ELSEVIER
SCIENCE DIRECT®
PHYSICS LETTERS B
Physics Letters B 579 (2004) 355–360
www.elsevier.com/locate/physletb

What is coherent in neutrino oscillations

Harry J. Lipkin [a,b,c,1]

[a] *Department of Particle Physics, Weizmann Institute of Science, Rehovot 76100, Israel*
[b] *School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv, Israel*
[c] *High Energy Physics Division, Argonne National Laboratory, Argonne, IL 60439-4815, USA*

Received 24 August 2003; received in revised form 19 October 2003; accepted 7 November 2003

Editor: H. Georgi

Abstract

Simple rigorous quantum mechanics with no hand waving nor loopholes clarifies the confusion between three contradictory descriptions of the coherence between different neutrino mass eigenstates that can give rise to oscillations: (1) The standard textbook description of oscillations in time produced by coherence between states with different masses and different energies. (2) Stodolsky's proof that interference between states having different energies cannot be observed in realistic experiments. (3) The description of a pion decay at rest into an observed muon and unobserved neutrino as a "missing mass" experiment where coherence between different neutrino mass eigenstates is not observable.

The known position in space of all realistic detectors is rigorously shown to provide the quantum-mechanical ignorance of the neutrino momentum needed to produce coherence between amplitudes from neutrino states with the same energy and different masses. Conditions are precisely formulated for the loss of coherence when mass eigenstate wave packets moving with different velocities separate. The example of Bragg scattering shows how quantum-mechanically imposed ignorance produces coherence.

1. How neutrinos with different masses can be coherent

1.1. Introduction

The standard textbook description shows that a coherent linear combination of neutrino eigenstates with the same momentum and different masses have different energies and oscillate in time. But such time oscillations and coherence between states having different energies are not observed in most realistic experiments [1,2]. Furthermore coherence or interference between different neutrino mass eigenstates cannot be observed in a "missing mass" experiment where the mass of an unobserved neutrino is uniquely determined by other measurements and momentum and energy conservation.

The resolution of these contradictions is just simple quantum mechanics. In any experiment which can detect neutrino oscillations, the position of the detector must be known with an error much smaller than the wave length of the oscillation to be observed.

E-mail addresses: harry.lipkin@weizmann.ac.il, lipkin@hep.anl.gov (H.J. Lipkin).

[1] Supported in part by grant from US–Israel Bi-National Science Foundation and by the US Department of Energy, Division of High Energy Physics, Contract W-31-109-ENG-38.

doi:10.1016/j.physletb.2003.11.013

The quantum-mechanical uncertainty principle therefore forces coherence between neutrino mass eigenstates having the same energy and different momenta. Time behavior, time measurements and stationarity in energy [2] are irrelevant for this conclusion. The location in space already says it all.

This simple physical argument is now spelled out rigorously with simple quantum mechanics and no hand waving. In all realistic experiments the product of the quantum fluctuations in the position of the detector and the momentum range over which coherence is established is a very small parameter. Expanding the exact transition matrix element for the neutrino detection in powers of this small parameter and taking the leading term gives the desired result.

1.2. No coherence in a missing mass experiment

When a pion decays at rest $\pi \to \mu\nu$ the energies E_μ, E_ν and momenta $\vec{p}_\mu$, $\vec{p}_\nu$ of the neutrino and muon can all be known. This is just a "missing mass" experiment. The neutrino mass M_ν is uniquely determined by $M_\nu^2 = (M_\pi - E_\mu)^2 - p_\mu^2$. So how can there be coherence and interference between states of different mass? We are guided to the resolution of this paradox by experience in condensed matter physics discussing which amplitudes are coherent in quantum mechanics [3–6].

The original Lederman–Schwartz–Steinberger experiment found that the neutrinos emitted in a π–μ decay produced only muons and no electrons. Experiments now show that at least two neutrino mass eigenstates are emitted in π–μ decay and that at least one of them can produce an electron in a neutrino detector. The experimentally observed absence of electrons can be explained only if the electron amplitudes received at the detector from different neutrino mass eigenstates are coherent and exactly cancel. This implies that sufficient information was not available to determine the neutrino mass from energy and momentum conservation. A missing mass experiment was not performed.

