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Matter and gravitons in the gravitational collapse

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ABSTRACT

We consider the effects of gravitons in the collapse of baryonic matter that forms a black hole. We first note that the effective number of (soft off-shell) gravitons that account for the (negative) Newtonian potential energy generated by the baryons is conserved and always in agreement with Bekenstein's area law of black holes. Moreover, their (positive) interaction energy reproduces the expected post-Newtonian correction and becomes of the order of the total ADM mass of the system when the size of the collapsing object approaches its gravitational radius. This result supports a scenario in which the gravitational collapse of regular baryonic matter produces a corpuscular black hole without central singularity, in which both gravitons and baryons are marginally bound and form a Bose–Einstein condensate at the critical point. The Hawking emission of baryons and gravitons is then described by the quantum depletion of the condensate and we show the two energy fluxes are comparable, albeit negligibly small on astrophysical scales.

1. Introduction

One of the main issues in gravity theory is to understand the formation of black holes from the gravitational collapse of compact objects. It is in fact a theorem in general relativity [1] that, provided the collapsing (massive) matter satisfies the weak energy condition and a trapping surface appears at some point, all the matter will eventually shrink into a space-time singularity (of infinite density). A simple and explicit example of this behaviour was illustrated long ago by Oppenheimer and Snyder [2]. However, such a singular final state of matter is clearly incompatible with the principles of quantum physics,[1] which immediately calls for a search of alternative end-points of the collapse within a fully quantum description of nature.

We consider here the gravitational collapse of a spherically symmetric object assuming the validity of the Hamiltonian constraint of general relativity, that is, of total energy conservation in Newtonian terms.[2] The only quantum gravity ingredient we shall employ is the description of the gravitational field in terms of "gravitons". In particular, we will include the effect of (negative energy) gravitons with a wavelength of the order of the size of the collapsing body in the total energy balance. These soft gravitons indeed appear in the quantum representation of the Newtonian potential by means of a coherent state coupled to the matter source [3,4], and one might speculate [5] that they could be associated with the recently advocated breaking of the BMS symmetry [6] precisely induced by the presence of localised matter. Our main result is that these (soft off-shell) gravitons satisfy Bekenstein's area law [7] and appear to produce the expected post-Newtonian correction [8] to the total energy of the system, which becomes a major contribution to the dynamics when the gravitational radius is approached. At that point, a black hole should form, mostly made of such soft gravitons (in a sense that will be clarified later on), in qualitative agreement with the corpuscular model of Refs. [9].

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[1] A similar conclusion follows from a quantum treatment of the gravitational radius of a quantum matter state [3].

[2] We should remark that in Newtonian physics the total energy of a system can be arbitrarily shifted by a constant, whereas in general relativity this operation is not allowed.

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2. Energy balance of self-gravitating objects

Let us consider a simple model for a compact stellar object made of N_B identical components, which we will call baryons for simplicity, of rest mass μ assembled in a spherically symmetric configuration of radius R. These baryons can interact gravitationally, and we assume their number does not depend on R [10]. We also neglect any emission of radiation, for simplicity, so that the total energy is conserved and always equals the Arnowitt–Deser–Misner (ADM) mass M of the system [11]. Energy conservation is granted in the Newtonian description of isolated systems, but let us also recall the same result holds in general relativity, where it is given by the Hamiltonian constraint associated with the freedom of time reparameterization. In an asymptotically flat space, the Hamiltonian constraint takes the form [11,12]

$$H \equiv H_B + H_G = M \, , \tag{1}$$

where H_B and H_G respectively denote the (super)-Hamiltonian of matter and gravity, obtained by varying the action with respect to the lapse function, and M emerges from boundary terms. For instance, one could consider configurations of a given constant $R = R_s$ as representing stable stars (for which the Hamiltonian constraint leads to the Tolman–Oppenheimer–Volkov equation), or let R reduce all the way down to form a black hole of size equal to the Schwarzschild gravitational radius[3]

$$R_H = 2\,\ell_p \frac{M}{m_p} \, . \tag{2}$$

If we ideally think of preparing the system when the N_B baryons are very far apart, the total energy is simply given by the baryonic rest mass,

$$H = E_B \equiv \mu\, N_B \simeq M \, . \tag{3}$$

Subsequently, as the radius R shrinks, the baryon energy will be decreased by the (negative) interaction potential energy U_{BG} between baryons mediated by the gravitons, and acquire kinetic energy K_B, so that

