



Z_3 flavor symmetry and possible implications

M.J. Luo

Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China

ARTICLE INFO

Article history:

Received 25 December 2008

Accepted 12 January 2009

Available online 15 January 2009

Editor: A. Ringwald

ABSTRACT

We show in this Letter that the Z_3 flavor symmetry, which can successfully produces the tri-bimaximal mixing and flavor pattern of neutrino sector, has a possible explanation in the framework of gauge symmetry by constructing a wavefunction of flavor state particles with the help of the Wilson loop. In this implementation of Z_3 flavor symmetry, we suggest that the flavor charge in weak interaction can be interpreted as topological charge. Its possible implications and generalizations to the quark sector are also discussed.

© 2009 Elsevier B.V. Open access under CC BY license.

1. Introduction

The Standard Model describes almost all laboratory data with 28 free parameters, most of them arise from the flavor and mass parameters in the Yukawa coupling y_{ij} between fermions and Higgs boson H , the Lagrangian is

$$\mathcal{L}_{\text{Yukawa}} = y_{ij}^u \bar{u}_i q_j H + y_{ij}^d \bar{d}_i q_j H^* + y_{ij}^e \bar{e}_i l_j H^*, \quad (1)$$

where $q, l, (\bar{u}, \bar{d}, \bar{e})$ are left (right) handed quarks and leptons. Although 17 in 22 of the flavor parameters are measured [1], to understand these free parameters is a great challenge. The dominant approach is constructing flavor symmetry to reduce the number of free parameters, e.g. [2].

Beyond the Standard Model, neutrino oscillation experiments [3], give us strong evidences that the neutrino also have non-zero masses and non-trivial mixing between mass eigenstates and flavor states

$$\nu_\alpha = \sum_{a=1}^3 U_{\alpha a} \nu_a, \quad (2)$$

in which a denotes the mass eigenstate and α the flavor state, $U_{\alpha a}$ is the MNS matrix [4] that has the form of nearly tri-bimaximal [5].

The request of understanding the tri-bimaximal mixing demands a theory of flavor, and the neutrino masses and mixing matrix have inspired many model buildings, e.g. non-Abelian discrete flavor symmetries [6], GUT \times discrete group models [7], shift symmetries [8] etc., in which a heuristic model is the Abelian Z_3 flavor symmetry, e.g. [9].

The observational facts that the mixing between mass eigenstates and flavor states, as well as the universality of the flavor

states in weak interaction imply that the elementary excitations of weak process are non-Fock [10], and have non-trivial structures. As it is known that the Fock quantization is closely related to the particle interpretation of non-interacting QFT in which momentum, energy (mass) and spin of particle are good quantum numbers. However, the flavor state does not carry definite mass and spin as its quantum numbers, but definite flavor charge which is instead seem as good quantum number for weak interaction. Actually, the flavor states are the eigenstates of the weak interaction and the Haag's theorem [11] states that Fock state does not exist for interacting QFTs.

Such non-Fock degrees of freedom (DoF) may be crucial for understanding the mixing phenomenon in weak processes. We suggest that the Z_3 flavor symmetry model for neutrino mixing is very heuristic, since the Wilson loop operator has a natural implementation of Z_3 symmetry and can be used as a guidance to construct the non-Fock elementary excitations. In this Letter, we will give a possible explanation of Z_3 symmetry in the framework of gauge symmetry with the help of Wilson loop by introducing it to each particle wavefunction. Then according to the non-Fock wavefunction, the eigenvalues are not related to the mass and spin, but rather a winding number of Wilson loop, so we could give the flavor charge a possible interpretation.

The Letter is organized as follows. We review a simple Z_3 flavor symmetry for neutrino in Section 2, our implementation of the Z_3 flavor symmetry is in Section 3 by using the Wilson loop, we give a topological quantum number interpretation to the flavor charge. The generalization to the quark sector, which will reproduce the Froggatt-Nielsen's scenario for the Yukawa couplings, is discussed in Section 4.