1.3. Why quantum-mechanically imposed ignorance is needed

Destruction of information by simple ignorance or stupidity cannot provide coherence. The experimental setup must forbid via the quantum-mechanical uncertainty principle the knowledge of the information necessary to determine the neutrino mass. This Letter analyzes the basic physics and presents a rigorous quantitative analysis of the hand-waving uncertainty principle argument. The knowledge of the position of any realistic neutrino detector is shown to be sufficient to provide the uncertainty in momentum needed to create coherence between the amplitudes carried to the detector by components in the neutrino wave function with the same energy, different masses and different momenta.

The initial state of the detector before the interaction with the neutrino is described by a many-body wave function that exists only in a finite region of configuration space. The probability is zero for finding any detector nucleon anywhere in space outside of this volume. This exact property of the exact initial state is rigorously shown below to prevent the detector from recognizing the difference between two incident neutrinos with the same energy and slightly different momenta. It ensures the quantum-mechanical ignorance needed to produce coherence. This physics can be handwaved and called the uncertainty principle. But it can also be proved rigorously [7].

2. The basic physics of neutrino detection

2.1. The neutrino wave packet

The neutrino wave packet traveling between source and detector vanishes outside some finite interval in space at any given time. At any point on its path it also vanishes outside some finite time interval. The packet therefore contains components with different momenta and different energies which are all coherent with well-defined phases to cancel out at all points in space and time where the probability of finding the neutrino vanishes.

However, not all the different kinds of coherence present in the wave packet are observable with a conventional detector. The detector is sensitive in very different ways to the different components in the wave packet [2,7,8].

2.2. The role of the neutrino detector

Neutrino absorption is a weak interaction described completely by the transition matrix of the weak interaction operator between the exact initial and final states of the lepton and detector, where the exact states include all strong interactions. This matrix element can be expanded in powers of a small parameter, the product of the displacement of the detector nucleon from the center of the detector and a momentum interval which includes all momenta of incident neutrinos having the same energy.

We shall now show that the leading term in this expansion gives the lepton flavor output for each energy component in the initial neutrino wave function as the coherent sum of the contributions from states with the same energy and different momenta. This is exact subject only to corrections of higher order in the small parameter which are negligible as long as the size of the detector is negligibly small in comparison with any neutrino oscillation wave length.

Consider the transition matrix element between an initial state $|i(E)\rangle$ with energy E of the entire neutrino—detector system and a final state $|f(E)\rangle$ of the system of a charged muon and the detector with the same energy E, where a neutrino ν_k with energy, mass and momentum E_ν, m_k and $\vec{P}_o + \delta\vec{P}_k$ is detected via the transition

$$\nu_k + p \to \mu^+ + n, \tag{2.1}$$

occurring on a proton in the detector. We express the neutrino momentum as the sum of the mean momentum $\vec{P}_o$ of all the neutrinos with energy E_ν and the difference $\delta\vec{P}_k$ between the momentum of each mass eigenstate and the mean momentum.

The transition matrix element depends upon the individual mass eigenstates k only in the momentum difference $\delta\vec{P}_k$ and a factor c_k for each mass eigenstate which is a function of neutrino mixing angles describing the transition amplitude for this mass eigenstate to produce a muon when it reaches the detector. The transition matrix element can thus be written in a factorized form with one factor T_o independent of the mass m_k of the neutrino and a factor depending on m_k.

$$\langle f(E)|T|i(E)\rangle = \sum_k \langle f(E)|T_o \cdot c_k e^{i\delta\vec{P}_k\cdot\vec{X}}|i(E)\rangle, \tag{2.2}$$

where $\vec{X}$ denotes the co-ordinate of the nucleon that absorbs the neutrino. Then if the product $\delta\vec{P}_k \cdot \vec{X}$ of the momentum spread in the neutrino wave packet and the fluctuations in the position of the detector nucleon is small, the exponential can be expanded and approximated by the leading term

$$\begin{aligned}\langle f(E)|T|i(E)\rangle &= \sum_k \langle f(E)|T_o \cdot c_k e^{i\delta\vec{P}_k\cdot\vec{X}}|i(E)\rangle \\ &\approx \sum_k \langle f(E)|T_o \cdot c_k|i(E)\rangle.\end{aligned} \tag{2.3}$$

The transition matrix element for the probability that a muon is observed at the detector is thus proportional to the coherent sum of the amplitudes c_k for neutrino components with the same energy and different masses and momenta to produce a muon at the detector. A similar result is obtained for the probability of observing each other flavor. The final result is obtained by summing the contributions over all the energies in the incident neutrino wave packet. But as long as the flavor output for each energy is essentially unchanged over the energy region in the wave packet, the flavor output is already determined for each energy, and is independent of any coherence or incoherence between components with different energies.

