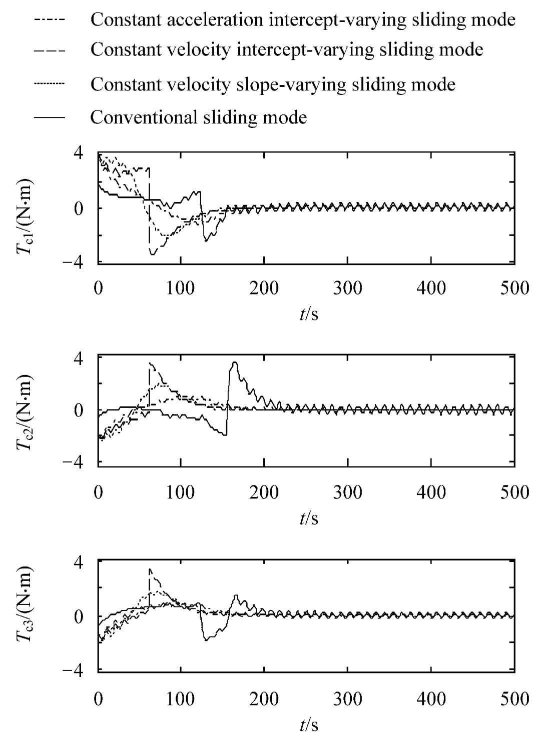

Fig.4 Time history of control torque.

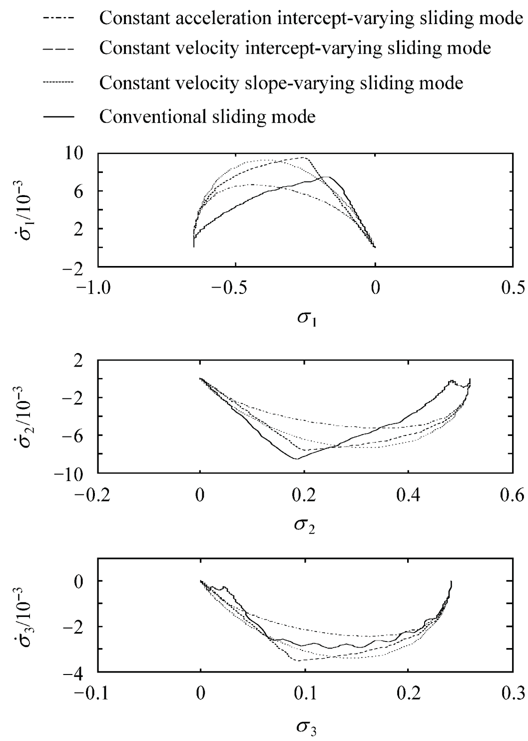

Fig.5 Three phase portraits of the system.

Table 2 Comparison of different algorithms in terms of control quality

Quality criterion	Conventional sliding mode	Constant acceleration intercept-varying sliding mode	Constant velocity intercept-varying sliding mode	Constant velocity slope-varying sliding mode
Reaching-phase duration/s	>0	0	0	0
Trajectory convergence time/s	200	120	98	92
Three-axis attitude point precision in steady state/(°)	0.023 0	0.004 6	0.004 6	0.004 6
Attitude stability in steady state/((°)·s^{-1})	0.01	0.02	0.02	0.02

(2) error convergence time;

(3) three-axis attitude point precision and three-axis attitude stability in steady state, which are both important to evaluate the performances of the attitude control of a spacecraft.

Table 2 evaluates the criteria (1)-(3) for each of the algorithms. Comparison shows that all three types of time-varying sliding mode controls could achieve better performances than the conventional sliding mode control. Closer examination of both Table 1 and Table 2 shows that although the constant velocity slope-varying sliding mode control yields nearly the same three-axis attitude point precision and attitude stability in steady state as others, it takes the shortest convergence time. This is responsible for the smallest index value of I in Table 1 and shows the better robustness against parameter uncertainty and external disturbances of time-varying sliding mode control.

6 Conclusions

To solve the problem of attitude tracking of a rigid spacecraft with an either known or measurable desired attitude trajectory, three types of time-varying sliding mode controls are presented. Of them, the first two use the intercept-varying sliding surfaces and the third the slope-varying sliding surface. Given the exact known initial values of the system, all three sliding surfaces pass the initial values of the system and then shift or rotate according to a certain scheme to attain the predetermined ones. Only in this way, besides elimination of the reaching phase, could improve the robustness against parameter uncertainty and external disturbances, which can not always be guaranteed by the conventional sliding mode control. Two parameters of the control law are optimized by using genetic algorithm to minimize an index which consists of two parts i.e. the weighted ISE of system and the weighted penalty term in case the constraint on the control input is violated. The simulation results show that when the constraints on control torque are all the time satisfied, time-varying sliding mode controls could ensure better robustness once the system starts working and achieve shorter setting time. Compared with the other two types of time-varying sliding mode controls, the constant velocity slope-varying one can achieve the minimum index.

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