ELECTRICAL ENGINEERING

Brushless DC motor tracking control using self-tuning fuzzy PID control and model reference adaptive control

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Abstract   This paper compares the performance of two different control techniques applied to high performance brushless DC motor. The first scheme is self-tuning fuzzy PID controller and the second scheme is model reference adaptive control (MRAC) with PID compensator. The purpose of the control algorithm was to force the rotor speed to follow the desired reference speed with good accuracy all time. This objective should be achieved for different speed/time tracks regardless of load disturbance and parameter variations. The simulation results presented show that the second control scheme has better performance.

1. Introduction

BLDC motors are a type of permanent magnet synchronous motors. They are driven by DC voltage, but current commutation is achieved by solid-state switches. The commutation instant is determined by the rotor position which is detected either by position sensors or by sensorless techniques [1]. BLDC motors have many advantages such as the following:

- Long operating life.
- High dynamic response.
- High efficiency.
- Better speed vs. torque characteristics.
- Higher speed range.
- Higher torque–weight ratio.

Therefore the BLDC motor has been used in many applications such as electric automotive, robotics and CD-ROMS and in such applications BLDC motor exposed to many kinds of load disturbances. Conventional control methods cannot achieve the desired speed tracking with good accuracy in case of sudden disturbance and parameter variations. This problem can be alleviated by implementing advanced control techniques such as adaptive control, variable structure control, fuzzy control and neural network [2]. The variable structure controller is simple, but it is difficult to implement. This is because of the possibility of the abrupt change in the control

signal, which might affect the system operation [3]. A neural-network-based motor control system has a strong ability to solve the structure uncertainty and the disturbance of the system, whereas it requires more computing capacity and data storage space [3]. Fuzzy control theory usually provides non-linear controllers that are capable of performing different complex nonlinear control action even for uncertain nonlinear systems [4]. Unlike conventional control designing, a FLC does not require precise knowledge of the system model such as the poles and zeroes of the system transfer function [5]. A fuzzy-logic control system based on expert knowledge database needs less calculations, but it lacks sufficient capacity for the new rules [1]. So, the combination between fuzzy and PID controller where the role of the fuzzy controller is to tune the PID controller parameters according to the error and change of error might be a good alternative compared to conventional PID control [3,4].

Adaptive control is one of the widely used control strategies to design advanced control systems for better performance and accuracy. Model reference adaptive control (MRAC) is a direct adaptive strategy with some adjustable controller parameters and an adjusting mechanism to adjust them [6]. As compared to the well-known and simple structured fixed gain PID controllers, adaptive controllers are very effective to handle the unknown parameter variations and environmental changes [7].

In this paper two advanced control strategies have been adopted and compared. The first control technique is self-tuning fuzzy PID controller which shows a good performance compared to conventional PID controller. The second technique is based on MRAC. Implementing MRAC shows a reasonable performance but it has high overshooting and continuous steady state error. Combining MRAC with PID control compensator will eliminate both the overshoots and steady state error.

The rest of this paper is organized as follows: the mathematical model of the BLDC motor introduced in Section 2. In Section 3, self-tuning fuzzy PID Controller is designed for BLDC speed control. In Section 4, the design steps of MRAC are shown. In Section 5 the simulation results are shown and in Section 6 the conclusion is shown.

2. State-space based modeling of BLDC motor

State-space equation method is a popular analysis method in modern control theory. The state-space method is becoming more and more popular in designing control systems with the fast development of computer techniques [8,9].

Suppose that the three-phase BLDC motor is controlled by the full-bridge driving in the two-phase conduction mode as in Fig. 1 [10].

Then the state space model of BLDC motor is in (1).

$$\begin{bmatrix} \dot{i}_A \\ \dot{i}_B \\ \dot{\omega} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 0 & 0 & 0 \\ 0 & -\frac{R}{L} & 0 & 0 \\ 0 & 0 & -\frac{B_v}{J} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ \omega \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{2}{3L} & \frac{1}{3L} & 0 \\ \frac{-1}{3L} & \frac{1}{3L} & 0 \\ 0 & 0 & \frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{AB} - e_{AB} \\ u_{BC} - e_{BC} \\ T_e - T_L \end{bmatrix}$$

$$(1)$$

where u_{AB}, u_{BC}: line voltage, e_{AB}, e_{BC}: line back emf, R: phase resistance of winding, L: phase inductance of winding, J: rotor
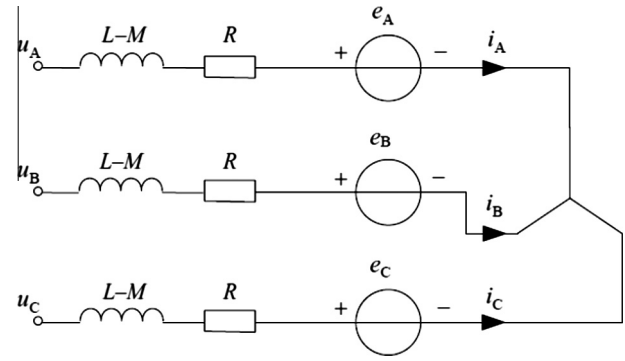

Figure 1   Equivalent circuit of the BLDC motor.

moment of inertia, T_L: load torque, ω: rotor speed, θ: rotor position, B_v: viscous friction coefficient, and i_A, i_B: phase current.

The validity of the state space model was verified by comparing the simulation results of this model with MATLAB model. The results of both models were identical.

Figs. 2 and 3 show the simulation results of open loop response of BLDC motor free running and subjected to sudden load with 50% of rated value at $t = 0.1$ s.

Fig. 2 shows the speed response of this test while Fig. 3 shows the phase current.

The parameters of the BLDC motor are listed in Table 1.

