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Inflation parameters from Gauss–Bonnet braneworld

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Abstract

We calculate the spectral index and tensor-to-scalar ratio for patch inflation arisen from the Gauss–Bonnet braneworld scenario. The patch cosmological models consist of Gauss–Bonnet (GB), Randall–Sundrum (RS), and 4D general relativistic (GR) cases. In order to compare with the observation data, we perform leading-order calculations for all patch models by choosing large-field, small-field, and hybrid potentials. We show that the large-field potentials are sensitive to a given patch model, while the small-field and hybrid potentials are insensitive to a given patch model. It is easier to discriminates between quadratic potential and quartic potential in the GB model rather than RS and GR models. Irrespective of patch models, it turns out that the small-field potentials are the promising models in view of the observation.

1. Introduction

There has been much interest in the phenomenon of localization of gravity proposed by Randall and Sundrum (RS) [1]. They assumed a positive three-brane embedded in the 5D anti-de-Sitter (AdS_5) spacetime. They obtained a localized gravity on the brane by fine-tuning a brane tension to a bulk cosmological constant. Recently, several authors have studied cosmological implications of the braneworld scenarios. We wish to mention that the brane cosmology contains some important deviations from the Friedmann–Robertson–Walker (FRW) cosmology [2,3].

On the other hand, it is generally accepted that curvature perturbations produced during inflation are considered to be the origin of inhomogeneities necessary for explaining cosmic microwave background (CMB) anisotropies and large-scale structures. The WMAP [4], SDSS [5,6], and other data put forward more constraints on cosmological models. These show that an emerging standard model of cosmology is the ΛCDM model. Further, these results coincide with theoretical predictions of the slow-roll inflation based on general relativity with a single inflaton. The latest data [6] shows a nearly scale-invariant spectrum with the spectral index $n_s = 0.98^{+0.02}_{-0.02}$, no evidence of the

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tensor-to-scalar ratio with $R < 0.36$, and no evidence of the running spectral index with $\alpha_s \simeq 0$.

If the brane inflation occurs, one expects that it provides us quite different results in the high-energy region [7–14]. Since the Gauss–Bonnet term modifies the Friedmann equation at high-energy significantly, its application to the brane inflation has been studied widely in the literature [15–21].

In this Letter, the patch cosmological models induced from the Gauss–Bonnet braneworld are introduced to study the brane inflation for large-field, small-field, and hybrid potentials. We use mainly the leading-order spectral index and tensor-to-scalar ratio to select which patch model with q is suitable for explaining the latest observation data.

The organization of this Letter is as follows. In Section 2 we briefly review the patch cosmology arisen from the Gauss–Bonnet braneworld scenario, and introduce relevant inflation parameters as observables. We introduce various potentials to compute their theoretical values and compare these with the observation data in Section 3. Finally we discuss our results in Section 4.

2. Patch cosmological models

We start with an effective Friedmann equation arisen from the Gauss–Bonnet brane cosmology by adopting a flat FRW metric as the background spacetime on the brane[1] [11,15,17]

$$H^2 = \beta_q^2 \rho^q, \tag{1}$$

where $H = \dot{a}/a$, q is a patch parameter labelling different models and β_q^2 is a factor with energy dimension $[\beta_q] = E^{1-2q}$. An additional parameter $\theta = 2(1 - 1/q)$ is introduced for convenience. We call the above defined on the q-dependent energy region as "patch cosmology". We summarize three different models and their parameters in Table 1. $\kappa_5^2 = 8\pi/m_5^3$ is the 5D gravitational coupling constant and $\kappa_4^2 = 8\pi/m_{\rm Pl}^2$ is the 4D gravitational coupling constant. $\alpha = 1/8g_s$ is the Gauss–Bonnet coupling with the string energy scale g_s and λ is the brane tension. Relationships between these are given by $\kappa_4^2/\kappa_5^2 = \tilde{\mu}/(1+\beta)$ and $\lambda = 2\tilde{\mu}(3-\beta)/\kappa_5^2$, where $\beta = 4\alpha\tilde{\mu}^2 \ll 1$, $\tilde{\mu} = 1/\ell$ with AdS_5 curvature radius ℓ. The RS case of $\tilde{\mu} = \kappa_4^2/\kappa_5^2$ is recovered when $\beta = 0$. We have to distinguish between GB ($\beta \ll 1$, but $\beta \neq 0$ exactly) and RS ($\beta = 0$) cases.