$$E_B(R) = M + K_B(R) + U_{BG}(R) + U_{BB}(R) \, , \tag{4}$$

where we explicitly wrote the dependence of all terms on the typical size R of the system, and $U_{BB} \geq 0$ is an additional (repulsive) interaction among baryons, which provides the pressure required for a static configuration at a given finite value of $R = R_s$ (corresponding to which we will have $K_B(R_s) = 0$). In a purely classical model, this would equal the total energy of the system, the ADM mass M, and one would thus find the classical equation of motion

$$K_B(R) + U_{BG}(R) + U_{BB}(R) = 0 \, . \tag{5}$$

But baryons are quantum, and a consistent description requires we also consider quantum features of the gravitational interaction. For this purpose, let us start from a simple estimate of the baryon total gravitational potential energy for a static configuration obtained from Newtonian physics, that is

$$U_{BG}(R) \simeq N_B\,\mu\,\phi_N(R) \simeq -N_B\,\mu\,\frac{\ell_p M}{m_p R} = -\frac{M^2\,\ell_p}{m_p R} \, , \tag{6}$$

where the classical Newtonian field ϕ_N satisfies the Poisson equation

$$\Delta\,\phi_N = \ell_p \frac{M}{m_p}\, j(r) \, , \tag{7}$$

with a static source profile such that $\int_0^R r^2\,dr\,j(r) = 1$. In the quantum theory, this field can be described by means of a coherent state of (virtual) gravitons [3,4], like a coherent state of (virtual) photons reproduces the Coulomb field around a static charge [4]. This can be easily seen from the momentum space form of Eq. (7),

$$k^2\,\phi_N(k) = -\frac{M}{m_p}\, j(k) \, , \tag{8}$$

where k is the dimensionless wave number, and expanding the graviton field operator in the corresponding radial modes, $\hat{\phi}_k \simeq (\hat{g}_k + \hat{g}^\dagger_{-k})/\sqrt{k}$. A coherent state is an eigenstate of the annihilation operators,

$$\hat{g}_k \mid g \,\rangle = g(k) \mid g \,\rangle \, . \tag{9}$$

In particular, we can choose

$$g(k) \simeq -\frac{M\, j(k)}{m_p\, k^{3/2}} \, , \tag{10}$$

which precisely reproduces the classical field,

$$\langle\, g \mid \hat{\phi}_k \mid g \,\rangle \simeq -\frac{M\, j(k)}{m_p\, k^2} \simeq \phi_N(k) \, . \tag{11}$$

The expectation value of the graviton number is now well-approximated by

$$N_G = \int k^2\,dk\,\langle\, g \mid \hat{g}^\dagger_k\,\hat{g}_k \mid g\,\rangle \simeq \frac{M^2}{m_p^2}\int dk\,\frac{j^2(k)}{k} \simeq \frac{M^2}{m_p^2} \sim \frac{R_H^2}{\ell_p^2} \, , \tag{12}$$

which is essentially Bekenstein's area law [7], but holds regardless of the actual size R of the matter source.

It is worth noting that, since M is constant in our approximation, the number N_G of gravitons is also conserved, like N_B. If we further write

$$U_{BG}(R) \simeq N_G\,\epsilon_G(R) \, , \tag{13}$$

we immediately conclude the typical graviton energy is in fact given by [3,4]

$$\epsilon_G \simeq -\frac{\ell_p}{R}\, m_p \, , \tag{14}$$

which is extremely small for a macroscopic source, but increases (in modulus) for decreasing R. The above relation tells us that ϵ_G is determined by the typical length of the quantum coherent state according to the de Broglie relation, but is of course negative. This feature is in agreement with gravity contributing to the general relativistic Hamiltonian constraint with a sign opposite to that of matter, and the non-relativistic view of the negative Newtonian energy. It also signals that the gravitons of a static potential are off-shell from the quantum field theory point of view.