3. Self-tuning fuzzy PID controller design

Selecting the proper PID controller parameters is very important. *Ziegler and Nichols* proposed the well-known method to find the coefficients of the PID controller. Although the performance of the PID controller can be improved by selecting the controller parameters based on one of the optimization techniques, both cannot guarantee to be effective [4]. For this reason this paper investigates the design of self-tuning fuzzy PID controller. The controller includes two parts conventional PID controller and fuzzy logic control (FLC) as shown in Fig. 4. In this case the parameters of the PID controller are adaptively changing using fuzzy logic algorithm.
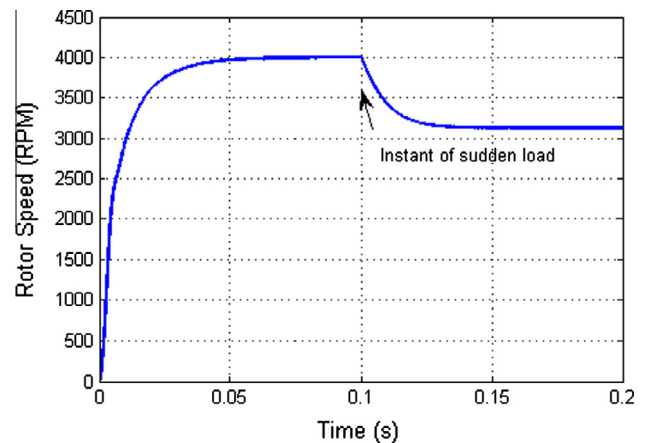

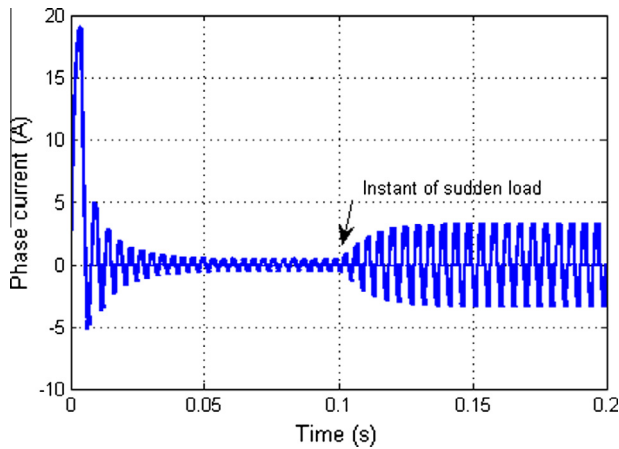**Figure 2**   Open loop speed response with sudden load at 0.1 s.

Figure 3 Open loop response of phase current with sudden load at 0.1 s.

Table 1 BLDC motor parameters per phase.

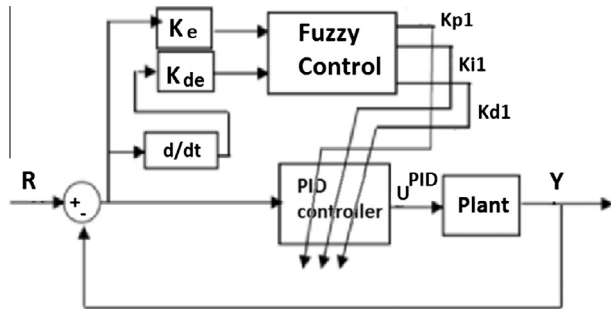| Rating	Symbol	Value	Units
DC resistance	R	0.57	Ω
Inductance	L	1.5	mH
Torque constant	K_t	0.082	N m/A
Number of poles	P	4	
Peak torque	T_p	0.42	N m
Rated voltage	V	36	V
Rotor inertia	J	$23e^{-6}$	kg m^2
Friction coefficient	B_v	0.0000735	N.M.S.
Rated speed	Ω	4000	RPM
Rated current	I	5	A

Figure 4 Structure of self-tuning PID fuzzy controller.

The PID controller parameters are updated according to the following equations:

$$K_{P2} = K_{p1} * K_P, \quad K_{i2} = K_{i1} * K_i \text{ and } K_{d2} = K_{d1} * K_d \qquad (2)$$

where K_{p1}: proportional modification coefficient, K_{i1}: integral modification coefficient, K_{d1}: derivative modification coefficient, K_p: initial proportional gain (constant value), K_i: initial integral gain (constant value), and K_d: initial derivative gain (constant value).

Hence, the control signal of self-tuning fuzzy PID controller can be described as follows:

$$U^{PID} = K_{p2}e(t) + K_{i2} \int e(t) + K_{d2} \frac{de(t)}{dt} \qquad (3)$$

where K_{p2}, K_{i2} and K_{d2} are the new gains of PID controller.

The general structure of fuzzy logic control is represented in Fig. 5 and comprises three principal components.

3.1. Fuzzification

This converts input data into suitable linguistic values. As shown in Fig. 5 there are two inputs to the controller: error and rate change of the error signals. For the system under study the universe of discourse for both $e(t)$ and $\Delta e(t)$ may be normalized from $[-1, 1]$, and the linguistic labels are {Negative Big, Negative medium, Negative small, Zero, Positive small, Positive medium, Positive Big} and are referred to in the rules bases as {NB, NM, NS, ZE, PS, PM, PB}, and the linguistic labels of the outputs are {Zero, Medium small, Small, Medium, Big, Medium big, very big} and referred to in the rules bases as {Z, MS, S, M, B, MB, VB}. Figs. 6 and 7 show the memberships of inputs and output of fuzzy logic control.

3.2. Rule base

A decision making logic is, simulating a human decision process. The rule base is simplified in Tables 2–5. The input e has 7 linguistic labels and Δe has 7 linguistic labels. Then we have $7 \times 7 = 49$ rule base. In this paper simplify 49 to 25 rule base [3].

3.3. Defuzzification

This yields a non-fuzzy control action from inferred fuzzy control action. The most popular method, center of gravity or center of area is used for defuzzification.

Fig. 8 shows the Simulink diagram of self-tuning fuzzy PID controller.

The controller of self-tuning fuzzy PID controller consists of two parts PID controller and fuzzy logic controller, which were tuned on-line the parameters of the PID controller.

4. Design steps of MRAC by MIT rule

4.1. MRAC

An adaptive controller consists of two loops, an outer loop or normal feedback loop and an inner loop or parameter adjustment loop as shown in Fig. 9.