Table 1
Three relevant models and their parameters classifying patch cosmological models

Model	q (θ)	β_q^2
GB	2/3 (−1)	$(\kappa_5^2/16\alpha)^{2/3}$
RS	2 (1)	$\kappa_4^2/6\lambda$
GR	1 (0)	$\kappa_4^2/3$

Before we proceed, we note that the Gauss–Bonnet braneworld affects inflation only when the Hubble parameter is larger than the AdS curvature scale ($H \gg \tilde{\mu}$). As a result, we have the two patch models of GB case with $q = 2/3$ and RS case with $q = 2$ case. For the other case of $H \ll \tilde{\mu}$, one recovers the 4D general relativistic (GR) case with $q = 1$. Furthermore, we assume that all of AdS curvature scale $\tilde{\mu}$, Gauss–Bonnet coupling α, and brane tension λ are stable, even if the vacuum energy on the brane is so large in the high-energy regions that $H \gg \tilde{\mu}$.

Let us introduce an inflaton ϕ whose equation is given by

$$\ddot{\phi} + 3H\dot{\phi} = -V', \tag{2}$$

where dot and prime denote the derivative with respect to time t and ϕ, respectively. The energy density and pressure are given by $\rho = \dot{\phi}^2/2 + V$ and $p = \dot{\phi}^2/2 - V$. From now on, we use the slow-roll formalism for inflation: an accelerating universe ($\ddot{a} > 0$) is being derived by an inflaton slowly rolling down its potential toward a local minimum. Then Eqs. (1) and

[1] For reference, here we add the action for the Gauss–Bonnet braneworld scenario:

$$S = \frac{1}{2\kappa_5^2}\int_{\rm bulk} d^5x\,\sqrt{-g_5}\,\big[R - 2\Lambda_5 + \alpha\big(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\big)\big] + \int_{\rm brane} d^4x\,\sqrt{-g}\,[-\lambda + \mathcal{L}_{\rm matter}]$$

with $\Lambda_5 = -3\tilde{\mu}^2(2-\beta)$ for an AdS_5 bulk and $\mathcal{L}_{\rm matter}$ for inflation. Its exact Friedmann equation is given by a complicated form: $2\tilde{\mu}(1 + H^2/\tilde{\mu}^2)^{1/2}[3 - \beta + 3\beta H^2/\tilde{\mu}^2] = \kappa_5^2(\rho + \lambda)$.

(2) take the following form approximately:

$$H^2 \approx \beta_q^2 V^q, \qquad \dot\phi \approx -\frac{V'}{3H}, \tag{3}$$

which are the background equations for patch cosmological models. It implies that the cosmological acceleration can be derived by a fluid with a vacuum-like equation of state $p \approx -\rho$. If $p = -\rho$ for $\dot\phi = 0$, this corresponds to a de Sitter inflation with $a(t) = a_0 e^{Ht}$. This is a model to obtain gravitational waves from the braneworld scenario [22]. In order for inflation to terminate and for the universe to transfer to a radiation-dominated universe, we need a slow-roll mechanism. To this end, we introduce Hubble slow-roll parameters (ϵ_1, δ_n) and potential slow-roll parameters $(\epsilon_1^q, \delta_n^q)$ as

$$\epsilon_1 \equiv -\frac{\dot H}{H^2} \approx \epsilon_1^q \equiv \frac{q}{6\beta_q^2}\frac{(V')^2}{V^{1+q}},$$
$$\delta_n \equiv \frac{1}{H^n\dot\phi}\frac{d^{n+1}\phi}{dt^{n+1}} \approx \delta_n^q, \tag{4}$$

with[2]

$$\delta_1^q = \frac{1}{3\beta_q^2}\left[\frac{(V')^2}{V^{1+q}} - \frac{V''}{V^q}\right],$$
$$\delta_2^q = \frac{1}{(3\beta_q^2)^2}\left[\frac{V'''V'}{V^{2q}} + \frac{(V'')^2}{V^{2q}} - \frac{5q}{2}\frac{V''(V')^2}{V^{2q+1}} + \frac{q(q+2)}{2}\frac{(V')^4}{V^{2q+2}}\right]. \tag{5}$$

Here the subscript denotes slow-roll- (SR-) order in the slow-roll expansion. A slow-roll parameter of $\epsilon_1^q \geqslant 0$ governs the equation of states $p = \omega_q \rho$ with $\omega_q = -1 + 2\epsilon_1^q/3q$, which implies that an accelerating expansion occurs only for $\epsilon_1^q < 1$ ($\omega_q < -1 + 2/3q$) [23]. $\epsilon_1^q = 0$ ($\omega_q = -1$) corresponds to de Sitter inflation. On the other hand, the end of inflation is determined by $\epsilon_1^q = 1$ ($\omega_q = -1 + 2/3q$). Hence, the allowed regions for inflation are different: $-1 \leqslant \omega < -1/3$ for GR models, $-1 \leqslant \omega < 0$ for GB model, and $-1 \leqslant \omega < -2/3$ for RS model. If one chooses the inflation potential V, then potential slow-roll parameters $(\epsilon_1^q, \delta_n^q)$ will be determined explicitly.