Since gravitons self-interact, we add the graviton interaction energy,

$$U_{GG}(R) \simeq N_G\,\epsilon_G(R)\,\phi_N(R) \simeq N_G\,\frac{M\,\ell_p^2}{R^2} \, , \tag{15}$$

which we note is positive and falls off with the size R of the source like $1/R^2$, two properties that characterise a typical post-Newtonian correction to the Newtonian potential [8]. Since

[3] We shall always write the Newton constant $G_N = \ell_p/m_p$ and the Planck constant $\hbar = \ell_p\, m_p$.

$$\left|\frac{U_{\rm GG}}{U_{\rm BG}}\right| \simeq \frac{R_{\rm H}}{R} \ll 1 \ , \tag{16}$$

this contribution is overall very small for a large star, but becomes crucial when $R \simeq R_{\rm H}$.

Finally, the complete Hamiltonian constraint (1) reads

$$\begin{aligned} M &= E_{\rm B} + U_{\rm GG} \\ &= M + K_{\rm B}(R) + U_{\rm BB}(R) + U_{\rm BG}(R) + U_{\rm GG}(R) \ , \end{aligned} \tag{17}$$

which should hold for any physically acceptable value of the size R of the system.

2.1. Black hole configuration

Let us now consider the case that the system can contract all the way down to $R \simeq R_{\rm H}$. At that point we find the interesting relation

$$U_{\rm GG}(R_{\rm H}) \simeq -U_{\rm BG}(R_{\rm H}) \simeq M \ , \tag{18}$$

which we can view precisely as the marginal bound condition for gravitons of Refs. [9]. It can also be recast in the form of the critical condition for a Bose–Einstein condensate of gravitons [9],

$$\alpha\, N_{\rm G} \simeq 1 \ , \tag{19}$$

where $\alpha \simeq \epsilon_{\rm G}^2/m_{\rm p}^2$ is the gravitational coupling for graviton–graviton scattering. Beside Eq. (12), which holds for any R and can be written as a scaling relation for the horizon radius,

$$R_{\rm H} \simeq \sqrt{N_{\rm G}}\,\ell_{\rm p} \ , \tag{20}$$

in the limit $R \simeq R_{\rm H}$ one also recovers the scaling relation for the effective graviton mass [9]

$$m = -\epsilon_{\rm G} \simeq \frac{m_{\rm p}}{\sqrt{N_{\rm G}}} \simeq \frac{M}{N_{\rm G}} \ . \tag{21}$$

The Hamiltonian constraint in the black hole configuration now yields

$$K_{\rm B} + U_{\rm BB} \simeq 0 \ . \tag{22}$$

However, since we reasonably assumed $U_{\rm BB} \geq 0$, the only possible solution allowing for the black hole formation is

$$K_{\rm B}(R_{\rm H}) \simeq U_{\rm BB}(R_{\rm H}) \simeq 0 \ , \tag{23}$$

which one can analogously view as the marginal bound condition for the baryons.

Indeed one could have considered the Oppenheimer–Snyder model of collapsing dust [2] from the beginning, so that $U_{\rm BB} = 0$ for all values of R, with a constant number $N_{\rm G}$ of soft gravitons. In this case $K_{\rm B}(R_{\rm H}) \simeq 0$ and the (quantum) matter stops collapsing. This of course represents a huge correction to the classical model in which the system ends into the singularity at $R = 0$. If we took the above Eq. (23) at face value, we could actually say that, since any kind of matter has $U_{\rm BB} > 0$ and $K_{\rm B} \geq 0$, the configuration $R \simeq R_{\rm H}$ should not even be reached. However, such a strong conclusion definitely requires a better analysis of all the terms in the Hamiltonian constraint (1) for $R \simeq R_{\rm H}$. In fact, we never explicitly considered the spatial distribution of the baryons, consequently the variable R could merely represents the typical (quantum) size of the collapsing object, rather then a sharp (classical) radius. The simple estimates (6) and (15) could then be improved by employing a better approximation for the potential of a self-gravitating system (for example, the harmonic [13] or Pöschl–Teller potential [14]) and one could also include effective quantum field theory corrections [15]. We leave such improvements for future investigations, and just remark that the effective number of soft gravitons in the Newtonian potential and in the black hole is much larger than the number of baryons. For example, let us consider a solar mass black hole ($M = M_\odot \sim 10^{38}\, m_{\rm p}$) made of neutrons ($\mu \sim 10^{-19}\, m_{\rm p}$). The number of neutrons in the system is $N_{\rm B} = M/\mu \sim 10^{57}$, whereas the number of gravitons $N_{\rm G} \simeq \mu\, N_{\rm B}^2/m_{\rm p} \sim 10^{95} \gg N_{\rm B}$, which is again consistent with the underling hypothesis of the corpuscular model for black holes.