The MIT rule is the original approach to model-reference adaptive control. The name is derived from the fact that it was developed at the Instrumentation Laboratory (now the Draper Laboratory) at MIT. Adjust parameters in such a way that the loss function is minimized [7,11].

$$J(\theta) = \frac{1}{2}e^2 \qquad (4)$$

To make J small, it is reasonable to change the parameters in the direction of the negative gradient of J, that is,

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma e \frac{\partial e}{\partial \theta} \qquad (5)$$
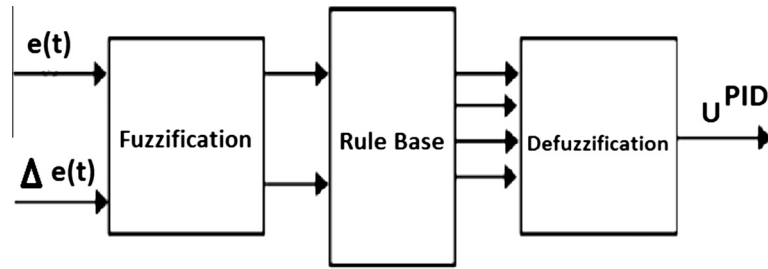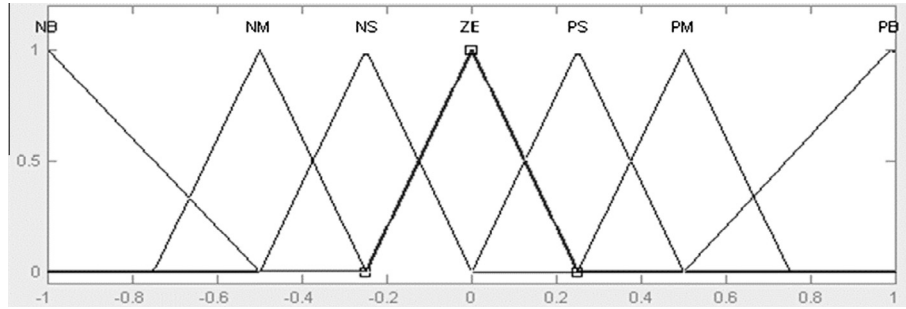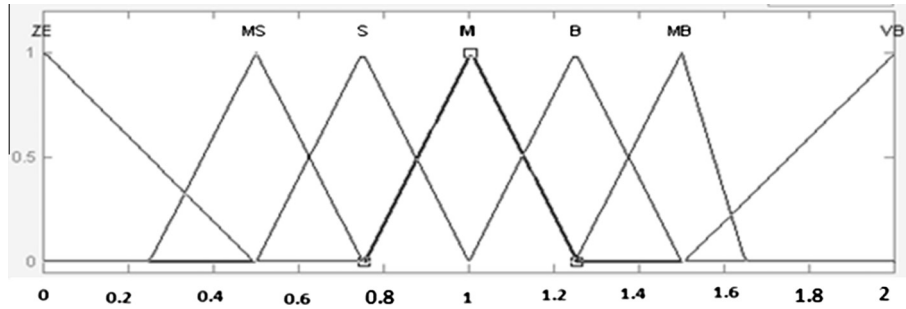
Figure 5   Fuzzy logic control structure.

Figure 6   Memberships functions of inputs (e, Δe).

Figure 7   Memberships functions of outputs (K_{p1}, K_{i1} and K_{d1}).

Table 2   The rule base of K_{p1}.

$\Delta e/e$	NB	NS	ZE	PS	PB
NB	VB	VB	VB	VB	VB
NS	B	B	B	MB	VB
ZE	ZE	ZE	MS	S	S
PS	B	B	B	MB	VB
PB	VB	VB	VB	VB	VB

Table 3   The rule base of K_{i1}.

$\Delta e/e$	NB	NS	ZE	PS	PB
NB	M	M	M	M	M
NS	S	S	S	S	S
ZE	MS	MS	ZE	MS	MS
PS	S	S	S	S	S
PB	M	M	M	M	M

where γ: adaptation rate, e: the error between the output speed of the BLDC motor and the model reference output, and θ: the controller parameter.

From Fig. 10 assume the BLDC motor is described by the single-input, single-output (SISO) system.

Table 4   The rule base of K_{d1}.

$\Delta e/e$	NB	NS	ZE	PS	PB
NB	ZE	S	M	MB	VB
NS	S	B	MB	VB	VB
ZE	M	MB	MB	VB	VB
PS	B	VB	VB	VB	VB
PB	VB	VB	VB	VB	VB

$$A \cdot y(t) = B(u(t) + v(t)) \tag{6}$$

where A & B are polynomials depend on the BLDC motor, $u(t)$: the output of controller, $y(t)$: the output speed of BLDC motor, and $v(t)$: the process disturbance.
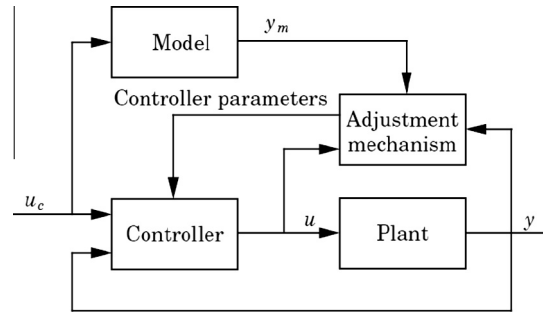
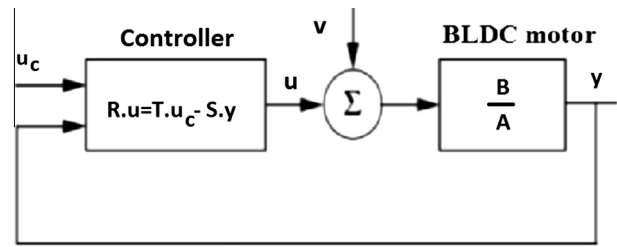**Figure 9** Block diagram of a model-reference adaptive system (MRAS).

Figure 10 A general linear controller with two degrees of freedom.

The controller is described in (7).

$$R \cdot u(t) = T \cdot u_c(t) - S \cdot y(t) \tag{7}$$

where R, T and S are controller polynomials and $u_c(t)$: the desired speed of BLDC motor.