We describe how inflation parameters can be calculated using the slow-roll formalism. Introducing a variable $u^q = a(\delta\phi^q - \dot\phi\psi^q/H)$ where $\delta\phi^q$ is a perturbed inflaton and ψ^q is a perturbed metric, its Fourier modes u_k^q in the perturbation theory satisfy the Mukhanov equation [24]:

$$\frac{d^2 u_k^q}{d\tau^2} + \left(k^2 - \frac{1}{z_q}\frac{d^2 z_q}{d\tau^2}\right)u_k^q = 0, \tag{6}$$

where τ is a conformal time defined by $d\tau = dt/a$, and $z_q = a\dot\phi/H$ encodes all information about a slow-roll inflation for a patch model with q. Asymptotic solutions are obtained as

$$u_k^q \to \begin{cases} \frac{1}{\sqrt{2k}} e^{-ik\tau} & \text{as } -k\tau \to \infty, \\ C_k^q z_q & \text{as } -k\tau \to 0. \end{cases} \tag{7}$$

The first solution corresponds to a plane wave on scale much smaller than the Hubble horizon of $d_H = 1/H$ (sub-horizon region), while the second is a growing mode on scale much larger than the Hubble horizon (super-horizon region). We consider a relation of $R_{c\mathbf{k}}^q = -u_{\mathbf{k}}^q/z_q$ together with $u_{\mathbf{k}}^q(\tau) = a_{\mathbf{k}} u_k^q(\tau) + a_{-\mathbf{k}}^\dagger u_k^{q*}(\tau)$. Using a definition of the power spectrum

$$P_{R_c}^q(k)\delta^{(3)}(\mathbf{k}-\mathbf{l}) = \frac{k^3}{2\pi^2}\left\langle R_{c\mathbf{k}}^q(\tau) R_{c\mathbf{l}}^{q\dagger}(\tau)\right\rangle,$$

one finds the power spectrum for curvature perturbations in the super-horizon region

$$P_{R_c}^q(k) = \left(\frac{k^3}{2\pi^2}\right)\lim_{-k\tau\to 0}\left|\frac{u_k^q}{z_q}\right|^2 = \frac{k^3}{2\pi^2}\left|C_k^q\right|^2. \tag{8}$$

Our next work is to find unknown coefficients C_k^q by solving the Mukhanov equation (6). In general, it is not easy to find a solution to this equation directly. Fortunately, we can solve it using either the slow-roll approximation [25,26] or the slow-roll expansion [27,28]. We find the q-power spectrum to the leading-order [18]

$$P_{R_c}^q = \frac{3q\beta_q^{2-\theta}}{(2\pi)^2}\frac{H^{2+\theta}}{2\epsilon_1} \to \frac{1}{(2\pi)^2}\frac{H^4}{\dot\phi^2}, \tag{9}$$

where the right-hand side should be evaluated at horizon crossing of $k = aH$. Using $d\ln k \simeq H dt$, the q-spectral index defined as

$$n_s^q(k) = 1 + \frac{d\ln P_{R_c}^q}{d\ln k} \tag{10}$$

[2] For another notation, we use $\delta_1^q = q\epsilon_1^q/2 - \eta_1^q$ with $\eta_1^q \equiv \frac{1}{3\beta_q^2}\frac{V''}{V^q}$.

is given by

$$n_s^q(k) = 1 - 4\epsilon_1^q - 2\delta_1^q. \quad (11)$$

The q-running spectral index is determined, to the leading-order, by

$$\frac{d}{d\ln k} n_s^q = -\frac{8(\epsilon_1^q)^2}{q} - 10\epsilon_1^q\delta_1^q + 2(\delta_1^q)^2 - 2\delta_2^q. \quad (12)$$

The tensor-to-scalar ratio R_q is defined by

$$R_q = 16\frac{A_{T,q}^2}{A_{S,q}^2}, \quad (13)$$

where the q-scalar amplitude is normalized by

$$A_{S,q}^2 = \frac{4}{25} P_{R_c}^q. \quad (14)$$

The GR ($q = 1$) tensor amplitude is given by

$$A_{T,\mathrm{GR}}^2 = \frac{1}{50} P_{T,\mathrm{GR}}, \quad (15)$$

where $P_{T,\mathrm{GR}} = (2\kappa_4)^2 (H/2\pi)^2$ because a tensor can be expressed in terms of two scalars like $\delta\phi$. Tensor spectra for GB and RS are known only for de Sitter brane with $p = -\rho$ [15,22]. It implies that tensor calculation should be limited to the leading-order computation. These are given by

$$A_{T,q}^2 = A_{T,\mathrm{GR}}^2 F_\beta^2(H/\tilde{\mu}), \quad (16)$$

where

$$F_\beta^{-2}(x) = \sqrt{1+x^2} - \left(\frac{1-\beta}{1+\beta}\right) x^2 \sinh^{-1}\left(\frac{1}{x}\right). \quad (17)$$

