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Leading order one-loop CP and P violating effective action in the Standard Model

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ABSTRACT

The fermions of the Standard Model are integrated out to obtain the effective Lagrangian in the sector violating P and CP at zero temperature. We confirm that no contributions arise for operators of dimension six or less and show that the leading operators are of dimension eight. To assert this we explicitly compute one such non-vanishing contribution, namely, that with three Z^0, two W^+ and two W^-. Terms involving just gluons and W's are also considered, however, they turn out to vanish in the P-odd sector to eighth order. The analogous gluonic term in the CP-odd and P-even (C-odd) sector is non-vanishing and it is also computed. The expressions derived apply directly to massive Dirac neutrinos. All CP-violating results display the infrared enhancement already found at dimension six.

1. Introduction

A full understanding of CP-violation remains a challenge and for this reason it is a fruitful field of research both in relation with the Standard Model of particle physics and in extensions thereof [1–9]. CP-violation enters in very different phenomena, like non-vanishing of the electric dipole moment of elementary particles, baryogenesis [10], or, assuming CPT invariance, the puzzling T-violation. Yet, in the Standard Model, CP-violation is rather elusive. There is no trace of it in the QCD sector, while in the electroweak sector it enters through a small parameter in the CKM matrix for quarks [11], and possibly also for leptons, for massive neutrinos [12]. Even in the electroweak sector manifestation of CP breaking involves a subtle combination, the Jarlskog determinant Δ, which requires order twelve in the quark (or leptons) masses and would vanish if two up-like or two down-like quarks were degenerated in mass [13]. In any case only through fermions CP can be broken in the Standard Model. The structure of the Standard Model action implies that integration of the fermions results in an effective Lagrangian of the form (we assume the unitary gauge throughout)

$$\mathcal{L}^{\text{eff}}(x) = \sum_\alpha g_\alpha \left(\frac{v}{\phi(x)}\right)^{d_\alpha - 4} \mathcal{O}_\alpha(x), \tag{1.1}$$

where $\mathcal{O}_\alpha(x)$ represents any possible operator, of mass dimension d_α, constructed as a Lorentz and gauge invariant product of the gauge fields, their derivatives and derivatives of the Higgs field. g_α is the operator coupling constant, with mass dimension $4 - d_\alpha$. $\phi(x)$ denotes the (unsubtracted) Higgs field and v its vacuum expectation value. The coupling constant (which may vanish for some operators) has two additive contributions, one from the quark loop and another from the lepton loop. In the CP-odd sector, g_α must contain the Jarlskog determinant. In terms of the Yukawa coupling this yields a tiny dimensionless number, Δ/v^{12}, of the order of 10^{-24}. This fact has occasionally been presented as an indication of an intrinsic limitation of the Standard Model to produce enough CP-breaking to account for observations, including the baryon asymmetry. While this might be true, qualitative arguments should eventually be supported by a detailed computation. Smit argued in [14] that the coupling g_α is just a homogeneous function of the quarks (or leptons) masses of the appropriate degree. This implies that $g_\alpha \sim \Delta \times I_\alpha$, where I_α has a large negative degree to compensate that of Δ. Both Δ and I_α depend only on the fermion masses and do not involve v. On the other hand, the various quark masses are very different and widely different result can be obtained by combining them at random. Actual calculations have been carried out in [15,16] for operators of dimension six, which is the first possible CP-violating contribution at one-loop. They show that $g_\alpha \sim J\kappa/m_c^2$ where is m_c the charm quark mass, $J = 2.9(2) \times 10^{-5}$ is the Jarlskog invariant [17] and κ is a dimensionless coefficient of the order of unity. Implications for cold electroweak baryogenesis have been considered in [18,19]. Unfortunately, these two calculations differ in that [15] finds such a dimension six contribution in the P-odd sector whereas [16] finds a contribution in the C-odd sector but none in the P-odd one.

