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A UTILITY APPROACH TO THE ANALYTIC HIERARCHY PROCESS

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Abstract—This paper attempts to examine the utility foundation of the Analytic Hierarchy Process (AHP). It identifies the conditions under which the selection of an alternative is consistent with the maximization of an underlying utility function, or more precisely, the conditions under which the AHP-recommended choice corresponds with the solution attained from maximizing the respondent's utility function.

INTRODUCTION

This paper attempts to examine the utility foundation of the Analytic Hierarchy Process (AHP). It identifies the conditions under which the selection of an alternative is consistent with the maximization of an underlying utility function, or more precisely, the conditions under which the AHP-recommended choice corresponds with the solution attained from maximizing the respondent's utility function.

In this analysis, we assume the existence of an underlying utility function, and abstract from the uncertainty about the alternatives and from the errors in preference responses. The paper focuses on the compatibility of the AHP and the utility (or more accurately, value) function, when the underlying utility function is of the types: uni-attribute, multi-attribute, additive and non-additive.

This examination shows that the AHP method is unconditionally consistent with the utility maximization criterion when the underlying utility function is uni-attribute. The consistency with the additive (but non-linear) function requires an additional assumption regarding the underlying utility function or, alternatively, a careful interpretation of pairwise comparisons of attributes. One very interesting result is that once we modify the procedure of aggregating the local weights into global weights, the AHP becomes unconditionally consistent with the utility maximization criterion in the cases of multiplicative utility functions. The unconditional nature of such a capability opens up an interesting and scarcely researched avenue for combining the AHP and utility theory in solving selection decision problems.

UNI-ATTRIBUTE UTILITY FUNCTIONS

Let us start with the general form of utility function $U(\mathbf{x})$, where $\mathbf{x} = (x_1, x_2, \ldots, x_m)$ constitutes the vector of m attributes. The utility maximization criterion requires the selection of alternative $\mathbf{a}$ such that:

$$\{\mathbf{a}: U(\mathbf{a}) = \max [U(\mathbf{x}): \mathbf{x} \in S]\}, \tag{1}$$

where S is the set of all alternatives under consideration.

A special case of expression (1) is when the decision maker bases the selection on only one attribute of alternatives. In such a case, $\mathbf{x}$ in expression (1) is scalar. This section shows that if one can elicit pairwise responses as utility ratios, the AHP and the utility maximization criterion lead to the same choice in the case of the uni-attribute function.

Define the following decision rule:

$$\{\mathbf{a}: \Phi(\mathbf{a}) = \max [\Phi(\mathbf{x}): \mathbf{x} \in S \quad \text{and} \quad \Phi(\mathbf{x}) \in M]\}, \tag{2}$$

where

$$M \equiv \left\{\Phi(\mathbf{x}) : \Phi(\mathbf{x}) = \sum_{\mathbf{y}\in S} [U(\mathbf{x})/U(\mathbf{y})] / \sum_{\mathbf{y}\in S} \Phi(\mathbf{y}), \mathbf{x}, \mathbf{y} \in S\right\},$$

and S is the same selection set as in expression (1).† Since we abstract from errors or inconsistencies which exist in preference responses, one can easily show that the decision rule (2) results in the same ordering of alternatives in S as the AHP estimation methods: the eigenvalue method [1], the simple row average method [2] and the mean transformation method [3]. The case of the simple row average is quite obvious because dividing each $\Phi(\mathbf{x})$ by a fixed number n (the number of alternatives) does not change the ordering of alternatives in M. In the absence of preference errors and inconsistencies, the identical outcome of the eigenvalue and the mean transformation methods with that of the simple row average has been noted previously [2, 3]. Hence, in this analysis, we will use decision rule (2) to represent the aggregation method of the AHP, and avoid the lengthy discussion of which of the other AHP computational methods may or may not be consistent with decision rule (2).

The following theorem establishes that decision rules (2) and (1) select the same alternative.

Theorem 1

Given a utility function U and a finite and discrete set of alternatives S, decision rules (1) and (2) have the same solution.