2.2. Quantum depletion of gravitons and baryons

In the above description, we neglected any possible emission of gravitons or baryons, but the black hole just formed should radiate according to Hawking's law [16]. In the corpuscular model, this effect is reproduced by the quantum depletion occurring because of the graviton–graviton scatterings [9]. We shall here consider the added contribution of graviton–baryon scatterings to the emission of gravitons, and the baryon–graviton scattering for the emission of baryons [17].

Because of the $N_{\rm B}$ baryons, the depletion law discussed in Refs. [9] will become

$$\begin{aligned} \dot N_{\rm G} &\simeq -N_{\rm G}^2\,\frac{1}{N_{\rm G}^2}\,\frac{1}{\ell_{\rm p}\sqrt{N_{\rm G}}} - N_{\rm G}\,N_{\rm B}\,\frac{1}{N_{\rm G}^2}\,\frac{1}{\ell_{\rm p}\sqrt{N_{\rm G}}} \\ &\simeq -\frac{1}{\ell_{\rm p}\sqrt{N_{\rm G}}}\left(1+\frac{N_{\rm B}}{N_{\rm G}}\right) \ , \end{aligned} \tag{24}$$

where in each of the two terms in the r.h.s. of the first line, the first factor accounts for the graviton and baryon multiplicity, the second factor is the gravitational coupling α^2 and the third factor comes from the typical energy m of the process. Also baryons will be scattered out of the collapsed object [17], according to

$$\begin{aligned} \dot N_{\rm B} &\simeq -N_{\rm G}\,N_{\rm B}\,\frac{1}{N_{\rm G}^2}\,\frac{1}{\ell_{\rm p}\sqrt{N_{\rm G}}} \\ &\simeq -\frac{N_{\rm B}}{N_{\rm G}}\,\frac{1}{\ell_{\rm p}\sqrt{N_{\rm G}}} \ , \end{aligned} \tag{25}$$

where we assumed the typical baryon energy will again be given by the typical graviton's energy, as predicted by Hawking. It is then important to notice that baryons with such a small energy can be emitted because Eq. (23) holds after the black hole is formed. Looking at the above emission rates, although $\dot N_{\rm G} \gg \dot N_{\rm B}$, it is clear that the corresponding energy fluxes are of the same magnitude.

$$m\,\dot N_{\rm G} \simeq \frac{M}{\ell_{\rm p}\,N_{\rm G}^{3/2}} \simeq \mu\,\dot N_{\rm B} \ . \tag{26}$$

This implies that both are indeed practically irrelevant for an astrophysical object with $M \gg m_{\rm p}$ (and thus $N_{\rm G} \gg N_{\rm B} \gg 1$), like one already expected from the standard expression of the Hawking effect.

3. Concluding remarks

We have shown, in rather general terms, that including the effect of soft gravitons in the description of the gravitational collapse of a compact object naturally leads to the expected post-Newtonian correction to the energy of the system and to the possible formation of a corpuscular black hole mostly made of gravitons. By this statement we precisely mean that the number of gravitons $N_{\rm G} \gg N_{\rm B}$ and their typical effective mass $m = -\epsilon_{\rm G}$ is such that $M \simeq N_{\rm G}\, m$ [9]. In astrophysical situations, where the Hawking radiation, here described as a depletion effect, is negligible, this state should represent the end-point of the collapse, with no central singularity at all.

Of course, our conclusion could be further refined by considering improved approximations for the various energy terms that appear in the Hamiltonian constraint, like we commented previously. Moreover, we did not solve for any specific dynamics, and it is thus possible that different energy terms will appear in a fully time-dependent analysis. Even if all the terms remained of the same functional form, numerical coefficients might very likely differ from those we employed, and this is the main reason we did not show many exact equalities in the present analysis. Of course, it would be extremely interesting to derive the macroscopic dynamics from the microscopic (quantum field theory) description of (corpuscular) black hole formation from graviton–graviton (and graviton–baryon) scatterings [18–20]. All that said, our findings still suggest to consider the possibility that the end-point of the gravitational collapse, or physical black holes, are quite different objects from those described by classical general relativity, and their quantum properties might therefore differ from the usual ones obtained from quantum field theory on classical black hole backgrounds.

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