In three different regimes, we approximate F_β^2 as F_q^2: $F_1^2 \approx F_\beta^2(H/\tilde{\mu} \ll 1) = 1$ for GR model; $F_2^2 \approx F_{\beta=0}^2(H/\tilde{\mu} \gg 1) = 3H/(2\tilde{\mu})$ for RS model; $F_{2/3}^2 \approx F_\beta^2(H/\tilde{\mu} \gg 1) = (1+\beta)H/(2\beta\tilde{\mu})$ for GB model. The tensor amplitude up to leading-order is given by

$$A_{T,q}^2 = \frac{3q\beta_q^{2-\theta}}{(5\pi)^2} \frac{H^{2+\theta}}{2\zeta_q} \quad (18)$$

with $\zeta_1 = \zeta_{2/3} = 1$ and $\zeta_2 = 2/3$ [16]. Finally, the tensor-to-scalar ratio is determined by

$$R_q = 16\frac{A_{T,q}^2}{A_{S,q}^2} = 16\frac{\epsilon_1}{\zeta_q} \quad (19)$$

in the patch cosmological models. Considering a relation for tensor spectral index $n_T^q = -(2+\theta)\epsilon_1$, one finds the consistency relations

$$R_1 = -8n_T^1 = 16\epsilon_1, \qquad R_2 = -8n_T^2 = 24\epsilon_1,$$
$$R_{2/3} = -16n_T^{2/3} = 16\epsilon_1. \quad (20)$$

The above shows that the RS consistency relation is equal to that for GR case, but it is different from that for GB case.

3. Inflation with potentials

A generic single-field potential can be characterized by two energy scales: a height of potential V_0 corresponding to the vacuum energy density during inflation and a width of the potential μ corresponding to the change of an inflaton $\Delta\phi$ during inflation. In general its form is given by $V = V_0 f(\phi/\mu)$. Different potentials have different f-forms. The height V_0 is usually fixed by normalization and thus the free parameter is just the width μ. We classify potentials into three cases: large-field, small-field, hybrid potentials.

3.1. *Large-field potentials*

It was shown that the quartic potential of $V = V_0\phi^4$ is under strong observation pressure (ruled out observationally) for GR and RS (GB) models, while the quadratic potential of $V = V_0\phi^2$ is inside of the 1σ-bound for GR and GB models with the range of e-folding number $50 \leqslant N \leqslant 60$. This is obtained from the likelihood analysis based on the leading-order calculations to n_s^q and R_q with patch cosmological models [19]. Here we choose the large-field potentials of $V^{LF} = V_0\phi^p$ with $p = 2, 4, 6$ for testing these with the patch cosmology. In this case potential slow-roll parameters are determined by

$$\epsilon_1^q = \frac{qp}{2} \frac{1}{[(q-1)p+2]N + \frac{qp}{2}}, \quad (21)$$

$$\delta_1^q = \frac{1}{2} \frac{(2-2p+qp)}{[(q-1)p+2]N + \frac{qp}{2}}. \quad (22)$$

Substituting these into Eqs. (11) and (19), one finds two inflation parameters. For a full computation of inflation parameters, see Ref. [20]. The LF-spectral index is given by

$$n_s^{\rm LF} = 1 - \frac{(3q-2)p+2}{[(q-1)p+2]N + \frac{qp}{2}} \tag{23}$$

in the leading-order calculation. The LF tensor-to-scalar ratio takes the form

$$R_q^{\rm LF} = \frac{8qp}{\zeta_q}\frac{1}{[(q-1)p+2]N + \frac{qp}{2}}. \tag{24}$$

Fortunately, there is no free parameter for large-field models. The numerical results [5,20] for large-field potentials are shown in Table 2.

3.2. Small-field potentials

In this subsection we choose the small-field potentials of $V^{\rm SM} = V_0[1-(\phi/\mu)^p]$ with $p = 2, 4, 6$. Here μ plays a role of the free parameter. For convenience, we treat $p = 2$ and $p > 2$ cases separately. For the $p = 2$ case, potential slow-roll parameters are determined by

$$\epsilon_1^q = \frac{q}{2}\left(\frac{\phi_f}{\mu}\right)^2 x_p^q e^{-x_p^q N}, \tag{25}$$

$$\eta_1^q = -\frac{x_p^q}{2}\left\{1 + q\left(\frac{\phi_f}{\mu}\right)^2 e^{-x_p^q N}\right\}, \tag{26}$$

with a dimensionless parameter

$$x_p^q \equiv \frac{2p}{3\beta_q^2\mu^2 V_0^{q-1}} \tag{27}$$

whose form is given explicitly by

$$x_p^{\rm GR} = \frac{p}{4\pi}\left(\frac{m_{\rm Pl}}{\mu}\right)^2, \qquad x_p^{\rm RS} = \frac{p}{2\pi}\left(\frac{m_{\rm Pl}}{\mu}\right)^2\frac{\lambda}{V_0},$$
$$x_p^{\rm GB} = \frac{2p}{3(2\pi)^{2/3}}\left(\frac{m_{\rm Pl}^{2/3}}{\mu}\right)^2\left(\frac{V_0\beta^2}{\tilde{\mu}^2}\right)^{1/3}. \tag{28}$$

