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Asymptotic limits of a discrete Kinetic Theory model of vehicular traffic

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ABSTRACT

The paper proposes a rigorous method to construct the hyperbolic asymptotic limit of the discrete Kinetic Theory model of vehicular traffic proposed in [8]. A second-order macroscopic model of the Payne–Whitham type is derived and the coefficients of the equations are obtained from the detailed description of the microscopic interactions developed in the kinetic model.

1. Introduction

Modelling of vehicles' traffic flow can be developed at different "scales of observation"; see review papers [1–3]. One is the *microscopic scale*, where the physical state of each vehicle is described by deriving the so-called microscopic "Follow-the-Leader" models. The other is the statistical or *mesoscopic scale*, where all the vehicles are seen as a system described through terms and tools of the statistical mechanics or Kinetic Theory, and where the evolution equations are derived by taking into account the microscopic interactions of each vehicle. In the *macroscopic scale*, analogously to the description of fluid mechanics, the evolution of the system of vehicles is derived under appropriate conservation assumptions of suitable averaged quantities. First-order macroscopic models consist of one conservation equation, corresponding to the density of vehicles, while second-order macroscopic models are obtained by adding a second evolution equation related to the vehicles' mean velocity.

Each representation scale has strongness and weakness in describing the evolution of the system and the problem of bridging different observation scales and linking them is an interesting question for the applications in traffic flow modelling which allows us to overcome some weak points that are specific to every modelling approach. Still few papers are devoted to this aim, among them, [4–6] are focused on the derivation of macroscopic equations from a kinetic model, [7] on the derivation of microscopic equations from a macroscopic modelling, and [8] on the derivation of macroscopic equations from a microscopic description.

This paper proposes a rigorous method to construct a hyperbolic asymptotic limit of the equations of the kinetic traffic flow model proposed in [9]. The reference model is a discrete velocity mathematical model for vehicular traffic where the discretization of the velocity variable takes into account the intrinsic granular nature of the flow of vehicles; moreover, interactions among vehicles are non-local and distributed on a visibility length, yielding to velocity transitions ruled by a density-dependent table of games. The proposed method allow us to recover the macroscopic description from the mesoscopic one; anyhow, due to an approximation (necessary in the proof) of the source term with a BGK term describing a relaxation toward equilibrium, e.g. [10], the resulting macroscopic model is different from the original kinetic model and

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describes an evolution dynamics different for any of the three models with discrete velocity proposed in the literature: [11,12,9].

Specifically, also due to the above mentioned approximation of the source term, the resulting system is analogous to the second-order Payne–Whitham model, [13,14], nevertheless the coefficients in the equations are obtained from the detailed description of the microscopic interactions of the reference kinetic model, in contrast to a fully macroscopic approach where they are phenomenologically assumed.

We have to recall that the second-order models of traffic flow of Payne–Whitham type has been seriously criticized by Daganzo in [15] and that Aw and Rascle proposed a model in [16] which overcome some of the highlighted problems. Nevertheless, in standard situations, see [17], there are no relevant differences between the Payne–Whitham model and the Aw–Rascle and the comparison between the two models is still an issue of reflection, [18]. Specifically, paper [18] focuses on the Daganzo's critics that characteristics waves faster than the vehicle speeds, as those of the Payne–Whitham model, are not realistic. The Aw and Rascle model overcomes this point having no characteristic waves both greater than the macroscopic vehicle speed. A detailed analysis on the linear instability of both models allows to conclude in [18] characteristic speeds faster that the average speed do not generate any theoretical inconsistency and thus it is not really necessary to overcome this critic. This conjecture is also supported by microscopic computer simulations where followers may accelerate (or decelerate) before their leaders. Moreover, as recalled in [8], the Payne–Whitham model can be derived as a macroscopic approximation of the microscopic "Optimal Velocity Model" that is validated by numerical simulations well in agree with empirical evidence and experimental data.

In details, the paper is organized in two sections which follow this introduction: Section 2 briefly summarizes the reference mathematical model and the equilibrium results in the spatially homogeneous case. Section 3 deals with the derivation of macroscopic equations for vehicles density and mean velocity from the kinetic equations through an hyperbolic asymptotic limit.