The purpose of this note is manifold. First, to reduce to the simplest and more transparent terms the calculation of these couplings constants. Second, to confirm that, although dimension six CP-odd and P-odd operators do exist, their coupling vanish in the

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Standard Model. Third, to verify that the order six cancellation is accidental, and non-vanishing contributions in the *CP*-odd and *P*-odd sector appear for the first time at dimension eight. The purely gluonic leading (eighth) order term is also computed since it is particularly simple. As it turns out, this term breaks *C* but not *P*. Lastly, to verify that the enhancement (as compared to the naive estimate) found at order six is displayed also at higher orders.

2. The method

We will integrate out the fermions in the Standard Model to extract the *CP* violating contribution of the resulting effective action. This is the one-loop approximation to the full effective action with one-particle irreducible bosonic lines and vertices. We work at zero temperature. Quarks will be explicitly considered. Leptons would not contribute to the *CP*-odd sector if neutrinos are assumed to be exactly massless. For massive Dirac neutrinos the contribution of the leptons will be completely analogous to the one obtained for quarks.

The quark-sector Lagrangian of the Standard Model, in its Euclidean version and in the unitary gauge, can be written as [20]:

$$\mathcal{L}(x) = \bar{q}(x)\mathbf{D}q(x) = (\bar{q}_L, \bar{q}_R)\begin{pmatrix} m & \not{D}_L \\ \not{D}_R & m \end{pmatrix}\begin{pmatrix} q_R \\ q_L \end{pmatrix}. \tag{2.1}$$

Here $q_{L,R}$ carry Dirac, generation (family), *ud* and color indices (*ud* space distinguishes the up-like from down-like quarks in each generation). Expanding further the matrices in *ud* space:

$$m = \begin{pmatrix} \frac{\phi}{v}m_u & 0 \\ 0 & \frac{\phi}{v}m_d \end{pmatrix}, \qquad \not{D}_L = \begin{pmatrix} \not{D}_u + \not{Z} + \not{G} & \not{W}^+C \\ \not{W}^-C^{-1} & \not{D}_d - \not{Z} + \not{G} \end{pmatrix},$$
$$\not{D}_R = \begin{pmatrix} \not{D}_u + \not{G} & 0 \\ 0 & \not{D}_d + \not{G} \end{pmatrix}. \tag{2.2}$$

Here $m_{u,d}$ are the diagonal matrices (in generation space) with the up-like and down-like quarks masses, respectively. G_μ is the gluon field, Z_μ the Z^0 field, $W^\pm_\mu$ the W boson fields, C is the CKM matrix, finally $(D_\mu)_{u,d} = \partial_\mu + q_{u,d}B_\mu$ where $q_u = 2/3$, $q_d = -1/3$, and B_μ is the weak hypercharge gauge connection. For convenience, in all cases the coupling constant has been included in the corresponding gauge connection. Further details can be found in [16].

After integration of the quark loop, the corresponding Euclidean effective action is just

$$\Gamma = -\operatorname{Tr}\log\mathbf{D}. \tag{2.3}$$

Γ is the sum of all the Feynman graphs with one quark loop and any number of bosonic legs, gauge fields and Higgs. This sum is written as a functional which will be expressed within a covariant derivative expansion of these bosonic fields.

Certainly, Γ can be computed following the efficient method outlined in [16] and based on [21], applied there to sixth order in the derivative expansion. However, one of our goals here is to present a derivation as transparent as possible, and closer to the method introduced in [22] on which the calculation of [15] is based. To this end, we will use the relation

$$\delta\Gamma = -\operatorname{Tr}\big(\delta\mathbf{D}\mathbf{D}^{-1}\big) = -\int d^4x\, \operatorname{tr}\big[\delta\mathbf{D}\langle x|\mathbf{D}^{-1}|x\rangle\big]. \tag{2.4}$$

In the second equality it has been used that the variation $\delta\mathbf{D}$, induced by the variation in the gauge and Higgs fields in $\mathbf{D}$, contains no derivatives. The method is then to choose a suitable variation of these fields, compute $\delta\Gamma$ in the desired sector, and subsequently seek a functional Γ fulfilling such a variation. The virtues of this approach are i) the current $\langle x|\mathbf{D}^{-1}|x\rangle$ is easier to obtain than the $\operatorname{Tr}\log\mathbf{D}$ itself, ii) the condition on $\delta\Gamma$ of being a consistent variation provides a non-trivial check of the calculation, and iii) even if one where to compute Γ directly, the simplest way to avoid integration by parts identities (i.e., redundant operators in the final expression) is to obtain its functional derivative, $\delta\Gamma/\delta\mathbf{D} = -\langle x|\mathbf{D}^{-1}|x\rangle$. This quantity is local and so free from x-integration by parts identities. This issue becomes increasingly important as the number of derivatives increases.