2. Z_3 flavor symmetry and neutrino mixing

In this section, we review a simple and heuristic model of neutrino mixing based on the Z_3 flavor symmetry. Consider the Z_3

E-mail address: mjluo@mail.ustc.edu.cn.

elements $(\omega, \omega^2, 1)$, where $\omega = e^{\frac{2\pi i}{3}}$. We assume neutrinos are Majorana particles, and a general Lagrangian of Majorana mass term is

$$\mathcal{L} = y \bar{\nu}^c \phi \nu, \quad (3)$$

where y is a coupling constant, c stands for the charge conjugation $\nu^c = C \bar{\nu}^T$. Since Z_3 is a one-dimensional Abelian group, so the transformation is purely multiplying an imaginary phase, $\omega^i \in Z_3$. The Lagrangian is invariant under the Z_3 transformation

$$\nu_i \rightarrow \omega^i \nu_i, \quad \bar{\nu}_i^c \rightarrow \bar{\nu}_i^c \omega^i, \quad \phi_i \rightarrow \omega^i \phi_i, \quad (4)$$

where the index i from 1 to 3, three Higgs fields are introduced in the model. After the transformation, the coupling takes the form

$$y(\omega^i \omega^j \omega^k) \bar{\nu}_i^c \phi_j \nu_k, \quad (5)$$

where $\omega^i \omega^j \omega^k$ in the parentheses should be an invariant of Z_3 , so it leads to $i + j + k = 0 \pmod{3}$, which constrains the relations between i, j, k . Expanding it into matrix in flavor basis, the texture of coupling then has the form

$$y(\bar{\nu}_1^c \quad \bar{\nu}_2^c \quad \bar{\nu}_3^c) \begin{pmatrix} \phi_1 & \phi_3 & \phi_2 \\ \phi_3 & \phi_2 & \phi_1 \\ \phi_2 & \phi_1 & \phi_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \quad (6)$$

It is easy to verify that the mass matrix can be almost diagonalized by the tri-bimaximal mixing matrix

$$U_{\text{tb}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \quad (7)$$

We have

$$U_{\text{tb}}^\dagger \phi U_{\text{tb}} = \begin{pmatrix} \phi_1 - \frac{\phi_2}{2} - \frac{\phi_3}{2} & 0 & \frac{\sqrt{3}}{2}(\phi_3 - \phi_2) \\ 0 & \phi_1 + \phi_2 + \phi_3 & 0 \\ \frac{\sqrt{3}}{2}(\phi_3 - \phi_2) & 0 & -\phi_1 + \frac{\phi_2}{2} + \frac{\phi_3}{2} \end{pmatrix}, \quad (8)$$

which is diagonalized when $\phi_2 = \phi_3$. The neutrino mass matrix is obtained by developing VEV for ϕ_i . Because of the non-zero (1, 3) and (3, 1) elements, the matrix needs further diagonalization which will give a deviation from the tri-bimaximal matrix and leads to a non-vanishing θ_{13} . The deviation or, equivalently, the non-vanishing θ_{13} will depend on the magnitude of the non-zero (1, 3) and (3, 1) elements, i.e. the difference of the VEV of ϕ_2 and ϕ_3 . Here, the VEVs of Higgs fields $\langle \phi_i \rangle$ does not necessary to be the electroweak scale for the masses of neutrino are small, in some models they could be a tiny scale, e.g. in Higgs triplet model $\langle \phi_i \rangle \sim 10^{-3}$ eV.

In the following sections, we will implement the transformation properties Eq. (4) of the discrete Z_3 symmetry in the framework of continuous gauge symmetry and construct similar Z_3 invariants as Eq. (5) to be the Yukawa couplings.

3. Implementation of Z_3 symmetry and possible implications

Now in general we discuss the discrete Z_N symmetry from the continuous gauge $SU(N)$ symmetry. The possible implications of Z_N flavor symmetry will be a good guidance for us to find the non-Fock wavefunction of the flavor state.

As it is well known that the gauge transformation on a particle, whose operator is near the identity element of gauge group $G = SU(N)$ (we call it small gauge transformation), it transforms as a fundamental representation of G ,

$$\Psi \rightarrow e^{i\theta^a t^a} \Psi \simeq (1 + i\theta^a t^a) \Psi, \quad (9)$$

where t^a is the generator of G , θ^a is the parameter of transformation and Ψ is the wavefunction of the particle. A gauge potential