Here we find useful inequalities: $x_p^{\rm GR} \ll 1$ for $\mu \gg m_{\rm Pl}$, $x_p^{\rm GR} \gg 1$ for $\mu \ll m_{\rm Pl}$; $x_p^{\rm RS} \ll 1$ for $\lambda/V_0 \to 0$, $x_p^{\rm RS} \gg 1$ for $\mu \ll m_{\rm Pl}$; $x_p^{\rm GB} \ll 1$ for $\beta \to 0$, $x_p^{\rm GB} \gg 1$ for $\mu \ll m_{\rm Pl}$. This means that the RS model is obtained from the RS braneworld in high-energy region, while the GB model is mainly determined from the Gauss–Bonnet term in the braneworld. Also the RS and GB models recover a result of the GR model in the low-energy limit of $m_{\rm Pl} \gg \mu$. From a relation for the number of e-folding: $N \simeq -3\beta_q^2\int_\phi^{\phi_f}(V^q/V')d\phi$, one finds a relation, $\phi = \phi_f e^{-x_p^q N/2}$. The SF-spectral index is given by

$$n_s^{\rm SF} = 1 - 6\epsilon_1^q + 2\eta_1^q$$
$$= 1 - x_p^q\left[1 + 4q\left(\frac{\phi_f}{\mu}\right)^2 e^{-x_p^q N}\right] \tag{29}$$

in the leading-order calculation. The SF tensor-to-scalar ratio is

$$R_q^{\rm SF} = \frac{16\epsilon_p^q}{\zeta_q} = \frac{8q}{\zeta_q}\left(\frac{\phi_f}{\mu}\right)^2 x_p^q e^{-x_p^q N}. \tag{30}$$

We determine ϕ_f from a condition of the end of inflation: $\epsilon_1^q(\phi_f) = 1$. For $p = 2$ case, one obtains a condition of $x_p^q < 1$ from $n_s^{\rm SF} < 1$. In the case of $r_p^q = qpx_p^q/4 \ll 1$, it provides $\phi_f \simeq \mu/(1+q)^{1/p}$ for numerical computation.

In the case of $p > 2$, we have different slow-roll parameters

$$\epsilon_1^q = \frac{qpx_p^q}{4\left[\left(\frac{\phi_f}{\mu}\right)^{2-p} + \frac{p-2}{2}x_p^q N\right]^{\frac{2(p-1)}{p-2}}}, \tag{31}$$

$$\eta_1^q = -\frac{(p-1)x_p^q}{2\left[\left(\frac{\phi_f}{\mu}\right)^{2-p} + \frac{p-2}{2}x_p^q N\right]^{\frac{2(p-1)}{p-2}}} - \frac{2(p-1)}{p}\epsilon_1^q. \tag{32}$$

A relation between ϕ and ϕ_f is given by $(\phi/\mu)^{2-p} = (\phi_f/\mu)^{2-p} + (p-2)x_p^q N/2$. In general, the SF-spectral index is given by

$$n_s^{\rm SF} = 1 - \frac{(p-1)x_p^q}{\left(\frac{\phi_f}{\mu}\right)^{2-p} + \frac{p-2}{2}x_p^q N} - \frac{5p-2}{8p}\zeta_q R_q^{\rm SF}, \tag{33}$$

where the SF tensor-to-scalar ratio is

$$R_q^{\rm SF} = \frac{4qpx_p^q}{\zeta_q\left[\left(\frac{\phi_f}{\mu}\right)^{2-p} + \frac{p-2}{2}x_p^q N\right]^{\frac{2(p-1)}{p-2}}}. \tag{34}$$

Also we get ϕ_f from a condition of the end of inflation: $\epsilon_1^q(\phi_f) = 1$. However, there is no constraint on x_p^q. In the case of $r_p^q(x_p^q) \ll 1$, we have $\phi_f \simeq \mu/(1+q)^{1/p}$, while for $r_p^q(x_p^q) \gg 1$, we find a connection

$$\phi_f \simeq \frac{\mu}{(r_p^q)^{1/2(p-1)}} = \frac{\mu}{(qpx_p^q/4)^{1/2(p-1)}}.$$