A convenient field to use as variation in Eq. (2.4) is Z which appears just in D^L_μ. We adopt such a choice, namely,

$$\delta\not{D}_L = \begin{pmatrix} \delta\not{Z} & 0 \\ 0 & -\delta\not{Z} \end{pmatrix} := \delta\hat{\not{Z}}, \qquad \delta\not{D}_R = \delta m = 0. \tag{2.5}$$

The Dirac operator can be written as $\mathbf{D} = P_L\not{D}_RP_R + P_R\not{D}_LP_L + P_RmP_R + P_LmP_L$ where $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$ project on the R or L spaces, respectively. $\mathbf{D}$ can be explicitly inverted in chiral blocks. In particular, for the RL block, $P_R\mathbf{D}^{-1}P_L = P_R(\not{D}_L - m\not{D}_R^{-1}m)^{-1}P_L$. Hence, from Eqs. (2.4) and (2.5),

$$\delta\Gamma = -\operatorname{Tr}\big(P_R\,\delta\hat{\not{Z}}\,(\not{D}_L - m\not{D}_R^{-1}m)^{-1}\big). \tag{2.6}$$

Therefore, if only the P-odd sector (i.e., that with a γ_5) is retained, we will have

$$\delta\Gamma^- = -\frac{1}{2}\operatorname{Tr}\big[\gamma_5\delta\hat{\not{Z}}(\not{D}_L - m\not{D}_R^{-1}m)^{-1}\big]$$
$$= -\frac{1}{2}\operatorname{Tr}\big[\gamma_5\delta\hat{\not{Z}}(\not{D}_L^{-1} + \not{D}_L^{-1}m\not{D}_R^{-1}m\not{D}_L^{-1} + \cdots)\big]. \tag{2.7}$$

It can be noted that each term of the expansion between parenthesis starts and ends with a label L. Moreover, as the chiral label propagates through the term it flips (from L to R and vice versa) at m but not at D_R or D_L. Keeping these rules in mind we can simply write

$$\delta\Gamma^- = -\frac{1}{2}\operatorname{Tr}\big[\gamma_5\delta\hat{\not{Z}}(\not{D}^{-1} + \not{D}^{-1}m\not{D}^{-1}m\not{D}^{-1} + \cdots)\big]$$
$$= -\frac{1}{2}\operatorname{Tr}\big[\gamma_5\delta\hat{\not{Z}}(\not{D} + m)^{-1}\big]. \tag{2.8}$$

The second equality follows from the fact that terms with an odd number of m's are automatically discarded since they cannot start and end with a label L.

A simple and convenient technique to compute $\langle x|(\not{D} + m)^{-1}|x\rangle$ is the method of symbols [16,23,24]. This method is suitable for computing one-loop Feynman graphs with external legs at zero momentum or more generally, for expansions around zero momentum. For an operator constructed with covariant derivatives, D, and other fields, M, the method gives

$$\operatorname{Tr} f(D, M) = \int d^4x\, \operatorname{tr}\langle x|f(D, M)|x\rangle = \int \frac{d^4x\, d^4p}{(2\pi)^4}\operatorname{tr} f(D + ip, M). \tag{2.9}$$

The integrand $f(D + ip, M)$ is a (pseudo)differential operator, however, after momentum integration all D_μ appear only in the form $[D_\mu, \,]$. Therefore, the subsequent x-integration is just on an ordinary function. In addition, the structure $[D_\mu, \,]$ ensures gauge invariance of the result. In the present case it takes the form

$$\delta\Gamma^- = -\frac{1}{2}\int \frac{d^4x\, d^4p}{(2\pi)^4}\operatorname{tr}\big[\gamma_5\delta\hat{\not{Z}}(\not{D} + i\not{p} + m)^{-1}\big]. \tag{2.10}$$

Note that the momentum variable p_μ, like $D^{R,L}_\mu$, does not introduce a flip in the chiral label.

