11 October 2001
PHYSICS LETTERS B
Physics Letters B 518 (2001) 171–180
www.elsevier.com/locate/npe

Soft supersymmetry breaking due to dimensional reduction over non-symmetric coset spaces

P. Manousselis [1], G. Zoupanos [2]

Physics Department, National Technical University, Zografou Campus, 157 80 Athens, Greece

Received 3 July 2001; received in revised form 10 August 2001; accepted 16 August 2001
Editor: G.F. Giudice

Abstract

A ten-dimensional supersymmetric E_8 gauge theory is compactified over six-dimensional coset spaces, establishing further our earlier conjecture that the resulting four-dimensional theory is a softly broken supersymmetric gauge theory in the case that the used coset space is non-symmetric. The specific non-symmetric six-dimensional spaces examined in the present study are $Sp(4)/(SU(2) \times U(1))_{\text{non-max}}$ and $SU(3)/U(1) \times U(1)$. © 2001 Published by Elsevier Science B.V.

1. Introduction

Supersymmetry has been one of the essential ingredients of most unification frameworks examined during the few last decades. This is not a surprising fact given that the hope of understanding in a unified manner particles with different spins and the aim that such a unified description should be free of ultraviolet divergencies have been in the core of most attempts. Supersymmetry by definition points to a fulfillment of the first hope, while already the first of the non-renormalization theorems in supersymmetric theories [1] guarantees improved ultraviolet properties of such theories. On the other hand the lack of any obvious sign of supersymmetry in the low energy physics that have been explored during the last decades, has risen the question of supersymmetry breaking to a fundamental issue comparable to the existence of supersymmetry itself.

Since the early days of supersymmetry several mechanisms such as the Fayet–Iliopoulos [2], the Fayet–O'Raifeartaigh [3] have been proposed, while the celebrated MSSM has been supplemented with a soft supersymmetry breaking (SSB) sector which was supposed to be inherited to the low energies by supergravity [4].

Concerning higher-dimensional supersymmetric theories, like those resulting in the field theory limit of superstrings, mostly two mechanisms have been employed. One assumes that $N = 1$ is preserved by the compactification process and supersymmetry breaking has its origin in the gaugino condensation taking place in the "hidden" sector of the theory which eventually is communicated to the observed sector. The other mechanism, called Scherk–Schwarz [5] breaks supersymmetry in the process of compactification. In Ref. [6] a new mechanism, based on the coset space dimensional reduction (CSDR) [7–9] has been proposed as the possible origin of the SSB sector of a four-dimensional supersymmetric theory.

E-mail addresses: pman@central.ntua.gr (P. Manousselis), george.zoupanos@cern.ch (G. Zoupanos).

[1] Supported by ΓΓΕΤ grand 97ΕΛ/71.
[2] Partially supported by EU under the RTN contract HPRN-CT-2000-00148 and the Humboldt Foundation.

PII: S0370-2693(01)01040-1

Specifically in Ref. [6] a ten-dimensional supersymmetric gauge theory based on the group E_8 was reduced over the six-dimensional non-symmetric coset space $G_2/SU(3)$ leading to an E_6 softly broken supersymmetric GUT in four dimensions. On the contrary the original supersymmetry of the theory was completely broken by the dimensional reduction procedure over the six-sphere $SO(7)/SO(6)$ which is a symmetric coset space. The conjecture of Ref. [6] was that the above findings have a wider validity. In the present work we establish further the conjecture of Ref. [6] that dimensional reduction over non-symmetric coset spaces leads *automatically* to softly broken supersymmetric four-dimensional theories, by studying the dimensional reduction of a ten-dimensional supersymmetric E_8 gauge theory over the rest two existing non-symmetric six-dimensional coset spaces. We find that the dimensional reduction over the non-symmetric coset spaces $Sp(4)/(SU(2)\times U(1))_{\text{non-max}}$ and $SU(3)/U(1)\times U(1)$ leads to softly broken supersymmetric gauge theories in four dimensions with a complete SSB sector, while no other term that could possibly spoil the ultraviolet properties of the theories appears.