For the case of two neutrinos with energy E and mass eigenstates m_1 and m_2 the relative phase of the two neutrino waves at a distance x is:

$$\phi_m^E(x) = (p_1 - p_2)\cdot x = \frac{(p_1^2 - p_2^2)}{(p_1 + p_2)}\cdot x = \frac{\Delta m^2}{2p}\cdot x, \tag{2.4}$$

where $\Delta m^2 \equiv m_2^2 - m_1^2$, and we have assumed the free space relation between the masses, m_i energy E and momenta.

The flavor output of the detector is thus seen to be determined by the interference between components in the neutrino wave packet with the same energy and different masses and momenta. All the relevant physics is in the initial state of the nucleon in the detector that detects the neutrino and emits a charged lepton, together with the relative phases of the components of the incident neutrino wave packet with the same energy.

This result (2.3), (2.4) is completely independent of the neutrino source and in particular completely independent of whether the source satisfies Stodolsky's stationarity condition [2]. No subsequent time measurements or additional final state interactions that mix energies can change this flavor output result.

The initial uncertainty in the momentum of the detector nucleon destroys all memory of the initial neutrino momentum and of the initial neutrino mass after the neutrino has been absorbed. The hand-waving justification of the result (2.3) uses the uncertainty principle and says that if we know where the detector is we do not know its momentum and cannot use momentum conservation to determine the mass of the incident neutrino. The above rigorous justification shows full interference between the contributions from different neutrino momentum states with the same energy as long as the product of the momentum difference and the quantum fluctuations in the initial position of the detector nucleon is negligibly small in the initial detector state.

This treatment of the neutrino detector is sufficient to determine the output of any experiment in which the incident neutrino wave packet is the same well-defined linear combination of mass eigenstates throughout the whole wave packet.

2.3. *At what distance is coherence lost?*

The above treatment has not considered the effects resulting from the different velocities of neutrino wave packets with different masses. The difference in velocity between components in two wave packets $(\delta v)_m$ with the same energy and different mass is just the difference in velocities $v = p/E$ for states with different momenta and the same energy,

$$(\delta v)_m = \frac{\partial}{\partial p} \cdot \left(\frac{p}{E}\right)_E \cdot (\delta p)_m = \frac{(\delta p)_m}{E}. \tag{2.5}$$

The packets will eventually separate and arrive at a remote detector at different separated time intervals. The detector then sees two separated probability amplitudes, each giving the probability that the detector observes a given mass eigenstate. All coherence between the different mass eigenstates is then lost. The question then arises when and where this occurs, i.e., at what distance from the source the coherence begin to be lost. Two different approaches to this problem give the same answer [9].

(1) The centers of the wave packets move apart with the relative velocity $(\delta v)_m$ given by Eq. (2.5). Thus the separation $(\delta x)_m$ between the wave packet centers after a time t when the centers are at a mean distance x from the source is

$$\begin{aligned}(\delta x)_m &= (\delta v)_m \cdot t = (\delta v)_m \cdot \frac{x}{v} \\ &= -\frac{\Delta m^2}{2pE} \cdot \frac{xE}{p} = -\frac{\Delta m^2}{2p^2} \cdot x.\end{aligned} \tag{2.6}$$

The wave packets will separate when this separation distance is comparable to the length in space of the wave packet. The uncertainty principle suggests that the length of the wave packet $(\delta x)_W$ and its spread in momentum space $(\delta p)_W$ satisfy the relation

$$(\delta x)_W \cdot (\delta p)_W \approx 1/2. \tag{2.7}$$

The ratio of the separation over the length is of order unity when

$$\left|\frac{(\delta x)_m}{(\delta x)_W}\right| \approx \left|\frac{\Delta m^2}{p^2}\right| \cdot (\delta p)_W \cdot x \approx 1. \tag{2.8}$$

(2) Stodolsky [2] has suggested that one need not refer to the time development of the wave packet, but only to the neutrino energy spectrum. The relative phase $\phi_m(x)$ between the two mass eigenstate waves at a distance x from the source depends upon the neutrino momentum p_ν as defined by the relation (2.4).