The covariant derivative expansion is just an expansion in powers of D_μ in Eq. (2.10). Only even orders contribute[1] (the space–time dimension being even). Besides, in the P-odd sector the effective action starts at fourth order in four dimensions, since a Levi-Civita pseudotensor must be present. Hence:

$$\delta\Gamma^- = \frac{1}{2}\int \frac{d^4x\, d^4p}{(2\pi)^4} \times \mathrm{tr}\left[\gamma_5 \delta\hat{\not{Z}} \sum_{n=0}^{\infty}\left(\frac{i\not{p}-m}{p^2+m^2}\not{D}\right)^{2n+3} \frac{i\not{p}-m}{p^2+m^2}\right]. \tag{2.11}$$

3. Sixth order *P*-odd terms

Taking the appropriate values of n in Eq. (2.11) and the appropriate contributions in $D^{R,L}$ and m, one can select the desired terms of the effective action. At least two W^+W^- pairs must be present in the CP-odd sector terms, since the quark loop must visit the three generations [11].[2] A term with just $(W^+W^-)^2$ would count as fourth order, however no CP-odd term can be constructed without introducing further fields or derivatives. To sixth order such a P-odd and CP-odd term can be written combining $(W^+W^-)^2ZD$. (D here refers to either D_u or D_d.) The question is whether this operator appears with a non-vanishing coefficient in the Standard Model or not. Ref. [15] claims that it does whereas the calculation in [16] concludes that it does not. Therefore we will start by reconsidering such a term within our present approach.

Under a variation of Z, the contribution of the candidate term to be found in Eq. (2.11) is of the form $\delta Z(W^+W^-)^2D$. We can set $\phi = v$, since we are not interested in contributions from Higgs, and likewise we can set Z_μ and G_μ to zero in $D_\mu^{R,L}$. Moreover, we can even set $D_\mu^u = D_\mu^d = \partial_\mu$ in $D_\mu^{R,L}$. The ordinary derivatives can be unambiguously replaced by covariant ones at the end without loss of information since no $F_{\mu\nu}$ tensor can be present in the term considered (which has just a single D operator).

The computation is tedious but straightforward using computer algebra software. Let us spell out the main steps in the calculation. We select terms with $n = 1$ (order six) in Eq. (2.11) and restore the L, R labels. We keep only terms starting and ending with the label L and only m introduces a chiral label flip $L \leftrightarrow R$. No flip is introduced by m^2, p or D. This yields terms of the type

$$\delta\Gamma^- = \int \frac{d^4x\, d^4p}{(2\pi)^4} \times \mathrm{tr}\left[\frac{1}{2}\gamma_5 \delta\hat{\not{Z}} N\not{p}\hat{\not{W}} Nm\not{\partial} Nm\hat{\not{W}} N\not{p}\hat{\not{W}} N\not{p}\hat{\not{W}} N\not{p} + \cdots\right] + \text{o.t.} \tag{3.1}$$

Here we have set $\not{D}_R = \not{\partial}$ and $\not{D}_L = \not{\partial} + \hat{\not{W}}$ where $\hat{\not{W}}$ represents the off diagonal (charged) part of $\not{D}_L$ as a matrix in ud space (see Eq. (2.2)). We have retained terms with precisely four $\hat{W}$'s and one derivative. Also we have introduced the quantity $N = (p^2 + m^2)^{-1}$. The dots in Eq. (3.1) refer to further terms of the same type (10 terms in all) while "o.t." refers to other terms which cannot have a contribution to the pattern $\delta Z(W^+W^-)^2D$.

Next, we expand the ud labels using Eq. (2.2) for m and $\hat{\not{W}}$, and Eq. (2.5) for $\delta\hat{Z}$. This produces

$$\delta\Gamma^- = \int \frac{d^4x\, d^4p}{(2\pi)^4} \times \mathrm{tr}\left[\frac{1}{2}\gamma_5 \delta\not{Z} N_u\not{p}\not{W}^+ CN_d m_d \not{\partial} N_d m_d \not{W}^- C^{-1} \times N_u\not{p}\not{W}^+ CN_d\not{p}\not{W}^- C^{-1} N_u\not{p} + \cdots\right] + \text{o.t.} \tag{3.2}$$

(20 terms in all), where $N_{u,d} = (p^2 + m_{u,d}^2)^{-1}$.