2. The coset space dimensional reduction

Given a gauge theory defined in higher dimensions the obvious way to dimensionally reduce it is to demand that the field dependence on the extra coordinates is such that the Lagrangian is independent of them. A crude way to fulfill this requirement is to discard the field dependence on the extra coordinates, while an elegant one is to allow for a non-trivial dependence on them, but impose the condition that a symmetry transformation by an element of the isometry group S of the space formed by the extra dimensions B corresponds to a gauge transformation. Then the Lagrangian will be independent of the extra coordinates just because it is gauge invariant. This is the basis of the CSDR scheme [7–9], which assumes that B is a compact coset space, S/R.

In the CSDR scheme one starts with a Yang–Mills–Dirac Lagrangian, with gauge group G, defined on a D-dimensional spacetime M^D, with metric g^{MN}, which is compactified to $M^4\times S/R$ with S/R a coset space. The metric is assumed to have the form

$$g^{MN}=\begin{bmatrix}\eta^{\mu\nu} & 0\\ 0 & -g^{ab}\end{bmatrix}, \tag{1}$$

where $\eta^{\mu\nu}=\text{diag}(1,-1,-1,-1)$ and g^{ab} is the coset space metric. The requirement that transformations of the fields under the action of the symmetry group of S/R are compensated by gauge transformations lead to certain constraints on the fields. The solution of these constraints provides us with the four-dimensional unconstrained fields as well as with the gauge invariance that remains in the theory after dimensional reduction. Therefore a potential unification of all low energy interactions, gauge, Yukawa and Higgs is achieved, which was the first motivation of this framework.

It is interesting to note that the fields obtained using the CSDR approach are the first terms in the expansion of the D-dimensional fields in harmonics of the internal space B and are massless after the first stage of the symmetry breaking which is geometrical. The effective field theories resulting from compactification of higher-dimensional theories contain also towers of massive higher harmonics (Kaluza–Klein) excitations, whose contributions at the quantum level alter the behaviour of the running couplings from logarithmic to power [10]. As a result the traditional picture of unification of couplings may change drastically [11]. Higher-dimensional theories have also been studied at the quantum level using the continuous Wilson renormalization group [12] which can be formulated in any number of spacetime dimensions with results in agreement with the treatment involving massive Kaluza–Klein excitations.

The group S acts as a symmetry group on the the extra coordinates. The CSDR scheme demands that an S-transformation of the extra d coordinates is a gauge transformation of the fields that are defined on $M^4\times S/R$, thus a gauge invariant Lagrangian written on this space is independent of the extra coordinates.

To see some of the details of the CDRS let us consider a D-dimensional Yang–Mills–Dirac theory with gauge group G defined on a manifold M^D which as stated will be compactified to $M^4\times S/R$, $D=4+d$, $d=\dim S-\dim R$:

$$A = \int d^4x\, d^dy \sqrt{-g}\Big[-\frac{1}{4}\operatorname{Tr}(F_{MN}F_{K\Lambda})g^{MK}g^{N\Lambda} + \frac{i}{2}\overline{\psi}\Gamma^M D_M\psi\Big], \tag{2}$$

where

$$D_M = \partial_M - \theta_M - A_M, \tag{3}$$

with

$$\theta_M = \frac{1}{2}\theta_{MN\Lambda}\Sigma^{N\Lambda} \tag{4}$$

the spin connection of M^D, and

$$F_{MN} = \partial_M A_N - \partial_N A_M - [A_M, A_N], \tag{5}$$

where M, N run over the D-dimensional space. The fields A_M and ψ are, as explained, symmetric in the sense that any transformation under symmetries of S/R is compensated by gauge transformations. The fermion fields can be in any representation F of G unless a further symmetry such as supersymmetry is required. So let ξ_A^α, $A = 1, \ldots, \dim S$, be the Killing vectors which generate the symmetries of S/R and W_A the compensating gauge transformation associated with ξ_A. Defining next the infinitesimal coordinate transformation as $\delta_A \equiv L_{\xi_A}$, i.e., the Lie derivative with respect to ξ, we obtain the following constraints for the scalar, vector and spinor fields,

$$\begin{aligned}
&\delta_A\phi = \xi_A^\alpha\partial_\alpha\phi = D(W_A)\phi,\\
&\delta_A A_\alpha = \xi_A^\beta\partial_\beta A_\alpha + \partial_\alpha\xi_A^\beta A_\beta = \partial_\alpha W_A - [W_A, A_\alpha],\\
&\delta_A\psi = \xi_A^\alpha\psi - \frac{1}{2}G_{Abc}\Sigma^{bc}\psi = D(W_A)\psi.
\end{aligned} \tag{6}$$