Proof. Since

$$\sum_{\mathbf{y}\in S} [U(\mathbf{x})/U(\mathbf{y})] = U(\mathbf{x}) \sum_{\mathbf{y}\in S} [1/U(\mathbf{y})] = A \cdot U(\mathbf{x})$$

and

$$\Phi(\mathbf{x}) = A \cdot U(\mathbf{x}) / \sum_{\mathbf{y}\in S} A \cdot U(\mathbf{y}) = C \cdot U(\mathbf{x}),$$

where

$$A \equiv \sum_{\mathbf{y}\in S} [1/U(\mathbf{y})]$$

and

$$C \equiv A / \sum_{\mathbf{y}\in S} A \cdot U(\mathbf{y}) = 1 / \sum_{\mathbf{y}\in S} U(\mathbf{y}).$$

In other words, C is the normalizing factor which makes the $\Phi(\mathbf{x})$s sum to 1. For any given set of S, C is fixed, and positive. The latter is due to the desirability, and hence, positive utility, of alternatives *in toto*. Thus, $\Phi(\mathbf{x})$ is a linear transformation of $U(\mathbf{x})$. Hence, the alternative **a** which maximizes rule (1) also maximizes rule (2).

Note that in decision rule (2), the aggregation rule of the pairwise scores is in the form of a summation, and it does admit averaging and other linear transformations. In other words, those AHP computational methods which conform to such aggregation lead to the maximization of the underlying utility function.

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† A more clear, and longer, specification in decision rule (2) is to define $\Phi(\mathbf{x})$ as $\Phi(\mathbf{x}) = \frac{\chi(\mathbf{x})}{\sum_{\mathbf{y}\in S} \chi(\mathbf{y})}$, where $\chi(\mathbf{x}) \equiv \sum_{\mathbf{y}\in S} \frac{U(\mathbf{x})}{U(\mathbf{y})}$. The formulation of decision (2) was adopted for the sake of notational brevity.

Let us clarify the development of this section with an example. Assume that the only decision criterion is the price of alternatives with the following underlying utility function:

$$U(p) = 1\,000\,000/p^2 \tag{3}$$

where p is the price attribute. Assume S, the alternative set, contains three alternatives with the price attributes \$10,000, \$12,000 and \$8000, respectively. Decision rule (1) leads to the $U(\mathbf{x})$ value 0.01, 0.0069 and 0.0156, respectively, recommending the selection of the third alternative. The pairwise comparisons of the alternatives, assuming away any inconsistency and errors in preference responses and considering pairwise scores as utility ratios, result in the following matrix:

$$B = \begin{pmatrix} 1 & 1.44 & 0.64 \\ 0.69 & 1 & 0.44 \\ 1.56 & 2.25 & 1 \end{pmatrix}.$$

Applying decision rule (2) attains $\Phi(\mathbf{x})$ values of 0.31, 0.21 and 0.48 for the three alternatives, respectively, and hence recommends the choice of the third alternative, a solution consistent with that of decision rule (1).

In the above analysis, we have assumed that the AHP elicits the relative utilities of alternatives. This assumption seems plausible given the repeated emphasis of the pairwise values as the relative preferences [2]. This, however, underlines the importance of the elicitation process for ensuring that respondents express their pairwise relative utilities of the attributes. Secondly, in such an analysis, the responses are not limited to the integer interval 1, 2, 3,...,9, as suggested [1], for respondents' convenience rather than any theoretical necessities.

ADDITIVE, NON-LINEAR, MULTI-ATTRIBUTE UTILITY FUNCTIONS

In this section, we explore the same question of the consistency of the AHP with the utility maximization criterion when the utility function has the additive, non-linear, multi-attribute form.

This time assume that the decision maker takes into account m attributes of alternatives to reach a selection decision. That is, the decision maker's underlying utility function is multi-attribute, i.e. $\mathbf{x}$ in decision rule (1) is a vector with m elements. Furthermore, assume that the form of the utility function is non-linear, but additive, as

$$U(\mathbf{x}) = \sum_{i=1}^{m} a_i \cdot U_i(x_i), \qquad a_i > 0, \tag{4}$$

where $U_i(x_i)$ is the utility of attribute i of alternative $\mathbf{x}$, and a_i is the weight related to attribute i. U_i may possess any non-linear form (of which the linear form is a special case). The following lemma and theorem establish the aggregation rules for inter-attribute scores that lead to the same selection as decision rule (1).

Lemma 1

If $\Psi(i,j) = a_i/a_j$ and $\Psi(i) \equiv \sum_{j=1}^{m} \Psi(i,j)$, then $\Psi(i)$ is a linear transformation of a_i.