At this point we can already factorize the trace between quantities which act only in generation space, namely, $N_{u,d}$, $m_{u,d}$ and C, and all the other quantities, which do not act on that space:

$$\delta\Gamma^- = \int \frac{d^4x\, d^4p}{(2\pi)^4} \times \left(\frac{1}{2}\mathrm{tr}\left[N_u C N_d^2 m_d^2 C^{-1} N_u C N_d C^{-1} N_u\right] \times \mathrm{tr}\left[\gamma_5\, \delta\not{Z}\, \not{p}\not{W}^+ \not{\partial}\not{W}^- \not{p}\not{W}^+ \not{p}\not{W}^- \not{p}\,\right] + \cdots\right) + \text{o.t.} \tag{3.3}$$

It is a general rule that m_u or m_d can only appear raised to even powers and therefore they can be eliminated in favor of N_u and N_d.[3] Eventually, all required momentum integrals and traces on 3×3 matrices in generation space can be cast in the form [16]

$$I^k_{a,b,c,d} = \int \frac{d^4p}{(2\pi)^4}(p^2)^k\, \mathrm{tr}\left[N_u^a C N_d^b C^{-1} N_u^c C N_d^d C^{-1}\right], \tag{3.4}$$

where the exponents k, a, b, c, d are non-negative integers. On the other hand, only the CP-odd contribution is of interest to us. This is the component antisymmetric under the exchange $C \to C^*$,

$$\hat{I}^k_{a,b,c,d} = i\,\mathrm{Im}\, I^k_{a,b,c,d}. \tag{3.5}$$

Due to cyclic and hermiticity properties of the trace and matrices involved, these integral satisfy

$$\hat{I}^k_{a,b,c,d} = -\hat{I}^k_{c,b,a,d} = -\hat{I}^k_{a,d,c,b}\,. \tag{3.6}$$

Such antisymmetry under exchange of the labels a and c, or b and d implies that many terms in Eq. (3.3) do not have a contribution to the CP-odd sector and this greatly alleviates the amount of subsequent computation.

Performing an angular average over the momentum and taking the color and Dirac traces in Eq. (3.3) yields then, for CP-odd terms ($N_c = 3$ is the number of colors)

$$\delta\Gamma^- = N_c \int d^4x \left(\frac{1}{3}\hat{I}^3_{1,1,2,2}\epsilon_{\mu\nu\alpha\beta}\delta Z_\mu W^-_\nu \partial_\alpha W^+_\beta W^-_\lambda W^+_\lambda + \cdots\right) + \text{o.t.} \tag{3.7}$$

There are 12 terms, all of them with the same coefficient $\hat{I}^3_{1,1,2,2}$. In this expression the derivative (∂_α) is still an operator. Next, all the derivative operators are moved to the right by repeated use of the identity $\partial X = X\partial + [\partial, X]$. After this is done, one indeed verifies that all terms with a ∂ operator at the right cancel with each other and so the integrand is just an ordinary function, as it should be. Using now the antisymmetry of the Levi-Civita tensor, it is found that all surviving terms actually cancel among them. Note that prior to bringing the derivatives to the form $[\partial, X]$ the factors in a term could not be reordered at will and so the cancellation was not manifest. However, as noted in [16], the final expression

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1 Note that $\delta\hat{\not{Z}}$ counts as order 1.

2 Of course, terms with a single W^+W^- pair are allowed beyond one-loop.

3 The label u or d does not change between two consecutive W's. This implies two consecutive D_L and so an even number of m's. That the presence of $m_{u,d}$ can be obviated follows also from Eq. (7.1) of [16].

of Γ must be symmetric under exchange of labels u and d. From this observation and the fact that $\hat{I}^3_{1,1,2,2}$ has the wrong symmetry under $u \leftrightarrow d$ it could already be inferred that the whole contribution would vanish [16].