$A = e A_\mu^a t^a dx^\mu$ induces a finite phase to the particle when it passes along a path $l(x, x')$ from x to x' ,

$$\Psi(x') = e^{i \int_{l(x, x')} A} \Psi(x). \quad (10)$$

The gauge potential A transforms as

$$A \rightarrow A^g = g^{-1} A g + \frac{1}{i} g^{-1} dg = A + \frac{1}{i} g^{-1} Dg, \quad (11)$$

where $g \in G$ defines a mapping from the base manifold to the gauge group, $g(x) = e^{i\theta^a(x)t^a}$, i.e. $g(x): M \rightarrow G$, and D is the covariant derivative $D = d + iA$. If the mapping g is non-trivial, then an extra phase appears which related to the gauge transformation whose operator does not contain the identity element, in other words, the gauge transformation has non-zero ‘‘winding number’’ and we call it large gauge transformation. We have

$$\Psi(x') = e^{i \int_{l(x, x')} A} e^{i \int_{l(x, x')} g^{-1} Dg} \Psi(x), \quad (12)$$

it gives a decomposition of a small gauge perturbation and large gauge transformation. The former responds to the local phase for a Fock-like particle, while the latter phase needs an extra quantum number to describe which has no effect on the local phenomenon.

When we consider that the path is closed to be a loop γ_x at a base point x , the first phase factor tends to vanish as the closed loop shrinks to the point x , while the second one remains for the obstacle in the non-simply connected space,

$$\Psi(x')|_{x' \rightarrow x} \rightarrow e^{i \int_{\gamma_x} g^{-1} Dg} \Psi(x). \quad (13)$$

Obviously, the trace of phase factor, which does not rely on the choice of the basis of the gauge group, is an invariant function under gauge transformation and independent with the spacetime metric, so it is expected to be an observable and be part of the wavefunction. The general form of the physical phase is written as the so-called Wilson Loop [12]

$$W_\gamma[A^g] = \text{tr}(P e^{i \int_\gamma A^g}), \quad (14)$$

where P denotes the path ordering along γ .

When the mapping g is non-trivial for some topological obstacle exist, e.g. all scalar fields form Z_N -vortices, the relevant gauge group $G = SU(N)/Z_N$ is not simply connected,

$$\pi_1(SU(N)/Z_N) = Z_N, \quad (15)$$

for the mapping $g(x): S^1 \rightarrow SU(N)/Z_N$, then g becomes multivalued, consider the closed loop γ parametrized by an angle θ with $0 \leq \theta \leq 2\pi$, and $\gamma(2\pi) = \gamma(0)$, we have $g(2\pi) = e^{2\pi ni/N} g(0)$ with $0 \leq n < N$. We say that the field has a winding number n in such a configuration.

As a consequence, there does not exist a global section to be the wavefunction in the case that the bundle is topological non-trivial, and each section differs by an extra phase representing a large gauge transformation. So according to the decomposition Eq. (12), the wavefunction of particle can be represented as a trivial wavefunction $\Psi_0(x)$ multiplying by an extra phase exhibited by the non-contractible Wilson loop at the base point x which characterized its topological class. In general each topological class of wavefunction can be constructed as

$$\Psi(x) = W_{\gamma_x}[A] \Psi_0(x). \quad (16)$$

Under the gauge transformation Eq. (11), we note that

$$W_\gamma[A^g] = \text{tr} e^{i \int_\gamma g^{-1} Dg} W_\gamma[A] = e^{2\pi n(\gamma)/N} W_\gamma[A], \quad (17)$$

where $n(\gamma)$ is the number of times the loop γ winding around the obstacle, which is topological stable against small perturbations,

$$n_\gamma[A] = n_\gamma[A + \delta A]. \quad (18)$$

Even if the $SU(N)$ symmetry is broken completely, the phase factor still values on the residual center of $SU(N)$, the Z_N , so we assume in this Letter, only the Z_N DoF are relevant to the wavefunction, which is transformed as the representation of Z_N group,

$$\begin{aligned}\Psi(x) \rightarrow \Psi'(x) &= W_\gamma[A^g]\Psi_0(x) = e^{2\pi ni/N} W_\gamma[A]\Psi_0(x) \\ &= e^{2\pi ni/N}\Psi(x),\end{aligned}\quad (19)$$

where $e^{2\pi ni/N}$ is the element of Z_N .