Proof. We have

$$\Psi(i) = \sum_{j=1}^{m} \Psi(i,j) = \sum_{j=1}^{m} a_i/a_j = a_i \sum_{j=1}^{m} 1/a_j = Ka_i.$$

Since K is fixed and positive for any given set S, the lemma holds.

Now, consider the following decision rule:

$$\{\mathbf{a}: \Phi(\mathbf{a}) = \max\,[\Phi(\mathbf{x}): \mathbf{x} \in S, \Phi(\mathbf{x}) \in M]\}, \tag{5}$$

where

$$M \equiv \left\{\Phi(\mathbf{x}) : \Phi(\mathbf{x}) \equiv \sum_{i=1}^{m} \Psi(i) \cdot \Phi_i(x_i)\right\}$$

and

$$\Phi_i(x_i) = \sum_{y_i} [U_i(x_i)/U_i(y_i)]/\sum_{y_i} \Phi_i(y_i),$$

and $\Psi(i)$ has the same definition as in Lemma 1.

Theorem 2

Given equation (4) as the utility function, decision rules (1) and (5) select the same alternative iff the utilities of each attribute sum to a fixed value.

Proof. From Theorem 1 and the fixed value for utility sums, we have

$$\Phi_i(x_i) = C \cdot U_i(x_i), \qquad C > 0,$$

and from Lemma 1,

$$\Psi(i) = Ka_i, \qquad K > 0.$$

Hence, $\Phi(\mathbf{x})$ is a linear transformation of $U(\mathbf{x})$ and decision rules (1) and (5) select the same alternative. This establishes the sufficiency of fixed value for C.

To show the necessity of equal C (and thus its inverse $1/C$) for all utilities of attributes, let us consider the following counterexample with $m = 2$, i.e. a two-attribute utility function. Assume that the application of decision rule (5) to alternative $\mathbf{x}$ has led to

$$\Phi(\mathbf{x}) = \Psi(1)\Phi_1(x_1) + \Psi(2)\Phi_2(x_2).$$

From Theorem 1 and Lemma 1, we have

$$\Phi(\mathbf{x}) = Ka_1C_1U_1(x_1) + Ka_2C_2U_2(x_2),$$

where $1/C_1$ and $1/C_2$ are the sums of utilities of attributes 1 and 2 of alternatives in the selection set S. Let $C_2/C_1 = f > 1$. If decision rule (1) selects the alternative **a**, decision rule (5) will select over **a** any alternative which has the same $U_1(x_1)$ and above $[(1/f)(U_2(x_2)]$ utility for x_2, hence the contradiction. Q.E.D.

Let us demonstrate the restriction implied by Theorem 2 with an example. Consider the selection of one of three cars with two attributes: price and gas mileage. Assume the following underlying utility function:

$$U(p, g) = a_1U_1(x_1) + a_2U_2(x_2) = 5(1\,000\,000/p^2) + 4(g^{0.5}). \qquad (6)$$

The three cars have prices equal to \$7000, \$8000 and \$10,000, and gas mileage of 24, 28 and 34, respectively. Using function (6) results in $U(\text{car}_1) = 19.7$, $U(\text{car}_2) = 21.24$ and $U(\text{car}_3) = 23.37$, which, based on decision rule (1), recommends the selection of the third car.

The AHP solution of the same problem requires one matrix of pairwise comparisons per attribute. To attain the entries of the two input matrices, one only needs to observe from function (6) that $U_1(x_1) = 1/49$, $U_1(x_2) = 1/64$, $U_1(x_3) = 1/100$, $U_2(x_1) = 24^{0.5}$, $U_2(x_2) = 28^{0.5}$ and $U_2(x_3) = 34^{0.5}$. Based on the assumption that the AHP elicits relative utilities of each attribute of every pair of alternatives, and abstracting from any inconsistencies in the responses, one can construct the two matrices of pairwise comparisons as follows:

$$B_p = \overset{\textit{Price}}{\begin{pmatrix} 1 & 1.31 & 2.04 \\ 0.76 & 1 & 1.56 \\ 0.49 & 0.64 & 1 \end{pmatrix}}, \qquad B_g = \overset{\textit{Gas mileage}}{\begin{pmatrix} 1 & 0.92 & 0.84 \\ 1.08 & 1 & 0.91 \\ 1.18 & 1.09 & 1 \end{pmatrix}}.$$

Using any of the AHP computational methods results in the following relative local weights for the three cars:

$$\Phi_p = \{0.44, 0.34, 0.22\} \quad \text{and} \quad \Phi_g = \{0.31, 0.33, 0.36\}.$$

Construction of the input of pairwise weights for the attributes requires an additional assumption regularly made in the literature [e.g. 4] that the higher level of the hierarchy recovers the relative weights of the attributes a_is in equation (4). In other words, combining (via multiplication) the local weights of attributes (at the higher level of the hierarchy) with the local relative weights of alternatives to arrive at the global weights in the AHP implicitly assumes that the local relative weights of attributes estimate a_i in equation (4). We too use this assumption in the following presentation. This allows us to note later the extreme importance of this assumption in the compatibility of the AHP with the additive utility functions.