W_A depend only on internal coordinates y and $D(W_A)$ represents a gauge transformation in the appropriate representation of the fields. G_{Abc} represents a tangent space rotation of the spinor fields. The variations δ_A satisfy, $[\delta_A, \delta_B] = f_{AB}^{C}\delta_C$ and lead to the following consistency relation for W_A's,

$$\xi_A^\alpha\partial_\alpha W_B - \xi_B^\alpha\partial_\alpha W_A - [W_A, W_B] = f_{AB}^{\;\;C} W_C. \tag{7}$$

Furthermore the W's themselves transform under a gauge transformation [8] as,

$$\widetilde{W}_A = gW_Ag^{-1} + (\delta_A g)g^{-1}. \tag{8}$$

Using Eq. (8) and the fact that the Lagrangian is independent of y we can do all calculations at $y = 0$ and choose a gauge where $W_a = 0$.

The detailed analysis of the constraints (6) given in Refs. [7,8] provides us with the four-dimensional unconstrained fields as well as with the gauge invariance that remains in the theory after dimensional reduction. Here we present only the results. The components $A_\mu(x, y)$ of the initial gauge field $A_M(x, y)$ become, after dimensional reduction, the four-dimensional gauge fields and furthermore they are independent of y. In addition one can find that they have to commute with the elements of the R_G subgroup of G. Thus the four-dimensional gauge group H is the centralizer of R in G, $H = C_G(R_G)$. Similarly, the $A_\alpha(x, y)$ components of $A_M(x, y)$ denoted by $\phi_\alpha(x, y)$ from now on, become scalars at four dimensions. These fields transform under R as a vector v, i.e.,

$$\begin{aligned}
&S \supset R,\\
&\operatorname{adj} S = \operatorname{adj} R + v.
\end{aligned} \tag{9}$$

Moreover $\phi_\alpha(x, y)$ act as an intertwining operator connecting induced representations of R acting on G and S/R. This implies, exploiting Schur's lemma, that the transformation properties of the fields $\phi_\alpha(x, y)$ under H can be found if we express the adjoint representation of G in terms of $R_G \times H$:

$$\begin{aligned}
&G \supset R_G \times H,\\
&\operatorname{adj} G = (\operatorname{adj} R, 1) + (1, \operatorname{adj} H) + \sum (r_i, h_i).
\end{aligned} \tag{10}$$

Then if $v = \sum s_i$, where each s_i is an irreducible representation of R, there survives an h_i multiplet for every pair (r_i, s_i), where r_i and s_i are identical irreducible representations of R.

Turning next to the fermion fields [7,8,13–15], similarly to scalars, they act as intertwining operators between induced representations acting on G and the tangent space of S/R, $SO(d)$. Proceeding along similar lines as in the case of scalars to obtain the representation of H under which the four-dimensional fermions transform, we have to decompose the representation F of the initial gauge group in which the fermions are assigned under $R_G \times H$, i.e.,

$$F = \sum (t_i, h_i), \tag{11}$$

and the spinor of $SO(d)$ under R

$$\sigma_d = \sum \sigma_j. \tag{12}$$

Then for each pair t_i and σ_i, where t_i and σ_i are identical irreducible representations there is an h_i multiplet of spinor fields in the four-dimensional theory. In order however to obtain chiral fermions in the effective theory we have to impose further requirements. We first impose the Weyl condition in D dimensions. In $D = 4n + 2$ dimensions which is the case at hand, the decomposition of the left-handed, say spinor under $SU(2) \times SU(2) \times SO(d)$ is

$$\sigma_D = (2, 1, \sigma_d) + (1, 2, \bar{\sigma}_d). \tag{13}$$

So we have in this case the decompositions

$$\sigma_d = \sum \sigma_k, \qquad \bar{\sigma}_d = \sum \bar{\sigma}_k. \tag{14}$$

