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Analysis Approach to Durability Based on Material Initial Fatigue Quality and *S-N* Curve

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Abstract

Based on probabilistic fracture mechanics approach, a new concept of material initial fatigue quality (MIFQ) is developed. Then, the relation between *S-N* curve and crack propagation curve is studied. From the study, a new durability analysis method is presented. In this method, *S-N* curve is used to determine crack growth rate under constant amplitude loading and evaluate the effects of different factors on durability and then the structural durability is analyzed. The tests and analyses indicate that this method has lower dependence on testing, and higher accuracy, reliability and generality and is convenient for application.

Keywords: probabilistic fracture mechanics approach; durability; *S-N* curve; crack propagation rate; initial fatigue quality

1 Introduction

Probabilistic fracture mechanics approach (PFMA) is a new technology specially used in durability analysis of structure details[1-2]. It can be used to investigate the rule of changing with time for the crack distribution of structure details and ensure the realization of structure durability according to the criterion of crack exceeded probability or servicing/replacing expense ratio[1-2]. However, this approach depends on experiments largely. In other words, different tests should be carried out for the different structure details, even if their materials and processing parameters are the same. Therefore, the method presented in Refs.[1-2] costs a lot and is not convenient for engineering application. Chen presented a simplified method for modeling initial fatigue quality (IFQ) which is depicted with only a group of test data and the corresponding material *S-N* curves[3]. However, crack growth rate still needed to be determined by tests. Barter, Yang and Manning also presented some improved methods which still depend on tests largely[4-8].

A new concept of material IFQ (MIFQ) is developed according to PFMA. And the relation between *S-N* curve and crack propagation curve is studied. Then, the new method predicting crack propagation rate and durability is presented based on this new concept and the studied relationship. Since MIFQ is generic and crack propagation rate can be determined with structure *S-N* curve, the new method is not depending on additional experiments and can evaluate the effects of different factors on the structure durability. The experiments demonstrate that this method is reasonable and applicable.

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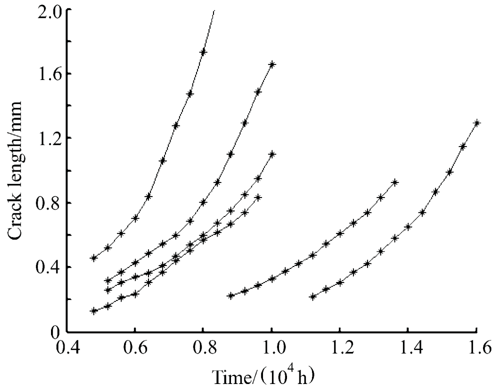
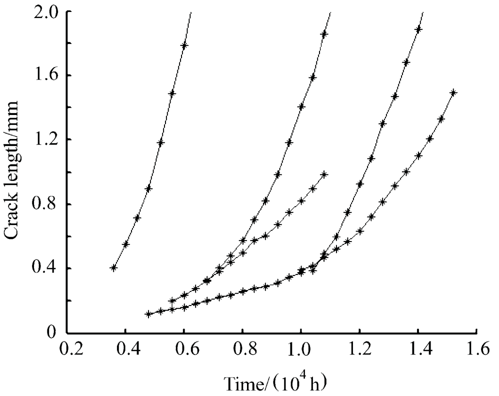
2 Material Initial Fatigue Quality

2.1 Durability tests

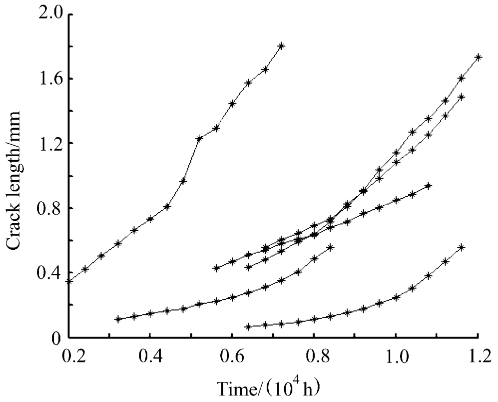
These tests are employed in Ref.[9], which were carried out under the loading spectra of F-16 fighter by US Air Force. The initial fractographic data of AFLR4, AFMR4A and AFHR4A data groups are shown in Fig.1 (the meanings of the symbols are shown in Table 1).