It is direct to check that the wavefunction Eq. (16) is the eigenfunction of the Dirac operator $\not{D}(A^g) = \not{D} + iA^g$. As it is well known that the gauge transformation of $\not{D}(A)$,

$$g^{-1}(x)\not{D}(A)g(x) = \not{D}(A^g),\quad (20)$$

consequently, every eigenfunction Ψ of $\not{D}(A)$,

$$\not{D}(A)\Psi = \lambda\Psi,\quad (21)$$

has an associated gauge transformed eigenfunction $g\Psi$,

$$\not{D}(A^g)g\Psi = \lambda g\Psi,\quad (22)$$

so is $W[A^g]\Psi$,

$$\not{D}(A^g)W[A^g]\Psi = \lambda W[A^g]\Psi,\quad (23)$$

since $W[A^g]$ valued on the center of group element g , without losing generality, taking the pure gauge $A^g = g^{-1}dg$, we have

$$W_\gamma[A^g] = \text{tr}(e^{\oint_\gamma g^{-1}dg}) = \text{tr}(g(2\pi)g^\dagger(0)).\quad (24)$$

In summary, if non-contractible loops exist (i.e. when the $\pi_1(G) \neq 0$ due to topological obstacle), the wavefunction of elementary excitation can be constructed as a topological trivial wavefunction multiplied by a non-trivial Wilson loop that exhibits its extra topological quantum number. So in this case, we suggest possible connections between such extra topological DoF and the flavor DoF:

- (1) The wavefunction Eq. (16) is the eigenfunction of the Dirac operator, so it is the eigenstate of interaction similar with the flavor state, but rather the energy eigenstate. The eigenvalue $e^{2\pi Qi/N} = \omega^Q$ classifies the equivalence class of the wavefunction, where the winding number measures topological charges Q that similar with the flavor indices i of ω^i in Eq. (4).
- (2) The flavors have similar formal behavior with the topological DoF under Z_N symmetry Eq. (19), the Z_N transformation is interpreted as a large gauge transformation and has no effects on local process. The local gauge quantum numbers assigned by gauge group for different flavors are the same.
- (3) The flavor charge seems stable against local gauge interactions similar with the behavior of Eq. (18), local flavor changing processes such as Lepton Flavor Violation (LFV) and Flavor Changing Neutral Current (FCNC) are highly suppressed in our observation by far.
- (4) Note that if we reverse the orientation of the loop γ and exchange the representation of G with its complex conjugate, the definition of Wilson loop is unchanged, it is equivalent to take the charge conjugation of the wavefunction.

These are important formal properties of the flavor DoF and the topological DoF of the non-Fock wavefunction Eq. (16). The wavefunction will have non-trivial consequences when we dealing with the VEVs of the Wilson loops in which the connection A is seen as dynamic field operator. In the following discussing, we will base on the non-trivial wavefunction Eq. (16) and take $N = 3$.

4. The quark sector

In this section, we will generalize the previous discussion to quantum version by seeing the wavefunction Eq. (16) as a field operator. In the standard treatment of effective QFT, the high energy DoF will be integrated out and contribute to the effective low energy DoF, we will find that the VEVs of the Wilson loops in particle's wavefunction will play a crucial role in their effective couplings which give a natural realization of strong hierarchy of quark masses.

First we briefly reconsider the neutrino case. Assume that the wavefunction of neutrino is almost identified with the trivial wavefunction $\Psi \simeq \Psi_0$, that is $\langle W[A] \rangle \simeq 1$, the VEV of the transformed Wilson loop in the neutrino wavefunction is then purely an imaginary phase valued on Z_3 ,

$$\langle W[A] \rangle \rightarrow \langle W[A^g] \rangle = \omega^n \langle W[A] \rangle \simeq \omega^n \in Z_3,\quad (25)$$

in other words, the wavefunction of neutrinos are the eigenstates of the Wilson loop operators with eigenvalues ω^n . Hence the result is the same as Section 2. The Lagrangian takes the similar form of Eq. (6), only the Higgs fields develop non-vanished VEVs with the scale of, e.g. in Higgs triplet model, $\langle \phi_i \rangle \sim 10^{-3}$ eV.