2. The reference model

In this section, we briefly recall from [9], the mathematical framework with the assumptions leading to the reference discrete kinetic model and the analysis of the spatially homogeneous problem. The evolution in time and space of traffic flow is described by introducing a distribution function over the independent variables (time, position and velocity), which is expressed in a dimensionless form and normalized in the interval [0, 1]. The velocity variable is discretized, introducing in $D_v = [0,\ 1]$ an equally spaced velocity grid $I_v = \{v_i\}_{i=1}^n$ corresponding to n velocity classes.

The classical macroscopic quantities are computed as moments of the distribution function. The *vehicles number density*:

$$\rho(t,x) = \sum_{i=1}^{n} f_i(t,x), \tag{2.1}$$

and the *vehicles number flux*:

$$q(t,x) = \sum_{i=1}^{n} v_i f_i(t,x), \tag{2.2}$$

are considered as dimensionless variables related to some characteristic values, that are identified as the maximum density allowed on the road according to the road capacity according to bumper to bumper conditions, and the corresponding maximum flux.

The class of mathematical models considered in [9] is a system of integro-differential equations with hyperbolic linear advection term which writes as follows:

$$\frac{\partial f_i}{\partial t} + v_i \frac{\partial f_i}{\partial x} = J_i^+[\mathbf{f}], \tag{2.3}$$

with:

$$\begin{aligned} J_i^+[\mathbf{f}] = {} & \sum_{h=1}^{n}\sum_{k=1}^{n} \int_x^{x+\zeta} \eta[\mathbf{f}](t,y) A_{hk}^i[\mathbf{f}](t,y) f_h(t,x) f_k(t,y) w(x,\ y)\, \mathrm{d}y \\ & - f_i(t,x) \sum_{h=1}^{n} \int_x^{x+\zeta} \eta[\mathbf{f}](t,y) f_h(t,y) w(x,\ y)\, \mathrm{d}y, \end{aligned} \tag{2.4}$$

where $\mathbf{f} = (f_1, \ldots, f_n)$ and $i = 1, \ldots, n$, and where interactions between vehicles are considered at the microscopic level defining, by phenomenological assumptions, the terms in Eq. (2.4). Specifically, the model assumes that:

(i) $\eta[\mathbf{f}]$ is the *interaction rate*, which gives the number of interactions per unit time among the vehicles and depends on the macroscopic conditions of traffic through the density ρ.

(ii) $A^i_{hk}[\mathbf{f}]$ defines the so-called *table of games*, which models the interactions at the microscopic scale among the vehicles by giving the probability that a vehicle with speed v_h adjusts its velocity to v_i after an interaction with a vehicle travelling at speed v_k. Two key ingredients are taken into account: the macroscopic local conditions of traffic by the density ρ and the quality of the road conditions by the parameter α.
(iii) Interactions are distributed over a dimensionless characteristic length $\zeta > 0$, which is interpreted as the *visibility length* of the drivers: a vehicle located at a point $x \in D_x$ is supposed to be affected, in average, by all vehicles comprised within a certain *visibility zone*, that in this context is identified with the interval $J_\zeta(x) = [x, x + \zeta]$.
(iv) $w(x, y)$ represents the function weighting the interactions over the above mentioned visibility zone.

The properties of the spatially homogeneous problem are analyzed in details in [9]. In the spatially homogeneous case, the distribution function $\mathbf{f}$ is independent of the variable x so that $f_i = f_i(t)$ and the system (2.3) reduces to:

$$\frac{\mathrm{d}f_i}{\mathrm{d}t} = J_i[\mathbf{f}], \tag{2.5}$$

with

$$J_i[\mathbf{f}] = \sum_{h=1}^{n}\sum_{k=1}^{n} \eta[\rho] A^i_{hk}[\rho] f_h f_k - f_i \sum_{h=1}^{n} \eta[\rho] f_h, \quad i = 1, \ldots, n, \tag{2.6}$$

where the density (2.1), due to the conservation of the number of the cars in the road, reduces to the constant value assumed at the initial time: $\rho = \rho(0)$.