(a) AFLR4



(b) AFMR4A



(c) AFHR4A

Fig.1 Initial fractographic data of data groups.

Table 1 Symbol meanings of data groups[9]

Symbol order	Test parameter	Symbol and its meaning
1	Material	A: 7475- T 7351 aluminum alloy
2	Loading spectra	F: block spectra equivalent to F-16 400 flight hours
3	Loading transfer	: no loading transfer X: 15% loading transfer Y: 30% loading transfer Z: 40% loading transfer
4	Stress level	L: low(220.8 MPa) M: middle(234.6 MPa) H: high (262.2 MPa)
5	Fastener type	R: MS-90353 rivet
6	Fastener size	3: 3/16 inch (1 inch=2.54 cm) 4: 1/4 inch
7	Thickness of specimen	For the structures without loading transfer, the thickness is 0.375 inch with no symbol or A, and 0.250 inch with B. For the structures with 15%, 30% and 40% loading transfer, the thickness is 0.375 inch

Based on these tests, the parameters of equivalent initial flaw size (EIFS) distribution, which depict IFQ of 7475-T7351 aluminium alloy, are calculated. The predicted results are shown in Table 2, where α, β, ε are time to crack initial (TTCI) distribution shape, scale, threshold parameter respectively, θ is crack growth rate parameter, and reference crack size of the testing specimen is 0.9 mm.

Table 2 EIFS distribution parameters of different data groups

Order	Data groups	Parameters		
		α	$Q\beta$	ε
1	AFLR4, AFMR4A, AFHR4A	2.80	4.00	0.80*
2	AFLR4, AFMR4B, AFHR4B	2.28	2.79	0.42
3	AFXLR4, AFXMR4, AFXHR4	2.01	1.93	0.58
4	AFYLR4, AFYMR4, AFYHR4	3.66	1.81	0.80*
5	AFXLR3, AFXMR3, AFXHR3	2.14	1.77	0.40
6	AFZLR4, AFZMR4, AFZHR4	2.73	1.69	0.80*
7	AFMR4A, AFXMR4, AFYMR4	3.21	2.56	0.78
8	AFHR4A, AFXHR4, AFYHR4	3.17	2.55	0.74
9	AFLR4, AFXLR4, AFYLR4	3.40	2.67	0.78

Note: * represents that there are no optimal solutions of paramenters.

2.2 IFQ of material

From Table 2, EIFS distributions of different data groups are diverse though the initial fractographic data are very sufficient. Theoretically, in Table 2, except for the IFQs of the second and fifth data groups, the IFQs of other data groups are uniform because the structure details are the same.

There are mainly three reasons for this situation. Firstly, some factors are stochastic and uncertain, such as surface roughness, case hardening, remaining stress, etc. Secondly, the error is induced by the method of calculating crack growth rate. As we know, loading level and order have significant effects on crack growth rate. If the true initial crack length is a_0, the calculated length will be a_1 in the low stress level and a_2 in the high stress level by the formula $a = a_0 \cdot \exp(QN)$ (see Fig.2, where N is the number of cycle, a_0 is initial crack length), and it may have $a_0 \neq a_1$, $a_1 \neq a_2$ and $a_0 \neq a_2$. So the estimated EIFS distributions are different. And thirdly, due to the certain stochastic nature of crack growth, the EIFS distribution parameters will be different with different reference crack sizes. And sometimes, there are no optimal estimates of EIFS distribution parameters with average rank method. So the EIFS distribution parameters of every data group are different.

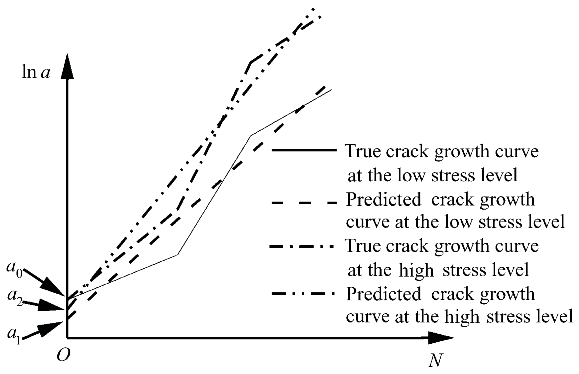


Fig.2 Effects of stress levels on IFQ.