Let us start from a vector-like representation F for the fermions. In this case each term (t_i, h_i) in Eq. (11) will be either selfconjugate or it will have a partner $(\bar{t}_i, \bar{h}_i)$. According to the rule described in Eqs. (11), (12) and considering σ_d we will have in four dimensions left-handed fermions transforming as $f_L = \sum h_k^L$. It is important to notice that since σ_d is non-selfconjugate, f_L is non-selfconjugate too. Similarly from $\bar{\sigma}_d$ we will obtain the right handed representation $f_R = \sum \bar{h}_k^R$ but as we have assumed that F is vector-like, $\bar{h}_k^R \sim h_k^L$. Therefore there will appear two sets of Weyl fermions with the same quantum numbers under H. This is already a chiral theory, but still one can go further and try to impose the Majorana condition in order to eliminate the doubling of the fermionic spectrum. Clearly if we had started with F complex, we should have again a chiral theory since in this case $\bar{h}_k^R$ is different from h_k^L (σ_d non-selfconjugate). Nevertheless starting with F vector-like is much more appealing and will be used in the following along with the Majorana condition. The Majorana condition can be imposed in $D = 2, 3, 4 + 8n$ dimensions and is given by $\psi = C(\bar{\psi})^T$, where C is the D-dimensional charge conjugation matrix. Majorana and Weyl conditions are compatible in $D = 4n + 2$ dimensions. Then in our case if we start with Weyl–Majorana spinors in $D = 4n + 2$ dimensions we force f_R to be the charge conjugate to f_L, thus arriving in a theory with fermions only in f_L. Furthermore if F is to be real, then we have to have $D = 2 + 8n$, while for F pseudoreal $D = 6 + 8n$.

Starting with an anomaly free theory in higher dimensions, in Ref. [16] was given the condition that has to be fulfilled in order to obtain anomaly free theories in four dimensions after dimensional reduction. The condition restricts the allowed embeddings of R into G [8,17]. For $G = E_8$ in ten dimensions the condition takes the form

$$l(G) = 60, \tag{15}$$

where $l(G)$ is the sum over all indices of the R representations appearing in the decomposition of the 248 representation of E_8 under $E_8 \supset R \times H$. The normalization is such that the vector representation in Eq. (9) which defines the embedding of R into $SO(6)$, has index two.

Next let us obtain the four-dimensional effective action. Assuming that the metric is block diagonal, taking into account all the constraints and integrating out the extra coordinates we obtain in four dimensions the following Lagrangian:

$$A = C \int d^4x \biggl(-\frac{1}{4} F^t_{\mu\nu} F^{t\,\mu\nu} + \frac{1}{2} (D_\mu \phi_\alpha)^t \bigl(D^\mu \phi^\alpha\bigr)^t + V(\phi) + \frac{i}{2} \bar{\psi} \Gamma^\mu D_\mu \psi - \frac{i}{2} \bar{\psi} \Gamma^a D_a \psi \biggr), \tag{16}$$

where $D_\mu = \partial_\mu - A_\mu$ and $D_a = \partial_a - \theta_a - \phi_a$ with $\theta_a = \frac{1}{2} \theta_{abc} \Sigma^{bc}$ the connection of the coset space, while C is the volume of the coset space. The potential $V(\phi)$ is given by

$$V(\phi) = -\frac{1}{4} g^{ac} g^{bd} \operatorname{Tr}\bigl(f_{ab}^C \phi_C - [\phi_a, \phi_b]\bigr) \times \bigl(f_{cd}^D \phi_D - [\phi_c, \phi_d]\bigr), \tag{17}$$

where, $A = 1, \ldots, \dim S$ and f's are the structure constants appearing in the commutators of the generators of the Lie algebra of S. The expression (17) for $V(\phi)$ is only formal because ϕ_a must satisfy the constraints coming from Eq. (6),

$$f_{ai}^D \phi_D - [\phi_a, \phi_i] = 0, \tag{18}$$

where the ϕ_i generate R_G. These constraints imply that some components ϕ_a's are zero, some are constants and the rest can be identified with the genuine Higgs fields. When $V(\phi)$ is expressed in terms of the unconstrained independent Higgs fields, it remains a quartic polynomial which is invariant under gauge transformations of the final gauge group H, and its minimum determines the vacuum expectation values of the Higgs fields [18,19].