Coherence will be lost in the neighborhood of the distance x where the variation of the phase over the momentum range $(\delta p)_W$ within the wave packet is of order unity. For the case of two neutrinos with energy E and mass eigenstates m_1 and m_2 the condition that the relative phase variation $|\delta\phi_m(x)|$ between the two neutrino waves is of order unity

$$\left|\delta\phi_m(x)\right| = \left|\frac{\partial\phi_m(x)}{\partial p_\nu}\right|\delta p_\nu \cdot x = \left|\frac{\Delta m^2}{2p_\nu^2}\right|(\delta p)_W \cdot x \approx 1. \tag{2.9}$$

We find that the two approaches give the same condition for loss of coherence.

3. How incomplete information provides coherence

3.1. *Bragg scattering*

Bragg scattering of photons by a crystal provides an instructive example of coherence arising from incomplete information on momentum conservation. Coherence between the photon scattering amplitudes from different atoms in the crystal produces constructive interference at the Bragg angles and gives peaks in the angular distribution. When a single photon is scattered from a crystal, momentum is transferred to the atom in the crystal that scattered the photon. If the recoil momentum is detected the atom that scattered the photon is identified and coherence is destroyed. Coherence arises when quantum mechanics prevents the measurement of the initial and final momenta of the individual atoms.

The initial and final states of the crystal are many-particle quantum states that are eigenstates of the Hamiltonian of the crystal. The dynamics of the crystal and the interaction with the incident photon allow elastic scattering, in which the photon is scattered by a single atom in the crystal but the quantum state of the crystal is unchanged. This is a purely quantum effect. Transferring momentum classically to an atom in a crystal must change the momentum and the motion of the particular atom and allow the identification of which atom scattered the photon.

The difference produced by quantum mechanics is simply seen in a toy model in which each atom is bound to its equilibrium position in the crystal by a harmonic oscillator potential. The atom that scatters the photon is initially in a definite discrete energy level in the potential. In contrast to the classical case, the atom cannot absorb the momentum transfer according to the energy and momentum kinematics of free particles. The final state of the atom in the potential must be one of the allowed energy levels, and there is a finite probability that the final state is the same as the initial state. In this case of elastic scattering, there is no information available on which atom scattered the photon, and the scattered amplitudes from all scattering atoms are coherent.

This example shows how amplitudes arising from different processes which would be classically distinguishable can be coherent. The quantum mechanics of bound systems can conceal the information which would be classically available from energy–momentum conservation for free particles.

3.2. Pion decay

This same effect conceals the mass of the neutrino emitted in pion decay. The initial pion in a beam stop cannot be strictly at rest; it is localized by its electrostatic interaction with the electric charges in the material where it was stopped. It is therefore in some kind of energy level of the bound system and described by a wave function which is a coherent linear combination of different momentum eigenstates. Measuring the energy of the muon determines the energy of the emitted neutrino, since the energy of the initial state is determined. But the momentum of the neutrino is not determined. In a simple toy model where the initial pion is bound by some external potential, it is described by a wave function which is a coherent wave packet in momentum space.

When the neutrino strikes a detector, the amplitudes produced by different mass eigenstates having the same energy and different momenta can be coherent. They are produced from the different momentum components in the initial pion wave function which are coherent with a definite relative phase. This can explain why no electrons are observed at a short distance from the detector.

If the neutrino amplitudes produced in this way propagate as free particles, these considerations determine completely the relative phase between the amplitudes for neutrinos having the same energy but different masses and different momenta. The phase change will produce neutrino oscillations with the same relation between mass differences and phase differences (2.4) that has been given by the standard treatments.