However, when we consider the quark sector, the quantum expectation values of Wilson loops will not be trivial, if the local DoF of quarks are confined. Writing the Yukawa coupling terms in ordinary form, where the gauge potential in the Wilson loop operators should be integrated out and the VEVs of such loop operators then make contributions to the effective Yukawa couplings, they formally become the correlation function of these Wilson loop operators, which are observables that gauge invariant under $SU(3)/Z_3$. They require their VEVs and the Lagrangian of quark mass terms take the form,

$$\langle W_{q_{aL}}^\dagger W_{H_c} H_c W_{q_{bR}} \rangle \bar{q}_{aL} q_{bR},\quad (26)$$

where the subscript a, b, c of the Wilson loop are integers representing the corresponding winding number of the loop, or equivalently in this Letter, the flavor species, we write it as

$$W_{n_a} = \text{tr} \left(P \exp i \oint_{\gamma^{(n_a)}} A \right),\quad (27)$$

where n_a is the winding number. Taking the number $\langle H_c \rangle$ out of the bracket, we have the expectation value of the correlation function of these three Wilson loops, which can be calculated by the standard Feynman path-integral, formally it can be written as

$$\langle W_a^\dagger W_c W_b \rangle = Z^{-1} \int DA W_a^\dagger W_c W_b e^{-S[A]}.\quad (28)$$

The symbol DA represents Feynman's integral over all gauge orbits, that is, all equivalence classes of connections modulo gauge transformations, and $S[A]$ is the action of gauge theory in 4 dimension with gauge group $G = SU(3)/Z_3$. The approximate behaviors of Eq. (28) can be given as follows. We assume that the loop operators are almost independent, the expectation value can be decomposed as

$$\langle W_a^\dagger W_c W_b \rangle \simeq \langle W_a^\dagger \rangle \langle W_c \rangle \langle W_b \rangle e^{i\delta(\gamma_a, \gamma_c, \gamma_b)},\quad (29)$$

in which $e^{i\delta(\gamma_a, \gamma_c, \gamma_b)}$ is an observable angle depending on three Wilson loops. A crucial properties of $\langle W_\gamma \rangle$ is that if the DoF of quarks are confined it has area law [13],

$$\langle W_{n_\gamma} \rangle = \text{tr} \exp \left(- \oint_\gamma dx^\mu \oint_\gamma dy^\nu \langle A_\nu(y) A_\mu(x) \rangle \right) \simeq e^{-\sigma \mathcal{A}_\gamma},\quad (30)$$

where $\langle A_\nu(y) A_\mu(x) \rangle$ is the propagator, \mathcal{A}_γ is the area of surface whose boundary is the loop, it is approximate that the area, which

the flux goes through, increases with the winding number, i.e. $\mathcal{A}_\gamma \simeq n_\gamma \mathcal{A}$, and σ is a fixed constant.

It gives quark a strong mass hierarchy. Neglecting the complex phase we have the Yukawa couplings

$$y_{ab} \simeq g_H e^{-\sigma(n_a \mathcal{A}_{q_L} + n_b \mathcal{A}_{q_R})}, \quad (31)$$

where only one Higgs field is involved and the $\langle W_H \rangle$ contributes to the coupling g_H , the VEV of Higgs field is of the electroweak scale $\langle H \rangle \simeq 246$ GeV.

Comparing it with the form of the Yukawa couplings resulting from approximate Froggatt–Nielsen [2] Abelian $U(1)$ Flavor Symmetries, in which the general scheme is a small symmetry breaking factor for each quark field, $\epsilon_{q,\bar{u},\bar{d}} \simeq \frac{\langle \theta_{FN} \rangle}{\Lambda}$ that leads to the Yukawa coupling elements

$$y_{ik}^u = g \epsilon_{\bar{u}}^i \epsilon_q^k, \quad y_{jk}^d = g' \epsilon_{\bar{d}}^j \epsilon_q^k, \quad (32)$$

in which $\langle \theta_{FN} \rangle$ is the VEV of an introduced field, Λ an energy scale and i, j, k are the integer flavor indices. Therefore, in our scenario it is natural to have the identification

$$\epsilon_{\bar{u}}^i = e^{-\sigma n_a \mathcal{A}_{u_L}}, \quad \epsilon_{\bar{d}}^j = e^{-\sigma n_b \mathcal{A}_{d_L}}, \quad \epsilon_q^k = e^{-\sigma n_c \mathcal{A}_{q_R}}, \quad (33)$$

where the integer flavor indices i, j, k identify with the winding number n_a, n_b, n_c . So it is a possible scenario to fit the flavor pattern of quark sector well.

5. Conclusion

In this Letter, we discussed a simple and heuristic Z_3 flavor symmetry model for neutrino masses, and gave it a possible realization by constructing a non-Fock wavefunction involving the Wilson loop, Eq. (16). In this scenario, we show that the flavor charge can be interpreted as topological charge. The flavor DoF has similar behavior with the topological DoF under Z_3 symmetry, and it is stable against local small gauge perturbation which is consistent with the fact that flavor changing processes are suppressed in our observations by far.