To make EIFS distribution be generalized well, the concept of MIFQ is given here, that the MIFQ is a token of material internal flaws and related to material properties, internal flaws (such as pores and inclusions) and loading types. The configuration, scale, processing parameters, stress level, etc. only have effects on crack propagation rate.

Similar to S-N curve, MIFQ is the intrinsic property of material. To obtain the MIFQ, standard specimen can be tested with constant amplitude loading to evaluate the EIFS distribution with the method in Ref.[10]. Since MIFQ is common for the structures with the same material, their EIFS distributions are also the same in spite of their scales, configurations, stress levels, etc.

3 Durability Analysis Method

3.1 Relation between crack propagation curve and *S*-*N* curve

S-N curve of structure expresses the relation between the stress level $\sigma_{\max}$ and structure fatigue life N_{f}, and can be depicted generally as follows

$$\sigma_{\max}^{m} N_{\mathrm{f}} = C_1 \tag{1}$$

where m and C_1 are material constants.

And crack propagation curve expresses the relation between crack propagation rate $\mathrm{d}a/\mathrm{d}N$ and stress strength factor amplitude, and is simplified in PFMA as follows[2]

$$\mathrm{d}a/\mathrm{d}N = Qa = \xi\sigma_{\max}^{\gamma} a \tag{2}$$

where ξ and γ are constants related to loading spectra and material characteristics.

Taking integral of Eq.(2), one obtains

$$\int_{a_0}^{a_{\mathrm{f}}} \frac{1}{a}\,\mathrm{d}a = \int_0^{N_{\mathrm{f}}} \xi\sigma_{\max}^{\gamma}\,\mathrm{d}N \Rightarrow C_2 = \sigma_{\max}^{\gamma} N_{\mathrm{f}} \tag{3}$$

where a_{f} equals to $\{K_{\mathrm{IC}}/[\Delta\sigma Y(a)]\}^2/\pi$ and expresses the limit crack length, $C_2=[\ln(a_{\mathrm{f}}/a_0)]/\xi$, K_{IC} is material fracture toughness, $Y(a)$ is the correctional factor of stress strength factor.

In Eq.(3), $\sigma_{\max}$ is the maximum stress acted on the structure and N_{f} is the corresponding fatigue life. Thus Eq.(3) expresses the relation between the maximum stress and fatigue life of structure, that is the S-N curve in conventional fatigue design. Eq.(1) and Eq.(3) indicate that there are correspondence between crack propagation curve and S-N curve, and one curve can be determined through the other curve. According to Eq.(1) and Eq.(3), the relationships between their parameters are concluded as follows

$$\left.\begin{aligned}\xi &= \frac{1}{C_1}\ln\frac{a_{\mathrm{f}}}{a_0}\\ \gamma &= m\end{aligned}\right\} \tag{4}$$

3.2 Effects of factors on crack growth rate

Generally, the effects of different factors on crack propagation are evaluated through costly experiments. In fact, the effects can be determined easily and conveniently after modeling the relationship of the two curves, because large amount of data can be obtained from the conventional stress approach.

If *S*-*N* curve can be modified by a factor as shown in Eq.(5),

$$\sigma_{\max}^{m'} N_{\mathrm{f}} = C'_1 \tag{5}$$

where m' and C'_1 are material constants.

Then, the corresponding parameters of crack propagation rate, ξ' and γ', can be modified as

$$\left.\begin{aligned}\xi' &= \frac{1}{C'_1}\ln\frac{a_{\mathrm{f}}}{a_0}\\ \gamma' &= m'\end{aligned}\right\} \tag{6}$$

From above, the key for evaluating the effects of multiple factors on durability is to determine the corresponding *S*-*N* curve. Once the modified relations in Eq.(6) are obtained, the effects of different factors on crack propagation can be evaluated and then the structure durability can be analyzed.

3.3 Durability analysis method

After the relations between the parameters of the two curves are established, the durability of structure can be analyzed through following steps.

Step 1 Calculate the initial crack length a_0 with given reliability according to EIFS distribution of material.

Step 2 Calculate the structure *S*-*N* curve parameters C_1 and m based on the *S*-*N* curve of corresponding material and the effects of different factors.

Step 3 Determine the fracture toughness K_{IC} of the material (referring appropriate handbook) to calculate the fracture length a_{f}.