The input matrix of the weights of attributes could be similarly constructed from utility function (6), due to the fact that $a_1 = 5$ and $a_2 = 4$, as:

$$B_a = \left(\begin{array}{cc} p & g \\ 1 & 5/4 \\ 4/5 & 1 \end{array}\right), \qquad \text{with the relative weights } \Psi = \{0.56, 0.44\}.$$

Aggregating the local weights leads to the following global weights:

$$\Phi(\mathbf{x}) = [0.56, 0.44]\begin{pmatrix} 0.44 & 0.34 & 0.22 \\ 0.31 & 0.33 & 0.36 \end{pmatrix} = \{0.38, 0.34, 0.28\},$$

which recommends the selection of the first car, which differs from the choice of the third car recommended by decision rule (1). This incompatibility is the consequence of lacking the necessity condition of Theorem 2: the necessity of having an underlying utility function in which the utilities of each attribute sum to a fixed value $1/C$. In the present methods of AHP, this fixed value is 1.

One may resolve the above incompatibility in one of the following two ways. The first is to assume that the underlying utility function is such that the utilities of each attribute sum to 1. The second approach consists of changing the interpretation of the relative weights of attributes.

(1) To show the first approach, let us change the form of the underlying utility function in the previous example such that the utilities of each attribute sum to 1, as follows:

$$U(p, g) = 5(1/0.046)(1\,000\,000/p^2) + 4(1/16.02)g^{0.5}, \tag{7}$$

where 0.046 and 16.02 are, respectively, the sum of price and gas mileage utilities of the three cars. Using function (7) in decision rule (1), we get $U(\text{car}_1) = 3.44$, $U(\text{car}_2) = 3.02$ and $U(\text{car}_3) = 2.54$, which recommends the selection of the first car. Taking (1/0.046) and (1/16.02) as the normalization factors of U_1 and U_2, the AHP solution using function (7) remains the same as that of function (6), i.e. the choice of the third car. Hence, the two decision rules (1) and (5) coincide in solution. In this way, the AHP assumes that the underlying utility function, if additive, depends on the alternatives under consideration. In other words, if another alternative is added to the set S, the coefficients of the underlying utility function change. Although this may indeed be the case in some decision problems, such an assumption would be in contrast with the popular view of the utility function as a choice-independent function.

(2) The second way to resolve the incompatibility depends on the interpretation of the relative weights of attributes. In Theorem 2, we interpreted the relative weights of attributes as the ratios of the coefficients a_is in the utility function. An alternative interpretation is to consider the pairwise comparison of weights as the ratios of collective utilities attained from one attribute against another. In the three-car example, the pairwise comparison of price and gas mileage attributes not only depends on the relative importance of one attribute against another, but also on the sum of the utilities of price attributes against that of the gas mileage of the three cars.

Let us show this point with the three-car example. When the price and gas mileage attributes are compared at the higher level of the hierarchy, one should ask the relative utility of the price and gas mileage of *all three cars* taken together. In such a case, the respondent would not just be comparing $a_1 = 5$ vs $a_2 = 4$, but $(0.046a_1) = 0.23$ vs $(16.02a_2) = 64.08$, where 0.046 and 16.02 are, as before, the sum of the price and gas mileage utilities of three cars. In such a case, the pairwise comparison of attributes would be

$$B_a = \begin{pmatrix} 1 & 0.004 \\ 278.6 & 1 \end{pmatrix} \quad \text{and} \quad \Psi(p, g) = \{0.004, 0.996\}.$$

Using the above local weights for attributes and the set of local weights of the alternatives arrived at before results in the global relative weights 0.31, 0.33 and 0.36, leading to the selection of the third car, a choice consistent with the utility maximization criterion of utility function (6).