Substituting (7) into (6) will result (8)

$$y(t) = \frac{BT}{AR + BS} u_c(t) + \frac{BR}{AR + BS} v(t) \tag{8}$$

The reference model is described by the single-input, single-output (SISO) system as follows:

$$A_m \cdot y_m(t) = B_m \cdot u_c(t) \Rightarrow y_m(t) = \frac{B_m}{A_m} u_c(t) \tag{9}$$

where A_m, B_m are polynomials depend on the reference model and $y_m(t)$: the output of model reference.

Assuming $v(t) = 0$ therefore:

$$y(t) = y_m(t) \Rightarrow \frac{BT}{AR + BS} = \frac{B_m}{A_m} \tag{10}$$

Let the transfer function of reference model is

$$\frac{y_m}{u_c} = \frac{b_m}{a_{m1} P^2 + a_{m2} P + a_{m3}} \tag{11}$$

where

$$P = \frac{d}{dt}$$

$a_{m1}, a_{m2}, a_{m3}, b_m$: the model reference transfer function coefficient.

Assume the transfer function of the BLDC motor is

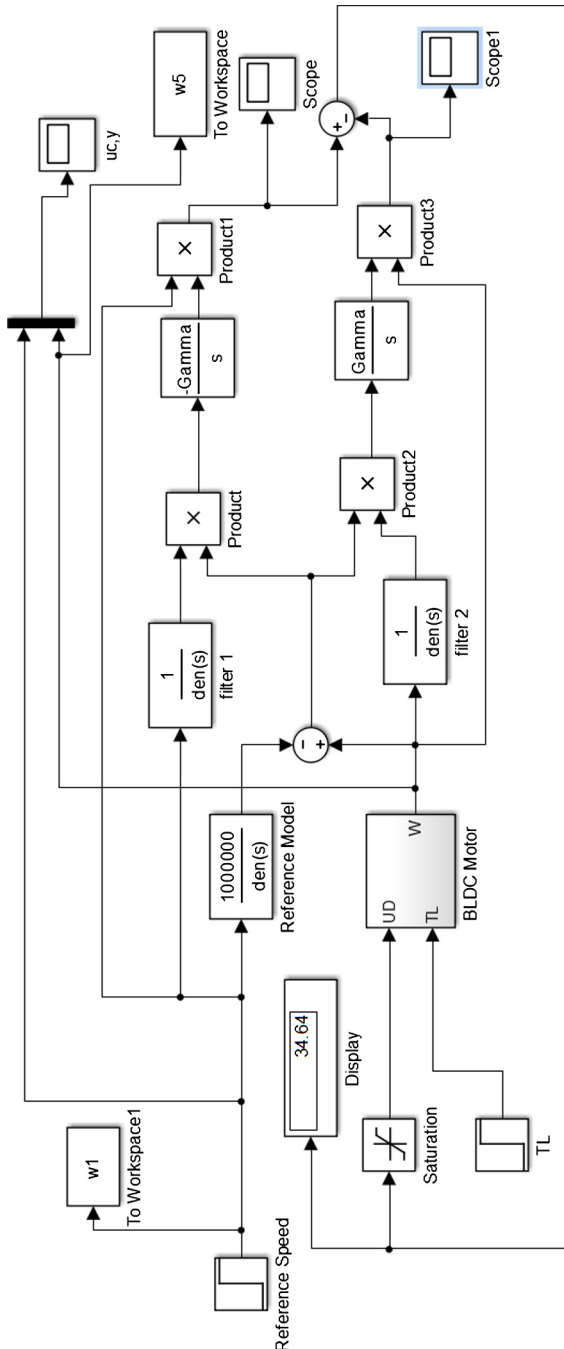$$\frac{y}{u} = \frac{b}{a_1 P^2 + a_2 P + a_3} \tag{12}$$

Figure 8 Simulink diagram of self-tuning fuzzy PID controller.

a_1, a_2, a_3, b: BLDC motor transfer function coefficient.

The diophantine equation is

$$AR + BS = A_0 A_m \qquad (13)$$

where $\quad A = a_1 P^2 + a_2 P + a_3,\ A_m = a_{m1} P^2 + a_{m2} P + a_{m3}$ and A_0 is a gain. R and S are controller polynomials.

$$\deg(S) = \deg(A) - 1 = 2 - 1 = 1 \ \Rightarrow\ S = s_0 + P s_1 \qquad (14)$$

where deg is the polynomial degree.

$$\deg(R) = \deg(S) \ \Rightarrow\ R = r_0 + r_1 P \qquad (15)$$

Figure 11   Simulink diagram of model reference adaptive control.
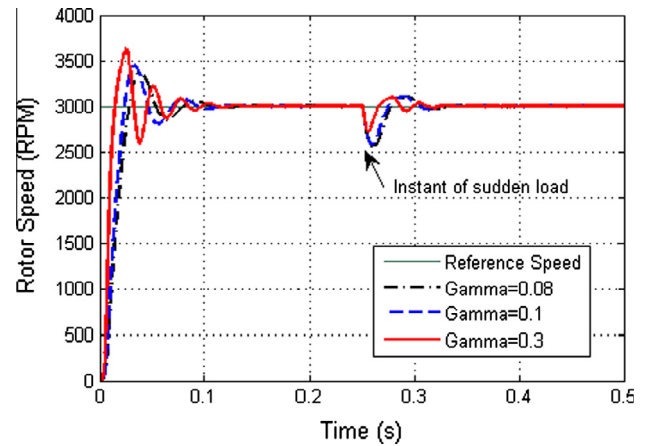

Figure 12   Speed response of MRAC at different values of adaptation rate gain.
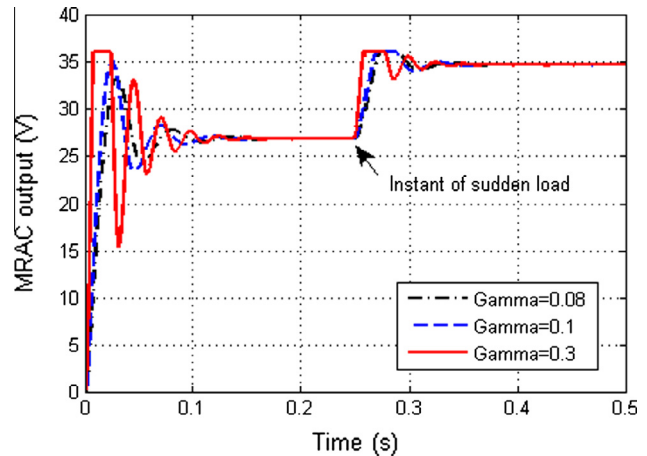

Figure 13   Controller output of MRAC at different values of adaptation rate.