Step 4 Calculate the parameters ξ and γ according to Eq.(4) or Eq.(6).

Step 5 Calculate economical life and damage degree and evaluate the durability of the structure.

4 Experiments

The tested data in Ref.[11] are employed. Nine levels of loads ((46.92 ± 22.77) MPa, (46.92 ± 30.36) MPa, (46.92 ± 37.95) MPa, (57.96 ± 22.77) MPa, (57.96 ± 30.36) MPa, (57.96 ± 37.95) MPa, (69.00 ± 22.77) MPa, (69.00 ± 30.36) MPa, and (69.00 ± 37.95) MPa) are acted on the 7075- T7351 aluminum alloy board with the dimension of 812.8 mm × 203.2 mm × 6.35 mm. The results of experiments are shown in Figs.3-4, where σ_{m} is the average stress level.

According to the stress-life tested results, the *S*-*N* curves are fitted (shown in Fig.3).Then, the parameters ξ and γ are predicted according to Eq.(3) and the crack propagation curves are predicted at different stress levels (shown in Fig.4). From Figs.3-4, the predicted results possess sufficient accuracy. The predicted results of fatigue life are all longer than the tested results for 1-2 times, it shows that this method is somewhat conservative.

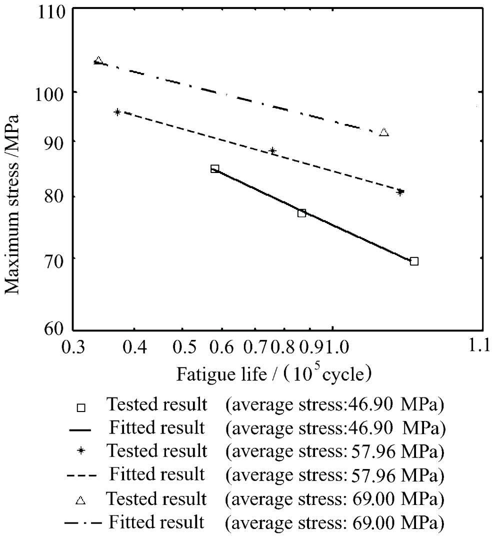
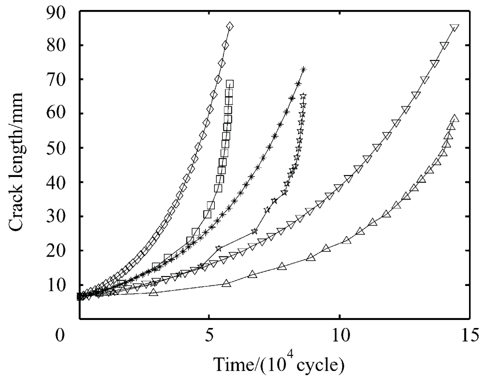
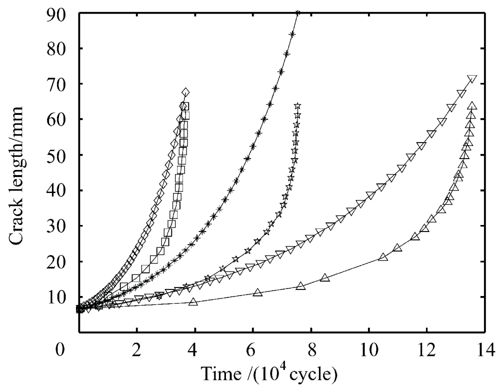


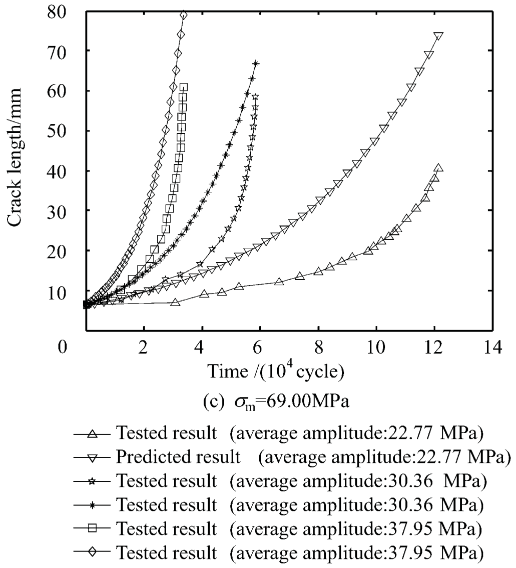
Fig.3 Fitted results of stress-life experiments.