The initial value problem is formalized linking the system of ordinary differential equations (2.5) in the unknowns $f_i : \mathbf{R}_+ \to \mathbf{R}_+$ to a suitable set of initial conditions:

$$f_i(0) = f_i^0 \in \mathbf{R}_+, \quad i = 1, \ldots, n. \tag{2.7}$$

The homogeneous stationary equation is: $J_i[\mathbf{f}] = 0$.

The existence and uniqueness of a local in time solution to problems (2.5)–(2.7) is proved applying Banach Fixed Point Theorem, and the existence of at least one equilibrium point is proved applying the Schauder Fixed Point Theorem. Moreover, for low dimensional cases ($n = 2$, $n = 3$ corresponding to 2 and 3 velocity classes), it is shown the existence and uniqueness of an internal equilibria, i.e. solutions such that more than one component of the distribution function $\mathbf{f}$ are different from zero. The evidence of an equilibrium distribution function $f_i^e(\rho)$ obtained as stationary solution of the spatially homogeneous case is confirmed by numerical simulations also for higher numbers of velocity classes, and it is exploited in the next section as a closure relation to derive the macroscopic equations. Moreover, the spatially homogeneous operator, (2.6) and the nonhomogeneous one, (2.4) will be used in next section to split contributions in the collision operators.

3. Derivation of macroscopic models—asymptotic limit

In this section, following some ideas outlined in [5,6], we present a rigorous method to derive the macroscopic equations for density and mean velocity from the kinetic equations, (2.3). Other equations for higher-order moments can be derived in a similar way. Coherently to the finite velocity of propagation of traffic waves, the method develops an hyperbolic asymptotic limit. Without any loss of generality, calculations to derive balance equations are developed setting equal to 1 the parameter α related to the quality of the road.

Multiplying the nonhomogeneous kinetic equation with $\phi(v_i) = 1$ and $\phi(v_i) = v_i$ and summing with respect to v_i, we obtain the following balance equations (for sake of brevity, here and in the following, we will omit $i = 1, \ldots, n$):

$$\frac{\partial}{\partial t}\sum_{i=1}^{n}(\phi(v_i) f_i(t,x)) + \frac{\partial}{\partial x}\sum_{i=1}^{n}(v_i \phi(v_i) f_i(t,x)) = \sum_{i=1}^{n}\phi(v_i) J_i^+(\mathbf{f})(t,x). \tag{3.1}$$

Recalling the expression of the macroscopic quantities ρ, (2.1), and $q = \rho u$, (2.2), we get from (3.1) with $\phi(v_i) = 1$, the continuity equation:

$$\frac{\partial \rho}{\partial t}(t,x) + \frac{\partial(\rho u)}{\partial x}(t,x) = 0. \tag{3.2}$$

To obtain the momentum equation, it is important to distinguish the flux due to local and distributed interactions. Thus, we split the collision operator as follows:

$$J_i^+(t,x) = J_i - (J_i - J_i^+), \tag{3.3}$$

where J is the spatially homogeneous operator, (2.6) and J^+ is the nonhomogeneous one, (2.4). Therefore, the momentum equation writes as follows:

$$\frac{\partial \rho u}{\partial t}(t,x) + \frac{\partial(P + \rho u^2)}{\partial x}(t,x) + E(t,x) = S(t,x), \tag{3.4}$$

where

$$P(t,x)=\sum_{i=1}^{n}(v_i-u(t,x))^2 f_i(t,x), \tag{3.5}$$

$$E(t,x)=\sum_{i=1}^{n} v_i\left(J_i[\mathbf{f}](t,x)-J_i^{+}[\mathbf{f}](t,x)\right), \tag{3.6}$$

and

$$S(t,x)=\sum_{i=1}^{n} v_i J_i[\mathbf{f}](t,x). \tag{3.7}$$

To obtain closed equations for ρ, (3.2) and u (3.4), we have to specify the dependence of P, E, and S on ρ and u. An explicit closure can be obtained from the solution of the stationary homogeneous case $f_i^e(\rho)$, analyzed in the previous section. Bearing in mind that we perform a closure of a nonhomogeneous problem with an equilibrium solution obtained in an homogeneous situation, now we consider a *local density* $\rho(t,x)$ which depends on time and space. Thus:

$$f_i^e(\rho)(t,x)=f_i^e(\rho(t,x)) \tag{3.8}$$

and the *equilibrium quantities* are defined as a consequence: the mean velocity writes:

$$u_e(\rho)=\frac{1}{\rho}\sum_{i=1}^{n} v_i f_i^e(\rho(t,x)), \tag{3.9}$$

and the traffic pressure writes:

$$P^e(\rho(t,x))=\sum_{i=1}^{n}[v_i-u^e(\rho(t,x))]^2 f_i^e(\rho(t,x)). \tag{3.10}$$

The *source term* is approximated as in a BGK model with a relaxation toward equilibrium, see e.g. [10]:

Assumption 3.1. The term S, defined in (3.7), using a relaxation-time approximation writes:

$$S\sim\nu\Big(\sum_{i=1}^{n} v_i(f_i-f_i^e)\Big)=\nu(\rho u-\rho u_e(\rho)). \tag{3.11}$$

Under some assumption the non-local terms (3.5)–(3.7), can be simplified as follows:

Lemma 3.1. *Let E be the operator given in (3.6), then we have the following:*

$$\begin{aligned}E(t,x)\sim\Bigg[&-\sum_{i=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{n} v_i f_h(t,x)\Bigg(\eta'[\rho]A_{hk}^i[\rho]+\eta[\rho](A_{hk}^i[\rho])'\frac{\partial\rho}{\partial x}f_k(t,x)\\&+\eta[\rho](t,x)A_{hk}^i[\rho](t,x)\frac{\partial f_k(t,x)}{\partial x}\Bigg)\\&+\sum_{i=1}^{n}\sum_{h=1}^{n} v_i f_i(t,x)\left(\eta'[\rho]f_h(t,x)\frac{\partial\rho}{\partial x}+\eta[\rho]\frac{\partial f_h(t,x)}{\partial x}\right)\Bigg]\int_0^{\zeta} r w(x,r+x)\mathrm{d}r,\end{aligned} \tag{3.12}$$

and at the equilibrium as $\zeta\longrightarrow 0$ we have:

$$E\sim\left(b_1^e(\rho)+b_2^e(\rho)(\rho u_e(\rho))\right)\frac{\partial\rho}{\partial x}, \tag{3.13}$$

where

$$b_1^e(\rho)=-\sum_{i=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{n} v_i f_h^e(\rho)\left((\eta'[\rho]A_{hk}^i[\rho]+\eta[\rho](A_{hk}^i[\rho])')f_k^e(\rho)+\eta[\rho]A_{hk}^i[\rho]\frac{\partial f_k^e}{\partial\rho}\right)\int_0^{\zeta} r w(x,r+x)\mathrm{d}r, \tag{3.14}$$

and

$$b_2^e(\rho)=\sum_{h=1}^{n}\left(\eta'[\rho]f_h^e(\rho)+\eta[\rho]\frac{\partial f_h^e}{\partial\rho}\right)\int_0^{\zeta} r w(x,r+x)\mathrm{d}r. \tag{3.15}$$

Proof. Using (3.6), then E can be written in the following form:

$$\begin{aligned}
E(t,x) &= \sum_{i=1}^{n} v_i \left(J_i[\mathbf{f}](t,x) - J_i^{+}[\mathbf{f}](t,x)\right) \\
&= \sum_{i=1}^{n} v_i \Bigg[\sum_{h=1}^{n}\sum_{k=1}^{n} \eta[\rho](t,x)A_{hk}^{i}[\rho](t,x)f_h(t,x)f_k(t,x) - f_i(t,x)\sum_{h=1}^{n}\eta[\rho](t,x)f_h(t,x) \\
&\quad - \sum_{h=1}^{n}\sum_{k=1}^{n}\int_0^{\zeta} \eta[\rho](t,r+x)A_{hk}^{i}[\rho](t,r+x)f_h(t,x)f_k(t,r+x)w(x,r+x)\mathrm{d}r \\
&\quad + f_i(t,x)\sum_{h=1}^{n}\int_0^{\zeta}\eta[\rho](t,r+x)f_h(t,r+x)w(x,r+x)\mathrm{d}r \Bigg] \\
&= \sum_{i=1}^{n} v_i \Bigg[\sum_{h=1}^{n}\sum_{k=1}^{n} f_h(t,x)\int_0^{\zeta}\Big(\eta[\rho](t,x)A_{hk}^{i}[\rho](t,x)f_k(t,x) \\
&\quad - \eta[\rho](t,r+x)A_{hk}^{i}[\rho](t,r+x)f_k(t,r+x)\Big)w(x,r+x)\mathrm{d}r \Bigg] \\
&\quad + \sum_{i=1}^{n} v_i f_i(t,x)\sum_{h=1}^{n}\int_0^{\zeta}\Big(\eta[\rho](t,r+x)f_h(t,r+x) - \eta[\rho](t,x)f_h(t,x)\Big)w(x,r+x)\mathrm{d}r \\
&= I[\mathbf{f}] + J[\mathbf{f}],
\end{aligned}$$

where $I[\mathbf{f}]$ is given by:

$$\begin{aligned}
I[\mathbf{f}] &= \sum_{i=1}^{n} v_i \Bigg[\sum_{h=1}^{n}\sum_{k=1}^{n} f_h(t,x)\int_0^{\zeta}\Big(\eta[\rho](t,x)A_{hk}^{i}[\rho](t,x)f_k(t,x) \\
&\quad - \eta[\rho](t,r+x)A_{hk}^{i}[\rho](t,r+x)f_k(t,r+x)\Big)w(x,r+x)\mathrm{d}r \Bigg].
\end{aligned}$$

Using

$$\begin{aligned}
&\eta[\rho](t,x)A_{hk}^{i}[\rho](t,x)f_k(t,x) - \eta[\rho](t,r+x)A_{hk}^{i}[\rho](t,r+x)f_k(t,r+x) \\
&\quad \sim -r\frac{\partial(\eta[\rho](t,x)A_{hk}^{i}[\rho](t,x)f_k(t,x))}{\partial x},
\end{aligned}$$

we get for $I[\mathbf{f}]$

$$\begin{aligned}
I[\mathbf{f}] &\sim -\sum_{i=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{n} v_i f_h(t,x)\frac{\partial(\eta[\rho](t,x)A_{hk}^{i}[\rho](t,x)f_k(t,x))}{\partial x}\int_0^{\zeta} r w(x,r+x)\mathrm{d}r \\
&\sim -\sum_{i=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{n} v_i f_h(t,x)\Bigg(\eta'[\rho]A_{hk}^{i}[\rho] + \eta[\rho](A_{hk}^{i}[\rho])'\frac{\partial\rho}{\partial x}f_k(t,x) \\
&\quad + \eta[\rho](t,x)A_{hk}^{i}[\rho](t,x)\frac{\partial f_k(t,x)}{\partial x}\Bigg)\int_0^{\zeta} r w(x,r+x)\mathrm{d}r.
\end{aligned} \tag{3.16}$$

In the same way, we can compute $J[\mathbf{f}]$, obtaining the following expression:

$$\begin{aligned}
J[\mathbf{f}] &= \sum_{i=1}^{n} v_i f_i(t,x)\sum_{h=1}^{n}\int_0^{\zeta}\Big(\eta[\rho](t,r+x)f_h(t,r+x) - \eta[\rho](t,x)f_h(t,x)\Big)w(x,r+x)\mathrm{d}r \\
&\sim \sum_{i=1}^{n} v_i f_i(t,x)\sum_{h=1}^{n}\left(\eta'[\rho]f_h(t,x)\frac{\partial\rho}{\partial x} + \eta[\rho]\frac{\partial f_h(t,x)}{\partial x}\right)\int_0^{\zeta} r w(x,r+x)\mathrm{d}r.
\end{aligned} \tag{3.17}$$