In summary, when the underlying utility function has an additive form, to reconcile the AHP with the utility maximization criterion, one must assume that the utilities of each attribute sum to 1, and hence, posit an underlying utility function dependent on the selection set S. Or, alternatively, one must interpret the relative weights of attributes such that the local relative weights of attributes account for the total utilities of alternatives, as well.

Both of the above two methods of reconciliation seem less than perfect. The present framework of investigation, however, permits us to explore a slightly different elicitation strategy to reconcile the two approaches more fully. Such a line of investigation, however, will take us beyond the focus of this paper, and may be found in Ref. [5].

NON-ADDITIVE, MULTI-ATTRIBUTE UTILITY FUNCTIONS

In this section, we explore cases where the underlying utility functions have non-additive forms. Let us start with the pure multiplicative form of the utility function:

$$U(\mathbf{x}) = \prod_{i=1}^{m} a_i U_i(x_i), \qquad a_i > 0 \quad \text{and} \quad U_i(x_i) > 0, \quad \text{for} \quad i = 1, 2, \ldots, m. \tag{8}$$

Since in this utility function, the product of the a_is is independent of the utilities, we can rewrite it as

$$U(\mathbf{x}) = F \prod_{i=1}^{m} U_i(x_i), \qquad F \equiv \prod_{i=1}^{m} a_i. \tag{9}$$

It can be immediately observed that the aggregation of inter-attribute scores (to arrive at the global relative weights) by taking the weighted averages does not lead to a selection compatible with function (9). However, one can define the inter-attribute aggregation according decision rule:

$$\{\mathbf{a} : \Phi(\mathbf{a}) = \max\,[\Phi(\mathbf{x}) : \mathbf{x} \in S, \Phi(\mathbf{x}) \in M]\}, \tag{10}$$

where

$$M \equiv \left\{\Phi(\mathbf{x}) : \Phi(\mathbf{x}) = \prod_{i=1}^{m} \Phi_i(x_i)\right\}$$

and

$$\Phi_i(x_i) = \sum_{y_i} U_i(x_i)/U_i(y_i), \quad \text{for} \quad i = 1, 2, \ldots, m.$$

The following lemma shows that if decision rule (10) is used in the aggregation of attribute scores (or local relative weights) of alternatives, the resultant solution equals the selection obtained from decision rule (1).

Lemma 2

Given the utility function (9), the alternative selected by applying decision rule (1) is the same as that of decision rule (10).

Proof. From Theorem 1, we have

$$\Phi_i(x_i) = A_i U_i(x_i), \qquad A_i > 0.$$

Incorporating the above in decision rule (10), we get

$$\Phi(\mathbf{x}) = \prod_{i=1}^{m} A_i U_i(x_i) = D \prod_{i=1}^{m} U_i(x_i),$$

where $D = \prod_{i=1}^{m} A_i$, which is positive and fixed for any given set of S. Hence, $\Phi(\mathbf{x})$ is a linear transformation of $U(\mathbf{x})$. Decision rules (1) and (10), thus, lead to the same selection decision.

Note that in the case of the underlying utility function with a pure multiplicative form such as function (9), one does not need to find the relative weights of attributes. Scoring the alternatives based on their attributes will suffice.

Let us examine the approach suggested by Lemma 2 with an example. Assume the three-car example of the previous section has the following utility function:

$$U(\mathbf{x}) = a_1 U_1(x_1) \cdot a_2 U_2(x_2) = 5(1\,000\,000/p^2)\,4(g^{0.5}). \tag{11}$$

Using the same price and gas mileage values (\$7000, \$8000, \$10,000; and 24, 28, 34, respectively) in function (11) yields the following utility values: $U(\text{car}_1) = 2.0$, $U(\text{car}_2) = 1.65$ and $U(\text{car}_3) = 1.17$. Thus, decision rule (1) selects the first car. Multiplying the scores: $\Phi_p = \{0.44, 0.34, 0.22\}$ and $\Phi_g = \{0.31, 0.33, 0.36\}$, obtained by applying the AHP computational method in the previous example, according to decision rule (10), we get: $\Phi(\text{car}_1) = 0.44(0.31) = 0.14$, $\Phi(\text{car}_2) = 0.34(0.33) = 0.11$ and $\Phi(\text{car}_3) = 0.22(0.36) = 0.08$, which leads to the selection of the same car recommended by decision rule (1), i.e. the first car. (One may, if so inclined, normalize the Φs of the cars to add up to 1.) Thus, when the underlying utility function is in a pure multiplicative form, the AHP method and the inter-attribute aggregation in the form of the multiplication of scores results, without any condition, in the selection of the same alternative as that of the utility maximization criterion. This is an interesting result because no utility estimation method can claim the same unconditional recovery capability. (The von Neumann–Morgenstern utility function, for example, constrains the utility of each attribute to the interval [0, 1], in addition to a number of other constraints for the multiplicative form [6, p. 341].)