4. Time measurements, momentum and energy

4.1. The possibility of time measurements

The preceding analysis does not consider experiments in which the transit time of the neutrino between source and absorber is measured. Experiments have been suggested in which the muon emitted together with the neutrino in a pion decay is observed at the neutrino source and the time that the muon is detected is measured precisely along with the time that the muon or electron is produced by absorbing the neutrino in the detector. The motivation is to use some kind of energy–time uncertainty to detect interference between components having different energies in the neutrino wave function.

However, in any realistic detector the quantum fluctuations in the position of the detector nucleon are small in comparison with the wave length of the neutrino oscillation. Thus the coherence and the relative phase of the components in the neutrino wave function having the same energy and different momenta are preserved. This relative phase completely determines the flavor output of the detector, i.e., the relative probabilities of producing a muon or an electron. In all realistic cases where the separation of wave packets moving with different velocities is negligible, Eqs. (2.9) and (2.8) show that these probabilities are essentially independent of energy over the relevant energy range. Thus the relative phases and coherence between components in the neutrino wave function with different energies is irrelevant. All energies give the same muon/electron ratio whether they add coherently or incoherently. Thus time measurements cannot change the muon/electron ratio observed at the detector.

Thus the flavor output from any time of flight experiment that uses a neutrino detector that preserves the coherence between states of the same energy and different momentum is already determined at the single energy level. It is unaffected by any interference between components of the neutrino wave function with different energies.

4.2. The difference between momentum and energy

Confusion tends to arise from thinking that momentum and energy should be on the same footing, particularly since relativity implies that they are components of the same four vector. But this is only true for isolated free particles. In any realistic neutrino experiment the neutrino is observed by a weak interaction with a detector. The detector, in its rest frame before the arrival of the neutrino, is in an initial state [2] described by a density matrix in which energy is diagonal and momentum is not. This is the critical difference between energy and momentum. There is no coherence and no well-defined relative phase between components in the detector density matrix with different energies. But there must be coherence and well-defined relative phases between components with different momenta, as shown rigorously by Eq. (2.2), because we know where the detector is in space and where it is not. The form factor (2.2) is seen to be negligibly different from unity as long as the quantum fluctuations in the position of the detector are small in comparison with the wave length of the oscillation being measured.

5. Conclusions

Coherence between amplitudes produced by neutrinos incident on a detector with different masses and the same energy has been shown to follow from the localization of the detector nucleon within a space interval much smaller than the wave length of the neutrino oscillation. Decoherence between different mass eigenstates results from the separation of wave packets moving with different velocities and is simply described also in terms of the energy dependence of the flavor output of a detector.

That coherence must exist in neutrinos emitted from π–μ decay follows from the original Lederman–Schwartz–Steinberger experiment which saw only muons and no electrons. We now know that at least two different neutrino mass eigenstates are emitted from π–μ decay and that at least one must couple to electrons. The only explanation for the absence of electrons at the detector is destructive interference from amplitudes produced by different mass eigenstates.

Acknowledgements

It is a pleasure to thank Eyal Buks, Maury Goodman, Yuval Grossman, Moty Heiblum, Yosef Imry, Boris Kayser, Lev Okun, Gilad Perez, David Sprinzak, Ady Stern, Leo Stodolsky and Lincoln Wolfenstein for helpful discussions and comments.

References

[1] H.J. Lipkin, Phys. Lett. B 348 (1995) 604.
[2] L. Stodolsky, Phys. Rev. D 58 (1998) 036006.
[3] Y. Aharonov, F.T. Avignone III, A. Casher, S. Nussinov, Phys. Rev. Lett. 58 (1987) 1173.
[4] H.J. Lipkin, Phys. Rev. Lett. 58 (1987) 1176.
[5] A. Stern, Y. Aharonov, Y. Imry, Phys. Rev. A 41 (1990) 3436.
[6] B. Kayser, Phys. Rev. D 24 (1981) 110.
[7] H.J. Lipkin, Phys. Lett. B 477 (2000) 195, and references therein, hep-ph/9907551.
[8] W. Grimus, P. Stockinger, S. Mohanty, Phys. Rev. D 59 (1999) 013011, hep-ph/9807442.
[9] H.J. Lipkin, hep-ph/9901399, in: Proceedings of the Europhysics Neutrino Oscillation Workshop (NOW'98), Amsterdam, 7–9 September 1998, see http://www.nikhef.nl/pub/conferences/now98/.