For the case of $x_p^q \ll 1$, two inflation parameters are given by

$$n_s^{\rm SF} = 1 - \frac{(p-1)x_p^q}{(1+q)^{\frac{p-2}{p}} + \frac{p-2}{2}x_p^q N} - \frac{5p-2}{8p}\zeta_q R_q^{\rm SF}, \qquad (35)$$

$$R_q^{\rm SF} = \frac{4qpx_p^q}{\zeta_q\left[(1+q)^{\frac{p-2}{p}} + \frac{p-2}{2}x_p^q N\right]^{\frac{2(p-1)}{p-2}}}. \qquad (36)$$

On the other hand, for the case of $x_p^q \gg 1$, the spectral index is

$$n_s^{\rm SF} = 1 - \frac{(p-1)x_p^q}{\left(\frac{qpx_p^q}{4}\right)^{\frac{2-p}{2(1-p)}} + \frac{p-2}{2}x_p^q N} - \frac{5p-2}{8p}\zeta_q R_q^{\rm SF}, \qquad (37)$$

and the SF tensor-to-scalar ratio takes the form

$$R_q^{\rm SF} = \frac{4qpx_p^q}{\zeta_q\left[\left(\frac{qpx_p^q}{4}\right)^{\frac{2-p}{2(1-p)}} + \frac{p-2}{2}x_p^q N\right]^{\frac{2(p-1)}{p-2}}}. \qquad (38)$$

In the limit of $x_p^q \to \infty$, we obtain the low-energy limit of GR case from RS and GB models. Also, in the limit of $x_p^{\rm GR} \to \infty (m_{\rm Pl} \gg \mu)$, one finds a well-known general relativistic case. These all lead to the same expression given by

$$n_s^{\rm SF} = 1 - \frac{p-1}{p-2}\frac{2}{N}, \qquad (39)$$

which is independent of the patch parameter q. In the limit of $x_p^q \to \infty$, one finds an asymptotic behavior for the tensor-to-scalar ratio

$$R_q^{\rm SF} \sim \frac{1}{(x_p^q)^{p/(p-2)}} \to 0. \qquad (40)$$

On the other hand, for $x_p^q \gg 1$, there exist upper limits for R_p^q such that [29]:

$$R_q^{\rm SF} < \bar{R}_q^{\rm SF}, \qquad (41)$$

with $\bar{R}_q^{\rm SF} = R_q^{\rm SF}|_{x_p^q=\bar{x}_p^q}$. Here

$$\{\bar{x}_p^q\} = \left\{\frac{p}{4\pi}, \frac{p}{2\pi}\frac{\lambda}{V_0}, \frac{2p}{3(2\pi)^{2/3}}\left(\frac{V_0\beta^2}{\tilde{\mu}^2}\right)^{1/3}\right\}$$

is the x_p^q-value for $m_{\rm Pl} = \mu$ in Eq. (28). The numerical results for $x_p^q \gg 1$ [5] and those from graphical analysis for $x_p^q \ll 1$ [10] are summarized at the last column in Table 2.

Table 2
The spectral index (n_s) and tensor-to-scalar (R). Here we choose $N = 55$ to find theoretical values for the large-field potentials (LF) and bounds for small-field potentials (SF). For SF case, each patch model in high-energy region is recovered when $x_p^q \ll 1$, while their low-energy limits are recovered when $x_p^q \gg 1$. The patch cosmological model is allowed only for $x_p^q \ll 1$ because for $x_p^q \gg 1$, it degenerates GR case

Patch	p	LF	SF ($x_p^q \ll 1$)	SF ($x_p^q \gg 1$)
GB ($q = 2/3$)	2	$n_s = 0.97$	$n_s \leqslant 1$	N/A
		$R = 0.14$	$R \leqslant 0.04$	N/A
	4	$n_s = 0.95$	$n_s \leqslant 1$	$0.95 \leqslant n_s \leqslant 1$
		$R = 0.56$	$R \leqslant 1.9 \times 10^{-3}$	$R \leqslant \bar{R}_{2/3}^4$
	6	N/A	$n_s \leqslant 1$	$0.96 \leqslant n_s \leqslant 1$
		N/A	$R \leqslant 6.0 \times 10^{-4}$	$R \leqslant \bar{R}_{2/3}^6$
GR ($q = 1$)	2	$n_s = 0.96$	$n_s \leqslant 1$	N/A
		$R = 0.14$	$R \leqslant 0.05$	N/A
	4	$n_s = 0.95$	$n_s \leqslant 1$	$0.95 \leqslant n_s \leqslant 1$
		$R = 0.29$	$R \leqslant 1.3 \times 10^{-3}$	$R \leqslant 9.5 \times 10^{-4}$
	6	$n_s = 0.93$	$n_s \leqslant 1$	$0.96 \leqslant n_s \leqslant 1$
		$R = 0.43$	$R \leqslant 3.0 \times 10^{-4}$	$R \leqslant 5.7 \times 10^{-4}$
RS ($q = 2$)	2	$n_s = 0.96$	$n_s \leqslant 1$	N/A
		$R = 0.22$	$R \leqslant 0.16$	N/A
	4	$n_s = 0.95$	$n_s \leqslant 1$	$0.95 \leqslant n_s \leqslant 1$
		$R = 0.29$	$R \leqslant 8.0 \times 10^{-4}$	$R \leqslant \bar{R}_2^4$
	6	$n_s = 0.94$	$n_s \leqslant 1$	$0.96 \leqslant n_s \leqslant 1$
		$R = 0.32$	$R \leqslant 1.0 \times 10^{-4}$	$R \leqslant \bar{R}_2^6$