Then, by using (3.16) and (3.17), we get (3.12).
Using (3.12), and that at the equilibrium the solution $f_i \sim f_i^{e}(\rho(t,x))$, we get (3.13). This completes the proof. □

The momentum equation, using (3.4), (3.11) and (3.13), writes:

$$\frac{\partial \rho u}{\partial t}(t,x) + \frac{\partial (P_e + \rho u^2)}{\partial x}(t,x) + \left(b_1^e(\rho) + b_2^e(\rho)(\rho u_e(\rho)) \right) \frac{\partial \rho}{\partial x} = \nu(\rho u - \rho u_e(\rho)). \tag{3.18}$$

By developing the momentum equation (3.18), and using the continuity equation (3.2), we get the equation satisfied by the local velocity:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial \rho}{\partial x}\left[\frac{\partial P_e}{\partial \rho} + b_1^e(\rho) + b_2^e(\rho)(\rho u_e(\rho)) \right] = \nu(u - u_e(\rho)). \tag{3.19}$$

The final evolution equations correspond to the second-order Payne–Whitham model, [13,14]:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial (\rho u)}{\partial x} = 0, \\ \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + \dfrac{a(\rho)}{\rho}\dfrac{\partial \rho}{\partial x} = \nu(u - u_e(\rho)), \end{cases} \tag{3.20}$$

with the so-called anticipation term

$$a(\rho) = \left[\frac{\partial P_e}{\partial \rho} + b_1^e(\rho) + b_2^e(\rho)(\rho u_e(\rho)) \right], \tag{3.21}$$

where the coefficients are computed explicitly above, (3.10), (3.14) and (3.15) and they are derived from the detailed microscopic modelling of microscopic interactions proposed in [9]. In order to compute explicitly these coefficients, we focus on specific models with fixed number of velocity classes considering, in particular, two velocity classes, $n = 2$ with two extreme (the worst and the best) road conditions. In this case, the non-null terms of the game table of the reference kinetic model [9] are: $A_{11}^2 = A_{12}^2 = A_{21}^2 = \alpha(1-\rho)$, $A_{22}^2 = 1 - \alpha\rho$.

•*Worst road conditions*. Passing is not allowed, and $\alpha = 0$. The equilibrium configuration is $f^e = (\rho, 0)$. Easy calculations allow to compute, from (3.10) and (3.14): $P_e = u_e(\rho) = b_1^e(\rho) = 0$, and, from (3.15), $b_2^e(\rho) = \frac{\zeta}{2(1-\rho)^2}$, and thus, the evolution equations write:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial (\rho u)}{\partial x} = 0, \\ \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} = \nu u. \end{cases} \tag{3.22}$$

•*Best road conditions*. The parameter $\alpha = 1$, and the equilibrium configuration is $f^e = (\rho^2, \rho - \rho^2)$. Easy calculations allow to compute, as before, the terms $P_e = \rho^2(1-\rho)$, $u_e(\rho) = 1-\rho$, $b_1^e(\rho) = \frac{\zeta\rho}{2}$, $b_2^e(\rho) = \frac{\zeta}{2(1-\rho)^2}$, and thus, the evolution equations write:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial (\rho u)}{\partial x} = 0, \\ \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + \left(2 - 3\rho + \dfrac{\zeta}{2}\dfrac{\rho}{1-\rho} \right)\dfrac{\partial \rho}{\partial x} = \nu[u - (1-\rho)]. \end{cases} \tag{3.23}$$

In conclusion, the proposed method allows to derive from the kinetic equations, Eqs. (2.3), the macroscopic equations, Eqs. (3.20), whose coefficients are specified considering different cases of interest, e.g. Eqs. (3.22) and (3.23). This method may be seen as a first step toward a consistent multiscale representation of traffic phenomena, which allows to shift from a model at a more detailed representation scale, as the mesoscopic one, to a model at a rougher representation scale, as the macroscopic one, according to the different physical situations, and depending on the needs of the analysis of the phenomena. This "adaptive" approach, where the representation scale is refined only when it is necessary, shows some advantages as for instance from a computational viewpoint for faster data screening.

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