So there are no terms of the type $(W^+W^-)^2 ZD$ in the CP-odd, P-odd sector of the Standard Model, in agreement with the alternative and more systematic calculation in [16].

4. Dimension eight operators

In this section we show that at eighth order in the derivative expansion there are non-vanishing contributions in the CP-odd and P-odd sector of the Standard Model. Concretely we consider terms of the form $(W^+W^-)^2 Z^3 D$, with no gluons nor derivatives of the Higgs field, and apply the technique just described. In this case we select terms with $n=2$ (order eight) in Eq. (2.11) and seek terms of the type $\delta Z(W^+W^-)^2 Z^2 D$. The calculation is analogous to the one shown previously, except that now Z is not set to zero in $\not{D}_L$, instead we use $\not{D}_L = \not{\partial} + \hat{\not{W}} + \hat{\not{Z}}$. Then we keep terms with precisely four $\hat{W}$'s, two $\hat{Z}$'s and one derivative. After restoring the L, R labels and u, d labels, and carrying out the momentum integration, the trace in color, Dirac and generation space, and applying the derivative to the right, one obtains:

$$\delta\Gamma^- = N_c \int d^4x \, \big(2\hat{I}^4_{1,1,2,4}\epsilon_{\mu\nu\alpha\beta}\delta Z_\mu Z_\lambda Z_{\nu\alpha} \times W^+_\beta W^+_\lambda W^-_\sigma W^-_\sigma + \cdots\big) + \text{o.t.} \tag{4.1}$$

Here $Z_{\nu\alpha}$ stands for the ν derivative of Z_α. To simplify the expression we have eliminated[4] the $\hat{I}^3_{a,b,c,d}$ in favor of $\hat{I}^4_{a,b,c,d}$ and have used identities involving $\delta_{\mu\nu}$ and $\epsilon_{\mu\nu\alpha\beta}$ to bring the expression to a canonical form.[5] The expression so obtained contains 62 operators, each one weighted with various integrals $\hat{I}^4_{a,b,c,d}$.

It remains to find out the effective action from which the variation in Eq. (4.1) derives. The method is just to propose all allowed independent operators of the form $(W^+W^-)^2 Z^3 D$ with arbitrary coefficients,[6] and take a first order variation with respect to Z to fix those coefficients. The fact that $\delta\Gamma^-$ turns out to be consistent is a non-trivial check of the computation. The Minkowski space result (see [16] for further details in the conventions) is

$$\begin{aligned}\mathcal{L}^{\text{eff}}(x) = \frac{N_c}{15}\frac{v^4}{\phi^4}\epsilon_{\mu\nu\alpha\beta}\big[&(12\hat{I}_1 - 16\hat{I}_2)Z_\mu Z^2_\lambda W^+_\nu W^+_\sigma W^-_\alpha W^-_{\beta\sigma} \\ &+ (4\hat{I}_1 + 23\hat{I}_2)Z_\mu Z^2_\lambda W^+_\sigma W^+_{\nu\alpha} W^-_\beta W^-_\sigma \\ &+ (6\hat{I}_1 - 23\hat{I}_2)Z_\mu Z^2_\lambda W^+_\nu W^+_{\alpha\beta} W^{-2}_\sigma \\ &+ (32\hat{I}_1 + 4\hat{I}_2)Z_\mu Z_\lambda Z_\sigma W^+_\nu W^+_{\alpha\lambda} W^-_\beta W^-_\sigma \\ &+ (16\hat{I}_1 - 38\hat{I}_2)Z_\mu Z_\lambda Z_\sigma W^+_\lambda W^+_{\nu\alpha} W^-_\beta W^-_\sigma \\ &+ (16\hat{I}_1 + 22\hat{I}_2)Z_\mu Z_\lambda Z_\sigma W^+_\lambda W^+_\sigma W^-_\nu W^-_{\alpha\beta} \\ &+ (-20\hat{I}_1 - 15\hat{I}_2)Z^2_\lambda Z_\sigma W^+_\mu W^+_\sigma W^-_\nu W^-_{\alpha\beta} \\ &+ 10\hat{I}_1 Z_\mu Z_\lambda Z_{\nu\alpha} W^{+2}_\sigma W^-_\beta W^-_\lambda \\ &- 20\hat{I}_2 Z_\mu Z_\lambda Z_{\nu\sigma} W^+_\alpha W^+_\lambda W^-_\beta W^-_\sigma + \text{c.c.}\big].\end{aligned} \tag{4.2}$$

Of the possible 21 independent operators, 18 have a non-vanishing coupling in the Standard Model. The expression is even under exchange of labels u and d, as it should be.