Table 5   PID control compensator parameter characteristics.

Parameter increase	Rise time	Overshoot	Settling time	Steady state error
K_p	Decrease	Increase	Small change	Decrease
K_i	Decrease	Increase	Increase	Great reduce
K_d	Small change	Decrease	Decrease	Small change

Figure 14 Simulink diagram of model reference adaptive control with PID compensator.

$$\deg(A_0) = \deg(A) + \deg(R) - \deg(A_m) = 2 + 1 - 2 = 1$$

$$A_o = P \tag{16}$$

Similarly

$$T = P \tag{17}$$

Substituting Eqs. (14)–(17) into Eq. (7) will result in (18)

$$(r_0 + r_1 P)u = P \cdot u_c - (s_1 P + s_0)y \tag{18}$$

$$u = \frac{P}{R(P)}u_c - \frac{S(P)}{R(P)}y \tag{19}$$

From Eq. (6) and assume $v(t) = 0$

$$(a_1 P^2 + a_2 P + a_3) = bu \tag{20}$$

Substituting (19) into (20) will result in (21)

$$(a_1 P^2 + a_2 P + a_3)y = b\left(\frac{T(P)}{R(P)}u_c - \frac{S(P)}{R(P)}y\right)$$

$$\Rightarrow \left((a_1 P^2 + a_2 P + a_3) + b\frac{S(P)}{R(P)}\right)y$$

$$= b\frac{T(P)}{R(P)}u_c \tag{21}$$

Eq. (21) may be written as follows:

$$y = \frac{bT(P)}{(a_1 P^2 + a_2 P + a_3)R(P) + b \cdot S(P)}u_c \tag{22}$$

$$e = y - y_m \tag{23}$$

Substituting Eqs. (11) and (22) into (23)

$$e = \left(\frac{bT(P)}{(a_1 P^2 + a_2 P + a_3)R(P) + bS(P)} - \frac{b_m}{a_{m1} P^2 + a_{m2} P + a_{m3}}\right)u_c \tag{24}$$

$$\frac{\partial e}{\partial T} = \frac{b}{(a_1 P^2 + a_2 P + a_3)R(P) + bS(P)}u_c \tag{25}$$

$$\frac{\partial e}{\partial S} = \frac{-b^2 T(P)}{((a_1 P^2 + a_2 P + a_3)R(P) + bS(P))^2}u_c \tag{26}$$

From Eq. (5)

$$\frac{\partial T}{\partial t} = -\gamma e \frac{b}{(a_1 P^2 + a_2 P + a_3)R(P) + bS(P)}u_c$$

$$\frac{\partial T}{\partial t} = -\gamma' e \frac{1}{(a_1 P^2 + a_2 P + a_3)R(P) + bS(P)}u_c \tag{27}$$

where

$$\gamma' = b\gamma \tag{28}$$

Similarly

$$\frac{\partial S}{\partial t} = -\gamma' e \frac{1}{a_{m1} P^2 + a_{m2} P + a_{m3}}y \tag{29}$$

$$\frac{B_m}{A_m} = \frac{\omega_n^2}{p^2 + 2\xi\omega_n p + \omega_n^2} \tag{30}$$
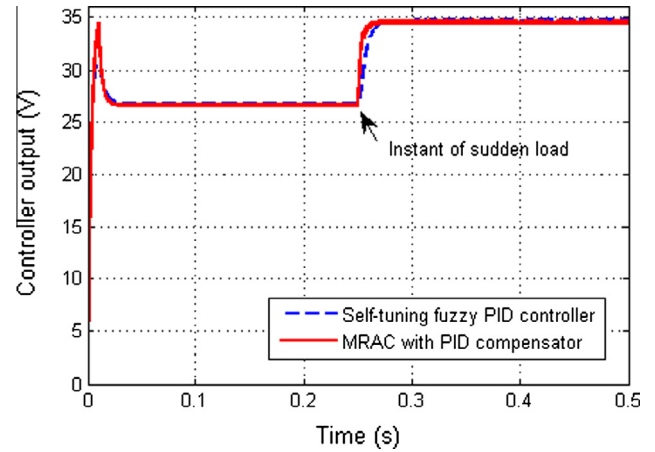

Figure 16 Controller output of self-tuning fuzzy PID controller and MRAC with PID compensator at sudden load.
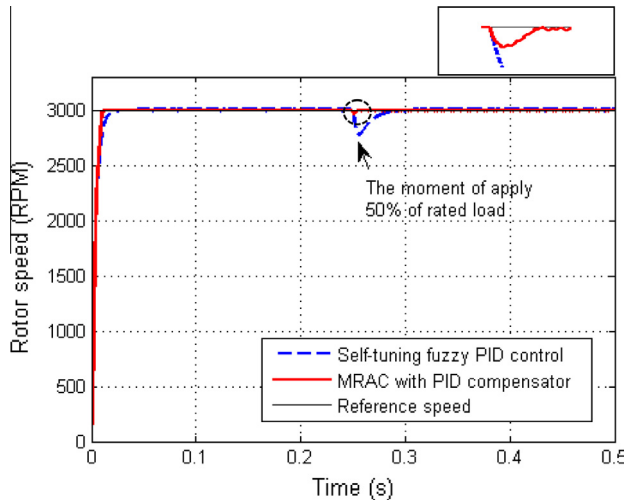