Another form of the multiplicative form for the utility function is

$$U(\mathbf{x}) = e^{\mathbf{y}}, \qquad \mathbf{y} = \prod_{i=1}^{m} U_i(x_i). \tag{12}$$

Define the following decision rule:

$$\{\mathbf{a}: U(\mathbf{a}) = \max\{\Phi(\mathbf{x}), \mathbf{x} \in S, \Phi(x) \in M\}, \tag{13}$$

where

$$M \equiv \left\{\Phi(\mathbf{x}), \Phi(\mathbf{x}) = \mathrm{e}^{z}, z \equiv \prod_{i=1}^{m} \Phi_i(x_i), \Phi_i(x_i) = \sum_{y_i} U_i(x_i)/U_i(y_i)\right\}.$$

It can be shown, with a similar reasoning to that in the proof of Lemma 2, that decision rules (1) and (13) select the same alternative.

Again, let us demonstrate the decision rule by continuing with the three-car example, and the following underlying utility function:

$$U(\mathbf{x}) = \mathrm{e}^{5(1\,000\,000/p^2)4(g^{0.5})}. \tag{14}$$

Using the same price and gas mileage values as before, we get the following for the utility values of the three cars: $U(\text{car}_1) = 7.39$, $U(\text{car}_2) = 5.23$ and $U(\text{car}_3) = 3.21$; with the first car as the best choice. The application of decision rule (13), using the AHP-computed local weights of $\Phi_p = \{0.44, 0.34, 0.22\}$ and $\Phi_g = \{0.31, 0.33, 0.36\}$, results in $\Phi(\text{car}_1) = \mathrm{e}^{0.44(0.31)} = 1.15$, $\Phi(\text{car}_2) = \mathrm{e}^{0.34(0.33)} = 1.12$ and $\Phi(\text{car}_3) = \mathrm{e}^{0.22(0.36)} = 1.08$, which recommends the selection of the first car, the same as that of the utility maximization criterion.

In decision rule (13), one may observe that the intra-attribute scoring (local relative weights) are based on the AHP, and the inter-attribute aggregation (arriving at the global relative weights) is based on the form of the underlying utility function. The unconditional correspondence of the two decision rules is quite striking, and underlines an aspect of the AHP which has scarcely been explored—aggregating the local relative weights based on an underlying utility function assumption.

Let us now examine other forms of non-additive utility functions. One such function is

$$U(\mathbf{x}) = \sum_{j=1}^{m} a_j U_j(x_j) + a_{m+1} U_{m+1}(x_i, x_k), \quad \text{for some } i, k, \leqslant m. \tag{15}$$

In such a case, one can define x_{m+1} as the attribute representing the interaction of x_i and x_k, or $x_{m+1} \equiv (x_i, x_k)$. Then, the results obtained in the previous section remain true for function (15), as well.

Another form of non-additivity is the case where some attributes have the additive form of equation (4) and some the multiplicative form of function (8). It can be shown [5] that even when the two aggregation rules of decision rules (5) and (10) are combined to accommodate such an extreme case of non-additivity, the conclusions arrived at in the additive utility function case in the previous section hold true for it, as well.

CONCLUDING REMARKS

This paper presents a framework for synthesizing the AHP and utility theory. It shows that the AHP maximizes the underlying utility function in the single attribute case. In multi-attribute cases, the AHP is unconditionally consistent with the utility maximization criterion where the underlying utility function has pure multiplicative forms. In the additive or multiplicative-additive cases, either an additional assumption about the underlying utility function or the careful interpretation of the relative weights of attributes would reconcile the AHP with the utility maximization criterion.

This analysis highlights the importance of aggregation methods used in combining local relative weights to arrive at global relative weights. It provides alternative aggregation methods which depend on the underlying utility function; the framework of this analysis opens up the possibility of exploring various aggregation processes which are in line with the functional form of the respondent's utility, hence synthesizing the AHP and utility theory to provide additional tools for solving selection problems.

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