3.3. Hybrid potentials

Finally we choose the hybrid-field potentials (HY) like $V^{\rm HY} = V_0[1 + (\phi/\mu)^p]$ with $p = 2, 4, 6$. In this case it requires an auxiliary field to end inflation. Here we separate $p = 2$ and $p > 2$ cases.

For the $p = 2$ case, the potential slow-roll parameters are determined by

$$\epsilon_1^q = \frac{q}{2}\left(\frac{\phi_f}{\mu}\right)^2 x_p^q e^{x_p^q N}, \qquad (42)$$

$$\eta_1^q = \frac{x_p^q}{2}\left\{1 - q\left(\frac{\phi_f}{\mu}\right)^2 e^{-x_p^q N}\right\}, \qquad (43)$$

with a dimensionless parameter $x_p^q = 2p/3\beta_q^2\mu^2 V_0^{q-1}$ defined in Eq. (28). From $N \simeq -3\beta_q^2 \int_\phi^{\phi_f}(V^q/V')\,d\phi$, one finds a relation, $\phi = \phi_f e^{x_p^q N/2}$ for $p = 2$. Here μ and ϕ_f are regarded as free parameters. The HY-spectral index is then given by

$$n_s^{\rm HY} = 1 - 6\epsilon_1^q + 2\eta_1^q = 1 + x_p^q\left[1 - 4q\left(\frac{\phi_f}{\mu}\right)^2 e^{x_p^q N}\right] \tag{44}$$

in the leading-order calculation. The HY tensor-to-scalar ratio is found to be

$$R_q^{\rm HY} = \frac{16\epsilon_p^q}{\zeta_q} = \frac{8qx_p^q}{\zeta_q}\left(\frac{\phi_f}{\mu}\right)^2 e^{x_p^q N}. \tag{45}$$

In order that Eqs. (44) and (45) be meaningful, we require a condition of $x_p^q < 1$. On the other hand, there is no way to determine ϕ_f from $\epsilon_1^q(\phi_f) = 1$ for HY case because ϕ_f is determined by other mechanism. Hence ϕ_f plays a role of the free parameter. Fortunately, $|(\phi/\mu)|^2 < 1$ is required because if $|(\phi/\mu)|^2 > 1$ in $V^{\rm HY}$, it is not much different from the large-field potentials. From Eq. (44), one finds a restrictive constraint $|(\phi/\mu)|^2 < \frac{1}{4q}$ which comes from the condition of $n_s^{\rm HY} > 1$.

In order to see a feature of the hybrid models, we need numerical results. In the case of $x_{p=2}^{\rm GR} = 0.04$ ($\mu = 2m_{\rm Pl}$), $\ln(\phi/\phi_f) = 1$ and $N = 50$, we have a blue spectral index $n_s^{\rm HY} \simeq 1 + x_p^q = 1.04$ but a small tensor-to-scalar ratio $R_{\rm GR}^{\rm HY} = 0.32(\phi/\mu)^2 < 0.08$.

In the case of $p > 2$, we have different slow-roll parameters

$$\epsilon_1^q = \frac{qpx_p^q}{4[(\frac{\phi_f}{\mu})^{2-p} - \frac{p-2}{2}x_p^q N]^{\frac{2(p-1)}{p-2}}}, \tag{46}$$

$$\eta_1^q = \frac{(p-1)x_p^q}{2[(\frac{\phi_f}{\mu})^{2-p} - \frac{p-2}{2}x_p^q N]^{\frac{2(p-1)}{p-2}}} - \frac{2(p-1)}{p}\epsilon_1^q. \tag{47}$$

In general, the HY-spectral index is given by

$$n_s^{\rm HY} = 1 + \frac{(p-1)x_p^q}{(\frac{\phi_f}{\mu})^{2-p} - \frac{p-2}{2}x_p^q N} - \frac{5p-2}{8p}\zeta_q R_q^{\rm HY}, \tag{48}$$

where the HY tensor-to-scalar ratio is

$$R_q^{\rm HY} = \frac{4qpx_p^q}{\zeta_q[(\frac{\phi_f}{\mu})^{2-p} - \frac{p-2}{2}x_p^q N]^{\frac{2(p-1)}{p-2}}}. \tag{49}$$

A HY-relation for $p > 2$ is given by $(\phi/\mu)^{2-p} = (\phi_f/\mu)^{2-p} - (p-2)x_p^q N/2 > 0$, which implies that an inequality of $N_{\rm max} > N$ exists with the definition of $N_{\rm max}$ as $(\phi_f/\mu)^{2-p} \equiv (p-2)x_p^q N_{\rm max}/2$.