A possible generalization to the quark sector is also discussed, in which we gave a possible scheme to compute the Yukawa couplings for quarks by calculating the correlation function of the Wilson loops in their wavefunctions. This scheme leads to strong hierarchy for the quark Yukawa couplings and reproduce the similar textures from the scenario of Froggatt–Nielsen's Abelian Flavor Symmetries.

References

- [1] W.M. Yao, et al., Particle Data Group, J. Phys. G 33 (2006) 1.
- [2] C.D. Froggatt, H.B. Nielsen, Nucl. Phys. B 147 (1979) 277.
- [3] SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011301;
For a review of Atmospheric experiment, see: C.K. Jung, et al., Annu. Rev. Nucl. Part. Sci. 51 (2001) 451;
KamLAND Collaboration, K. Eguchi, et al., Phys. Rev. Lett. 90 (2003) 021802;
K2K Collaboration, M.H. Ahn, et al., Phys. Rev. Lett. 90 (2003) 041801.
- [4] Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28 (1962) 870.
- [5] P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B 530 (2002) 167;
Z.Z. Xing, Phys. Lett. B 533 (2002) 85;
P.F. Harrison, W.G. Scott, Phys. Lett. B 535 (2002) 163;
X.G. He, A. Zee, Phys. Lett. B 560 (2003) 87.
- [6] For A_4 see: E. Ma, G. Rajasekaran, Phys. Rev. D 64 (2001) 113012, hep-ph/0106291;
E. Ma, Mod. Phys. Lett. A 17 (2002) 627;
K.S. Babu, E. Ma, J.W.F. Valle, Phys. Lett. B 552 (2003) 207, hep-ph/0206292;
M. Hirsch, E. Ma, A. Villanova del Moral, J.W.F. Valle, Phys. Rev. D 72 (2005) 091301, hep-ph/0507148;
M. Hirsch, E. Ma, A. Villanova del Moral, J.W.F. Valle, Phys. Rev. D 72 (2005) 119904, Erratum;
G. Altarelli, F. Feruglio, Nucl. Phys. B 720 (2005) 64, hep-ph/0504165;
G. Altarelli, F. Feruglio, Nucl. Phys. B 741 (2006) 215, hep-ph/0512103;
E. Ma, Phys. Rev. D 73 (2006) 057304, hep-ph/0511133;
E. Ma, H. Sawanaka, M. Tanimoto, Phys. Lett. B 641 (2006) 301, hep-ph/0606103, and references therein.
- [7] For $GUT \times S_4$: D.G. Lee, R.N. Mohapatra, Phys. Lett. B 329 (1994) 463, hep-ph/9403201;
C. Hagedorn, M. Lindner, R.N. Mohapatra, JHEP 0606 (2006) 042, hep-ph/0602244;
Y. Cai, H.-B. Yu, Phys. Rev. D 74 (2006) 115005, hep-ph/0608022;
For $GUT \times A_4$: S. Morisi, M. Picariello, E. Torrente-Lujan, Phys. Rev. D 75 (2007) 075017, hep-ph/0702034.
- [8] T.D. Lee, hep-ph/0605017;
T.D. Lee, Chinese Phys. 15 (2006) 1009;
R. Friedberg, T.D. Lee, High Energy Phys. Nucl. Phys. 30 (2006) 591, hep-ph/0606071.
- [9] T. Miura, E. Takasugi, M. Yoshimura, Phys. Rev. D 63 (2000) 013001;
Y. Koide, H. Nishiura, Phys. Rev. D 66 (2002) 093006;
E. Ma, hep-ph/0312192;
B. Hu, F. Wu, Y.L. Wu, Phys. Rev. D 75 (2007) 113003.
- [10] C. Giunti, C.W. Kim, U.W. Lee, Phys. Rev. D 45 (1992) 2414;
C. Giunti, Eur. Phys. J. C 39 (2005) 377, hep-ph/0312256.
- [11] R. Haag, Dan. Mat. Fys. Medd. 29 (12) (1955);
R. Haag, Local Quantum Physics, Springer, 1996.
- [12] K. Wilson, Phys. Rev. D 10 (1974) 2445.
- [13] G. 't Hooft, Nucl. Phys. B 138 (1978) 1;
Yu.M. Makeenko, A.A. Migdal, Phys. Lett. B 97 (1980) 235.