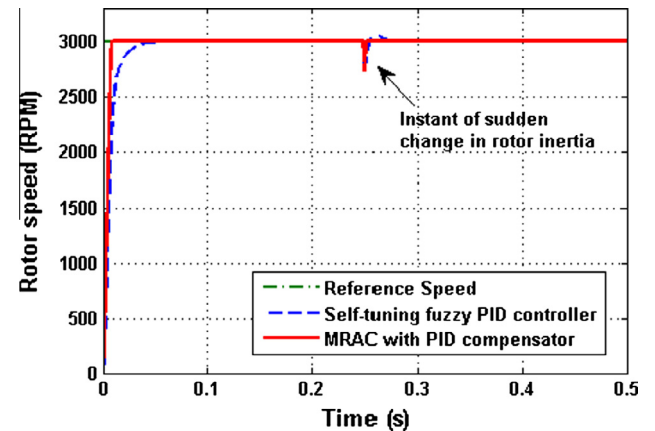**Figure 15** Speed response of MRAC with PID compensator and self-tuning fuzzy PID controller at sudden load.

Figure 17 Speed response of MRAC with PID compensator and self-tuning fuzzy PID controller with sudden change in inertia.

where ξ (*damping ratio*) = 1 and ω_n (*natural frequency*) = 1000 (selected by designer).

Fig. 11 shows the Simulink block diagram of the whole drive system including MRAC.

The performance of MRAC is investigated at speed regulation with sudden change in load. Fig. 12 shows the ability of MRAC to withstand the sudden increase in the load torque by 50% of rated torque at 0.25 s. This test is carried out at different values of adaptation rate. It can be noted that the higher adaptation rate will give better performance (less rise time), high overshooting and high steady state error. Fig. 13 shows the corresponding MRAC output.

4.2. MRAC with PID compensator

MRAC is designed to eliminate the difference between the output of model reference and the actual speed. It does not take into account the error between reference speed and actual speed. This disadvantage can be alleviated by adopting PID compensator. The parameters of the PID compensator are selected by trial and error. Table 5 shows the effect of changing each of the PID compensator parameter on the system performance.

Fig. 14 shows the Simulink diagram of MRAC with PID compensator.

5. Simulation results

Several tests have been carried out to compare the performance of self-tuning fuzzy PID controller and MRAC with PID compensator.

5.1. Speed regulation at sudden change in load

Fig. 15 shows a comparison between self-tuning fuzzy PID controller and MRAC with PID compensator. The comparison shows that the MRAC with PID compensator has better performance. The rise time is less also in case of sudden load
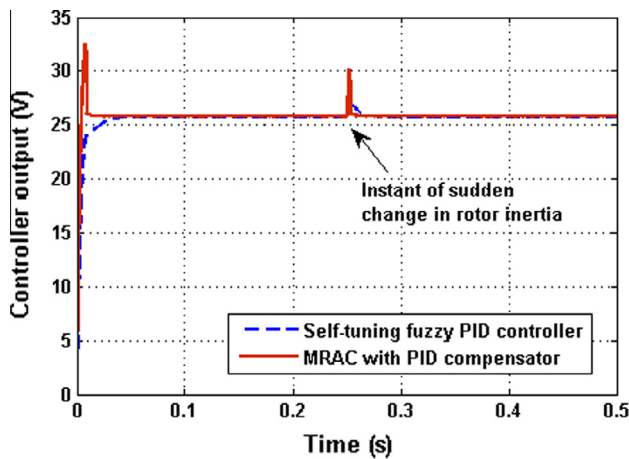

Figure 18 Controller output of self-tuning fuzzy PID controller and MRAC with PID compensator at sudden change in rotor inertia.
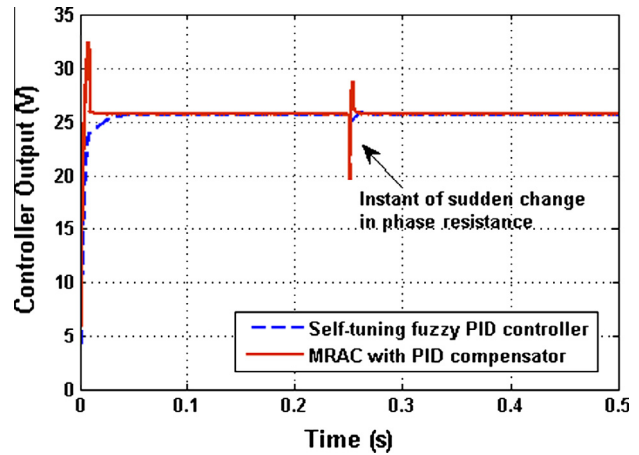

Figure 20 Controller output of self-tuning fuzzy PID controller and MRAC with PID compensator at sudden change in phase resistance.
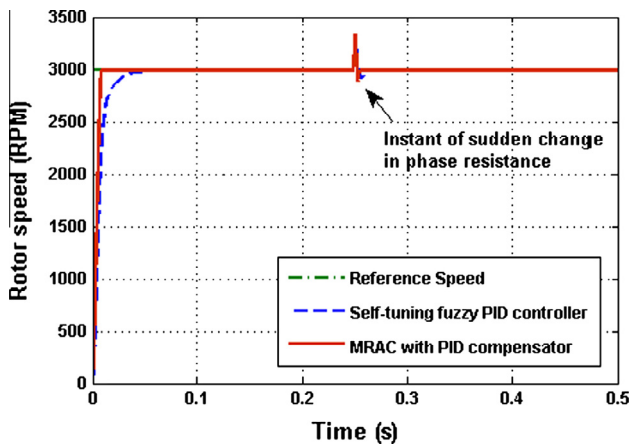

Figure 19 Speed response MRAC with PID compensator and self-tuning fuzzy PID controller at sudden change in phase resistance.
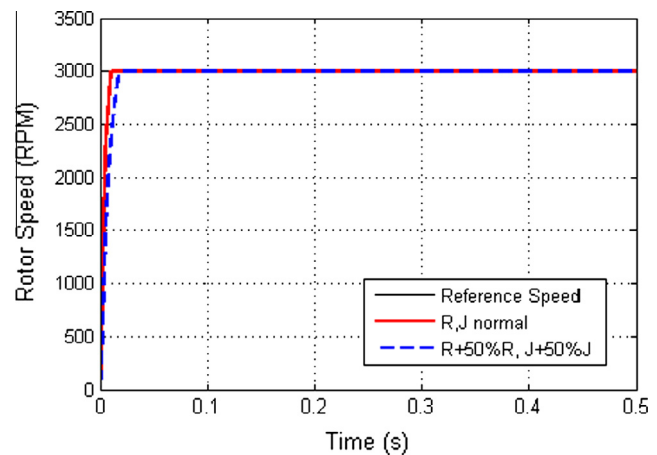

Figure 21 The effect of change in phase resistance and inertia on the speed response of MRAC with PID compensator.

disturbance and the speed can recover its desired value much faster, while Fig. 16 shows the controller output of both controllers.