Consequently, the HY-spectral index is given by

$$n_s^{\rm HY} = 1 + \frac{p-1}{p-2}\frac{2}{N_{\rm max} - N} - \frac{5p-2}{8p}\zeta_q R_q^{\rm HY}, \tag{50}$$

where the HY tensor-to-scalar ratio is

$$R_q^{\rm HY} = \frac{4qp}{\zeta_q (x_p^q)^{\frac{p}{p-2}}[\frac{(p-2)N_{\rm max}}{2}]^{\frac{2(p-1)}{p-2}}}\frac{1}{[1 - \frac{N}{N_{\rm max}}]^{\frac{2(p-1)}{p-2}}}. \tag{51}$$

4. Discussions

We introduce various potentials which are classified into large-field, small-field, and hybrid types for the ordinary inflation in the GR case. Using the patch cosmological models together with various potentials, we compute the two cosmological observables, spectral index and tensor-to-scalar ratio.

In large-field models without free parameter, the spectral index n_s and tensor-to-scalar ratio R depend on the e-folding number N only. Actually this simplicity provides strong constraints on large-field models. Further, combining the Gauss–Bonnet braneworld with large-field potentials provides more tighten constraints than the 4D general relativistic case. The GB case is regarded as the promising model for testing the large-field potentials because it accepts the quadratic potential. On the other hand, it rejects the quartic potential because theoretical points are far outside the 2σ-bound [19]. Actually, the GB cosmological model improves the theoretical values predicted by the GR model, whereas the RS model provides indistinctive values more than the GR case. As is shown in Table 2, the GB model splits large-field potentials into three distinct regions clearly: for $N = 55$, $n_s^{\rm GB} = 0.97 \to 0.95$ ($p = 2 \to p = 4$), $R_{\rm GB} = 0.14 \to 0.56$, and a power-law inflation with $p = 6$: $n_s^{\rm PI} = 1 - [(2r - 1)/2r^4]^{1/3}$, $R_{\rm PI} = 16/r$ [18]. Contrastively, we have $n_s^{\rm GR} = 0.96 \to 0.95 \to 0.93$ ($p = 2 \to p = 4 \to p = 6$), $R_{\rm GR} = 0.14 \to 0.29 \to 0.43$, whereas $n_s^{\rm RS} = 0.96 \to 0.95 \to 0.94$, $R_{\rm RS} = 0.22 \to 0.29 \to 0.32$. Theoretical points predicted by the RS model lie very close to the border between the regions allowed and disallowed by observation. Consequently, the large-field models depend on critically which model is used for calculation.

In small-field potentials, one has a free parameter $\mu(x_p^q)$ related to the potential shape. It is thus difficult to constrain inflation parameters, in compared to the large-field potentials. However, combining the graphical analysis with the data [10], we find useful bounds. For $x_p^q \ll 1$, there is no constraint on the lower-bound for the spectral index, but all of its upper bounds are given by 1. This means that patch cosmological model is not useful for testing small-field potentials. For $x_p^q \gg 1$ and $p > 2$, one finds lower-bounds for the spectral indices. Furthermore, there is no constraint on the spectral indices for $p = 2$ case. This implies that although n_s is independent of the patch parameter q, the small-field potentials are in good agreement with the observational data [5,6]. Combining the Gauss–Bonnet braneworld with small-field potentials, there exist unobserved differences in the upper-bound of the tensor-to-scalar ratio R.

Concerning the hybrid models, we find a blue spectral index with $n_s^{\mathrm{SF}} > 1$. However, there is no actual difference between patch cosmological model because one more free parameter is necessary to determine the end of inflation (ϕ_f), in addition to $\mu(x_p^q)$. It implies that the scheme of inflation (q) is less important than mechanism of the hybrid inflation (μ, ϕ_f). As a result, we do not find any new result when combining the Gauss–Bonnet braneworld with the hybrid potentials.

In conclusion, the GB model is still a promising one to discriminate between the quadratic and quartic potentials in the large-field type by making use of the observation data. Although the small-field potential are insensitive to patch cosmological models, these are considered as the promising potentials in view of the observational data. Finally, we do not find any new result when combining the Gauss–Bonnet braneworld with the hybrid potentials. This implies that it is not easy for hybrid type to compare with the data [5,6].

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