In the fermion part of the Lagrangian the first term is just the kinetic term of fermions, while the second is the Yukawa term [20]. Note that since ψ is a Majorana–Weyl spinor in ten dimensions the representation in which the fermions are assigned under the gauge group must be real. The last term in Eq. (16) can be written as

$$L_Y = -\frac{i}{2}\bar{\psi}\Gamma^a\left(\partial_a - \frac{1}{2}f_{ibc}e^i_\gamma e^\gamma_a \Sigma^{bc} - \frac{1}{2}G_{abc}\Sigma^{bc} - \phi_a\right)\psi = \frac{i}{2}\bar{\psi}\Gamma^a\nabla_a\psi + \bar{\psi}V\psi, \tag{19}$$

where

$$\nabla_a = -\partial_a + \frac{1}{2}f_{ibc}e^i_\gamma e^\gamma_a \Sigma^{bc} + \phi_a, \tag{20}$$

$$V = \frac{i}{4}\Gamma^a G_{abc}\Sigma^{bc}, \tag{21}$$

and we have used the full connection with torsion [8] given by

$$\theta^a{}_{cb} = -f^a{}_{ib}e^i_\alpha e^\alpha_c - \left(D^a{}_{cb} + \frac{1}{2}\Sigma^a{}_{cb}\right) = -f^a{}_{ib}e^i_\alpha e^\alpha_c - G^a{}_{cb} \tag{22}$$

with

$$D^a{}_{cb} = g^{ad}\frac{1}{2}\left[f_{db}{}^e g_{ec} + f_{cb}{}^e g_{de} - f_{cd}{}^e g_{be}\right] \tag{23}$$

and

$$\Sigma_{abc} = 2\tau(D_{abc} + D_{bca} - D_{cba}). \tag{24}$$

We have already noticed that the CSDR constraints tell us that $\partial_a\psi = 0$. Furthermore we can consider the Lagrangian at the point $y = 0$, due to its invariance under S-transformations, and as we mentioned $e^i_\gamma = 0$ at that point. Therefore Eq. (20) becomes just $\nabla_a = \phi_a$ and the term $\frac{i}{2}\bar{\psi}\Gamma^a\nabla_a\psi$ in Eq. (19) is exactly the Yukawa term.

Let us examine now the last term appearing in Eq. (19). One can show easily that the operator V anticommutes with the six-dimensional helicity operator [8]. Furthermore one can show that V commutes with the $T_i = -\frac{1}{2}f_{ibc}\Sigma^{bc}$ (T_i close the R-subalgebra of $SO(6)$). In turn we can draw the conclusion, exploiting Schur's lemma, that the non-vanishing elements of V are only those which appear in the decomposition of both $SO(6)$ irreps 4 and $\bar{4}$, e.g. the singlets. Since this term is of pure geometric nature, we reach the conclusion that the singlets in 4 and $\bar{4}$ will acquire large geometrical masses, a fact that has serious phenomenological implications. In supersymmetric theories defined in higher dimensions, it means that the gauginos obtained in four dimensions after dimensional reduction receive masses comparable to the compactification scale. However as we shall see in the next sections this result changes in presence of torsion. We note that for symmetric coset spaces the V operator is absent since in that case f^c_{ab} vanish by definition.

3. Soft supersymmetry breaking by dimensional reduction over non-symmetric coset spaces

Recently a lot of interest has been triggered by the possibility that superstrings can be defined at the TeV scale [21]. The string tension became an arbitrary parameter and can be anywhere below the Planck scale and as low as TeV. The main advantage of having the string tension at the TeV scale, besides the obvious experimental interest, is that it offers an automatic protection to the gauge hierarchy [21], alternative to low energy supersymmetry [22], or dynamical electroweak symmetry breaking [23–25]. However the only vacua of string theory free of all pathologies are supersymmetric. Then the original supersymmetry of the theory, not being necessary in four dimensions, could be broken by the dimensional reduction procedure.