We have defined $\hat{I}_1 = \hat{I}^4_{1,1,2,4} - \hat{I}^4_{1,1,4,2}$ and $\hat{I}_2 = \hat{I}^4_{1,2,2,3} - \hat{I}^4_{2,1,3,2}$, and "c.c." refers to complex conjugate; Z_μ is real, $(W^\pm_\mu)^* = W^\mp_\mu$ and $\hat{I}_{1,2}$ are imaginary. The (underivated) Higgs field has been restored using that it scales as the mass dimension of $\hat{I}_{1,2}$. Also the derivative includes the field B_μ when it acts on the W's [16]. Numerically,[7]

$$\hat{I}_{1,2} = \frac{iJ}{(4\pi)^2}\frac{\kappa_{1,2}}{m_s^2 m_c^2}, \quad \kappa_1 = 0.226, \quad \kappa_2 = 0.456. \tag{4.3}$$

We can see that the values of these coefficients are considerably larger than simple estimates based on the Jarlskog determinant divided by the appropriate power of v. At sixth order the enhancement is driven by the small mass of the light quarks and so this can be considered as a kind of chiral enhancement. The possibility of such an effect was first pointed out in [14] and confirmed in [15,16].

The momentum integrals $\hat{I}^k_{a,b,c,d}$ are completely explicit but rather complicated homogeneous functions of the fermion masses [16]. At sixth order these integrals are not continuous at $\bar{m}_u, \bar{m}_d, m_s = 0$, yet one can take the limit $\bar{m}_u, \bar{m}_d \to 0$ and subsequently $m_s \to 0$ and this approximation gives a value fairly close to the exact one [16]. To discuss the situation at eighth order we will consider the simpler case of $m_b, m_t \to \infty$ which is a quite good approximation for $\kappa_{1,2}$. At eighth order the momentum integrals are more ultraviolet convergent and also more infrared divergent than at sixth order. Specifically, $\hat{I}_2$ diverges as $1/m_s^2$ when $\bar{m}_u = \bar{m}_d = 0$, with $\kappa_2 = 1/2$. The other integral, $\hat{I}_1$, is more infrared divergent: in the same limit κ_1 depends on the ratio $\bar{m}_u/\bar{m}_d$, varying continuously between $-1/6$ for $\bar{m}_d \ll \bar{m}_u$ to $3/2$ for $\bar{m}_u \ll \bar{m}_d$. In Eq. (4.3) we have used $\bar{m}_u = 2.55$ MeV and $\bar{m}_d = 5.04$ MeV.

We have also considered CP-violating terms containing only W's and gluons. Such terms appear for the first time at eighth order since (at one loop) at least four W's are needed to violate CP and two $G_{\mu\nu}$ are required to make a color singlet. In the P-odd sector one can write three independent operators. Remarkably, we find that they have zero coupling in the Standard Model. On the other hand, in the P-even (C-violating) sector, there are also three operators of which one has zero coupling while the other two terms result in the following effective Lagrangian (in Minkowski space)[8]

$$\mathcal{L}^{\text{eff}}(x) = -\frac{4}{3}\frac{v^4}{\phi^4}\hat{I}^2_{1,1,2,2}\big(W^{+2}_\lambda W^-_\mu W^-_\nu G^a_{\mu\alpha}G^a_{\nu\alpha} - \text{c.c.}\big). \tag{4.4}$$

Numerically, $\hat{I}^2_{1,1,2,2} = iJ\kappa_3/((4\pi)^2 m_s^2 m_c^2)$, with $\kappa_3 = 3.76$. This coefficient diverges logarithmically as $\bar{m}_u, \bar{m}_d \to 0$. Note that, at the order considered, the dimension four gluon condensate[9] does not induce a CP-violating interaction between the four W's. Such interaction vanishes identically, as it should be, since no CP-odd term can be written using just W's without derivatives or other fields.