5.2. Speed regulation at parameter variation

The proposed controller is investigated by changing the motor parameters. Three tests are carried out for this purpose. These tests are summarized as follows.

5.2.1. Sudden change in inertia

In this test the inertia of BLDC motor will be increased 10% at 0.25 s. Fig. 17 shows the ability of MRAC with PID controller compensator to accommodate the speed disturbance in a short time compared to self-tuning fuzzy PID controller, while Fig. 18 shows the corresponding controller output of both controllers.
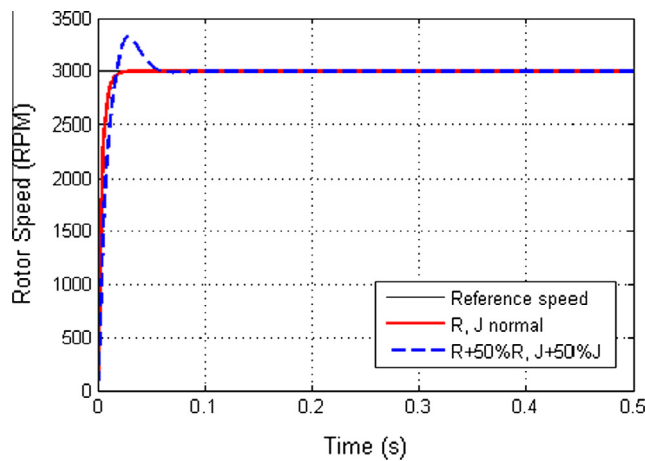
5.2.2. Sudden change in phase resistance

In this test the phase resistance of BLDC motor will be decreased 10% at 0.25 s. Figs. 19 and 20 show the results of this test. Fig. 19 shows a comparison of the performance of self-tuning fuzzy PID controller and MRAC with PID compensator. It can be noted that both controllers are robust where in case of the sudden change in phase resistance both controllers can track the reference speed at a short time. Fig. 20 shows both controllers output at sudden change in phase resistance.

5.2.3. Effect changes of both phase resistance and the rotor inertia

In this test both the phase resistance and the inertia of BLDC motor will be increased simultaneously by 50% from its original values. It can be noted in Fig. 21 that the MRAC with PID control compensator has a small change from its original

Figure 22 The effect of change in phase resistance and inertia on the speed response of self-tuning fuzzy PID controller.
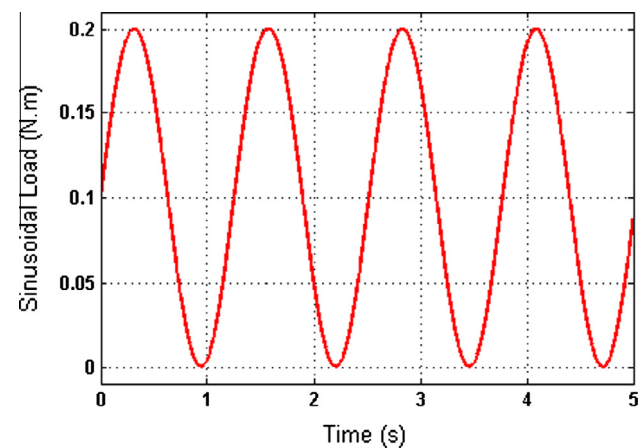

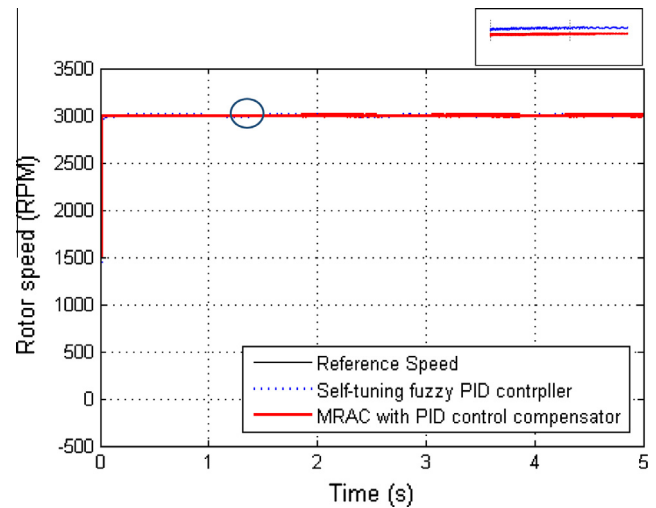**Figure 24** Speed response of MRAC with PID compensator and self tuning fuzzy PID controller at sinusoidal load.

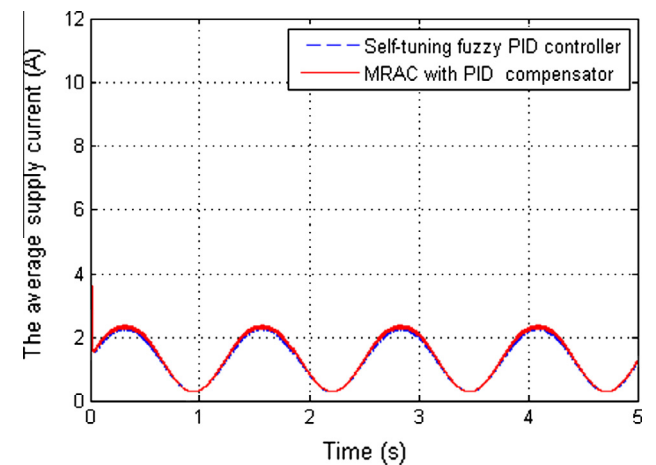**Figure 23** Sinusoidal load torque varies between 0% and 50% of peak torque.