(a) σ_m=46.92 MPa



(b) σ_m=57.96 MPa



(c) σ_m=69.00MPa

Fig.4 Tested and predicted results of crack growth rate.

The reasons for the differences between the predicted and tested results are as follows.

The first is due to the crack propagation behavior. Generally, during small crack stage, crack is propagated slowly. Then the crack is growing steadily. At the last stage, the crack grows unsteadily and then the structure is damaged. So, the experimental curve is concave (see Fig. 5).

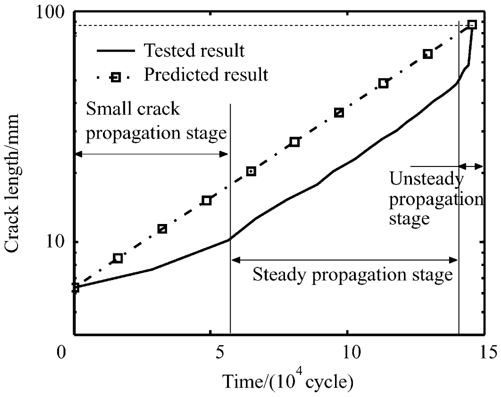


Fig.5 Error analysis.

However, the method presented in this paper predicts the crack growth based on *S*-*N* curve. Because

$$Q = \xi\sigma_{\max}^{\gamma} = \frac{1}{C_1}\ln\frac{a_f}{a_0}\sigma_{\max}^{m} = \frac{1}{N_f}\ln\frac{a_f}{a_0} \tag{7}$$

the predicted crack growth rate is only related to the parameters of crack during initial and final states and is the average value for that of the whole crack growth process. From Fig.5, at the small crack stage, the predicted crack growth rate (the slope of the curve in Fig.5) is larger than the tested result. At the steady growth stage, the two curves are nearly parallel to each other and at the last stage, the predicted results are significantly smaller than the tested results. So, in the range of fatigue life, the predicted are all larger than the tested.

The second is due to the precision of *S*-*N* curve parameters. From Eq.(4), the precision of *S*-*N* curve parameters are directly connected to the accuracy of crack growth rate curve. That is, the predicted results will match the tested results well if the accuracy of *S*-*N* curve is high.

The third is due to the precision of $Y(a)$. $Y(a)$ is a complicated function with certain error. So, the calculated a_f will have some error.

5 Discussions

(1) Since the *S*-*N* curves are generally measured with constant amplitude load, the presented method can be applied to structures under constant

amplitude stress.

(2) *S*-*N* curve can also be expressed as follows

$$\sigma = A\left(1+\frac{C}{N_{\mathrm{f}}^{\alpha}}\right) \text{ or } (N_{\mathrm{f}}+B)(\sigma-A)^{m}=K \qquad (8)$$

where A, B, C, K, m and α are material constants.

In this case, before using the new method, it is needed to transfer Eq.(8) into Eq.(1).

(3) When there are significant errors for fitting the whole *S*-*N* curve with Eq.(1), only fitting local *S*-*N* curve may be applicable.

6 Conclusions

(1) Based on large amount of tested data and analysis, the effects of the different factors on IFQ of structure are analyzed and a new concept of MIFQ, which can be used widely, is presented.

(2) The relation between the crack propagation curve and *S*-*N* curve is studied. Based on this study, the new method of determining crack growth rate under constant amplitude loading is presented. Then, the corresponding approach to durability is developed.

(3) The reasons for the occurrence of the differences between the predicted results and tested results are analyzed. And the certain conservativeness of this new method is pointed out as well.

(4) The method can be used to analyze the durability of structures and evaluate the effects of different factors on durability under constant amplitude loading without additional experiments. So, it can be used widely and is convenient for application.

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Biography:

Yang Moucun Born in 1979, he received M.S. from Jiangsu University in 2004 and Ph.D. from Nanjing University of Aeronautics and Astronautics in 2007, and now is a teacher of Nanjing University of Technology. He has published several scientific papers in various periodicals. His main research interests include fatigue design and dynamics of structure.

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