The strict expansion in derivatives considered here is formal. At zero temperature hadrons, rather than quarks, are the actual relevant degrees of freedom due to confinement. This means that gluon contributions of higher order in the loop expansion will dominate the effective action. A way to estimate this effect is to

[4] The various $\hat{I}^k_{a,b,c,d}$ are not linearly independent due to integration by parts in momentum space.

[5] Such identities are increasingly more complicated as the order increases. We have cut this Gordian knot by simply expanding the Lorentz indices explicitly. This direct method identifies all linearly dependent combinations by construction.

[6] One can write 30 operators of which 23 are independent using δ–ϵ identities, and only 21 using integration by parts.

[7] $\bar{m}_u$, $\bar{m}_d$, m_c, m_s, m_t, m_b denote the quark masses.

[8] The gluon field strength tensor has been normalized according to $[D_\mu, D_\nu] = i(\lambda_a/2)G^a_{\mu\nu}$.

[9] In the presence of the gluon condensate $G^a_{\mu\nu}G^a_{\alpha\beta} = \frac{1}{12}(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha})\langle (G^a_{\lambda\sigma})^2\rangle +$ fluctuations.

replace current quark masses by masses of the order of the constituent ones, about 300 MeV for u and d quarks. The new values of the various coefficients κ introduced above depend on how the replacement is implemented, but the rule is to decrease those coefficients by about two or three orders of magnitude, as a consequence of the attenuation in the infrared divergences. Alternatively, one could consider going to higher temperatures, sufficiently above the deconfinement transition which takes place at a temperature around 200 MeV. The main effect for the momentum integrals, and so for the coefficients κ, would be to replace the energy variable by the Matsubara frequencies, being the lowest one, πT, the most relevant one. Simple estimates yield a similar quenching as that produced by the constituent mass, and for the same reasons. On the other hand, even at zero temperature no confinement exists for leptons. In the electroweak sector loop corrections are expected to be small. Our calculation applies directly to the leptonic sector with massive Dirac neutrinos, replacing the quark masses by the lepton ones. Due to the infrared behavior noted for the momentum integrals, even very small neutrino masses would translate into finite (and even large) contributions to the coupling constants of the corresponding operators. In this case the largest source of quenching for these couplings will come from the finite four-momentum of the external fields, which has been assumed to be small throughout the present calculations. (Formally, finite four-momentum of the external fields corresponds to higher orders in the expansion in Eq. (1.1) and a resummation of the terms would be needed to account for it.)

5. Conclusions

We have shown that, to one-loop and at zero temperature, the leading P-violating CP-odd operators in the effective action of the Standard Model are of dimension eight. We have computed explicitly the couplings for the operators of the form $Z^3(W^+W^-)^2$ plus one covariant derivative, Eq. (4.2). These operators come from Feynman graphs with one quark-loop, four W legs, three Z legs and zero or one $B \sim Z + \gamma$ leg. In principle, dimension six operators could develop beyond one-loop or at finite temperature due to the breaking of Lorentz invariance. Purely gluonic operators of dimension eight have also been computed, Eq. (4.4), and they are C-odd and P-even.

Remarkably the coupling constants we find are not vanishingly small, rather they have a natural scale related to intermediate mass quarks times the Jarlskog invariant. These results show that Standard Model CP-violation is not parametrically suppressed at zero temperature. This may be relevant for theories of electroweak baryogenesis of the early universe.

All formulas derived for quarks extend directly to massive Dirac leptons. This implies that even if the neutrino masses are small their contribution to the CP-violating couplings needs not be small, due to infrared sensitivity in the momentum integrals on which the couplings depend. As a consequence, such couplings will be strongly dependent on the mass ratios between neutrinos of the different generations. The calculation does not directly apply to massive Majorana neutrinos. Therefore, for this type of neutrinos the possibility is still open for genuine contributions from CP-odd and P-odd operators of dimension six at zero temperature.

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