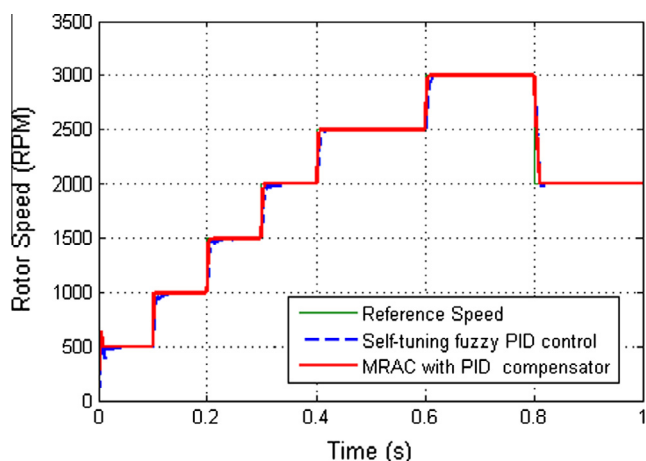**Figure 25** The average supply current of BLDC motor at sinusoidal load.

Figure 26 MRAC with PID compensator and self-tuning fuzzy PID controller response at speed tracking.
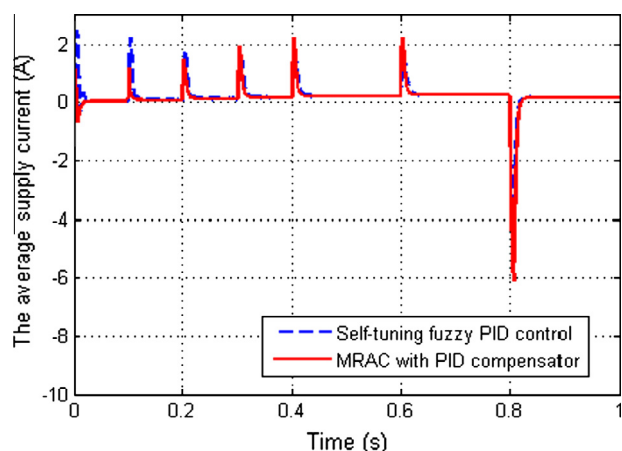

Figure 27 The average supply current of BLDC motor at different commands of speed.

response. In contrast in Fig. 22 the self-tuning fuzzy PID controller has a big change especially in the overshoot.

5.3. Speed regulation at sinusoidal load

The robustness of the proposed controller is tested by loading the BLDC motor with load torque which is continuously changing in sinusoidal form as shown in Fig. 23.

Fig. 24 shows that the motor speed is oscillated around the reference speed in both controllers in case of the self-tuning fuzzy PID controller and the maximum speed deviation is 0.41%, while the MRAC with PID compensator has maximum speed deviation 0.043%.

Fig. 25 shows the average supply current when the BLDC motor is driving a sinusoidal load torque. It is clearly noted that the average supply current is varying sinusoidally with the same frequency of the load.

5.4. Speed tracking

Fig. 26 shows that the MRAC with PID compensator has faster response than self-tuning fuzzy PID controller at different commands of reference speed. Fig. 27 shows the average supply current of BLDC motor at different commands of speed. It can be noted that the average current is increased at each new command of reference speed.

6. Conclusion

In this paper two different control techniques are implemented in order to achieve good speed regulation/tracking regardless of the presence of external disturbances and parameter variations. The first technique is the fuzzy PID in which the controller parameters are not constant and continuously updated adaptively according to the error and the change of error in order to achieve the required speed tracking. The second technique uses model reference adaptive control with PID compensator. The performance of MRAC without PID compensator suffers from high overshooting and high S.S error. By combining PID compensator with MRAC the performance will be improved. Several tests are carried out to investigate the performance of both controllers. These tests include sudden disturbance and parameter variations. Simulation results show that both controllers are robust and suitable for high performance drive applications. It is also clear that the performance of MRAC is much better than self-tuning fuzzy PID controller especially for the systems subjected to sudden disturbances.

References

[1] Xia C-L. Permanent magnet brushless DC motor drives and controls. Singapore: John Wiley & Sons Singapore Pte. Ltd.; 2012.

[2] Mohan L, Scholar M, Electronics P. Comparison of PI and adaptive fuzzy PID controllers for speed control of BLDC motor. Int J Innov Res Electr Electron Instrum Control Eng 2014;2:1050–4.

[3] Shamseldin MA, El-samahy AA. Speed control of BLDC motor by using PID control and self-tuning fuzzy PID controller. 15th int work res educ mechatron 2014.

[4] Thamizmani S, Narasimman S. Design of fuzzy PID controller for brushless DC motor. Int J Emerg Res Manage Technol 2014;9359:66–75.

[5] Agarwal P, Bose A. Brushless Dc motor speed control using proportional-integral and fuzzy controller. J Electr Electron Eng 2013;5:68–78.

[6] Jain P. Design of a model reference adaptive controller using modified MIT rule for a second order system. Adv Electron Electr Eng 2013;3:477–84.

[7] Barash MM. Adaptive control. J Manuf Syst 1986;5:145. http://dx.doi.org/10.1016/0278-6125(86)90040-3.

[8] Control P. Modeling and control of a brushless Dc motor. National Institute of Technology 2007.

[9] Thesis M. BLDC motor modelling and control – a Matlab®/Simulink® implementation; 2005.

[10] Press I, Shafer L, Arnold GW, Jacobson D. Analysis of electric machinery and drive systems; n.d.

[11] Swarnkar P. Effect of adaptation gain in model reference adaptive controlled second order system. Eng Technol Appl Sci Res 2011;1:70–5.

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