The weakly coupled ten-dimensional $E_8 \times E_8$ supersymmetric gauge theory is one of the few to possess the advantage of anomaly freedom [26] and has been extensively used in efforts to describe quantum gravity along with the observed low energy interactions in the heterotic string framework [27]. In addition its strong coupling limit provides an interesting example of the realization of the brane picture, i.e., E_8 gauge fields and matter live on the two 10-dimensional boundaries, while gravitons propagate in the eleven-dimensional bulk [28].

In the following sections we shall be reducing a supersymmetric ten-dimensional gauge theory based on E_8 over the six-dimensional coset spaces $Sp(4)/(SU(2)\times U(1))_{\text{non-max}}$ and $SU(3)/U(1)\times U(1)$ and examine the consequences of the resulting four-dimensional theory mostly as far as supersymmetry breaking is concerned.

3.1. Supersymmetry breaking by dimensional reduction over $Sp(4)/(SU(2)\times U(1))_{\text{non-max}}$

In the present study we start with a ten-dimensional supersymmetric gauge theory based on the group E_8 and reduce it over the non-symmetric coset $Sp(4)/(SU(2)\times U(1))_{\text{non-max}}$. Therefore in the terminology of Section 2 we have chosen $G=E_8$, $B=Sp(4)/(SU(2)\times U(1))_{\text{non-max}}$, $D=10$ and Weyl–Majorana fermions belonging in the adjoint of G. We start by giving the decompositions to be used,

$$E_8\supset SU(3)\supset SU(2)\times U(1)\times E_6.$$

The decomposition of 248 of E_8 under $SU(3)\times E_6$ is given by

$$248=(8,1)+(1,78)+(3,27)+(\bar{3},\overline{27}),$$

while under $(SU(2)\times U(1))\times E_6$ is the following:

$$\begin{aligned}248={}&(3_0,1)+(1_0,1)+(1_0,78)+(2_3,1)\\&+(2_{-3},1)+(2_1,27)+(2_{-1},\overline{27})\\&+(1_{-2},27)+(1_2,\overline{27}).\end{aligned}\tag{25}$$

In the present case R is chosen to be identified with the $SU(2)\times U(1)$ of the latter of the above decompositions. Therefore the resulting four-dimensional gauge theory is based on the group

$$H=C_{E_8}\big(SU(2)\times U(1)\big)=E_6\times U(1),$$

where the $U(1)$ appears since the $U(1)$ in R centralizes itself. The $R=SU(2)\times U(1)$ content of $Sp(4)/(SU(2)\times U(1))_{\text{non-max}}$ vector and spinor are $2_1+2_{-1}+1_2+1_{-2}$ and $2_1+1_0+1_{-2}$, respectively. Thus applying the CSDR rules (9), (10) and (11), (12) we find that the surviving fields in four dimensions can be organized in a $\mathcal{N}=1$ vector supermultiplet V^α which transforms as 78 of E_6, a $\mathcal{N}=1$ $U(1)$ vector supermultiplet V and chiral supermultiplets (B^i, C^i), transforming as $(27,1)$, and $(27,-2)$ under the gauge group $E_6\times U(1)$.

We find that the potential of the four-dimensional theory, in terms of the physical scalar fields β^i, and γ^i is given by

$$\begin{aligned}&V\big(\beta^i,\gamma^j\big)\\&\quad=\text{const}-\frac{6}{R_1^2}\beta^i\beta_i-\frac{4}{R_2^2}\gamma^i\gamma_i\\&\qquad+\left[4\sqrt{\frac{10}{7}}R_2\left(\frac{1}{R_2^2}+\frac{1}{2R_1^2}\right)d_{ijk}\beta^i\beta^j\gamma^k+\text{h.c.}\right]\\&\qquad+6\big(\beta^i(G^\alpha)_i^j\beta_j+\gamma^i(G^\alpha)_i^j\gamma_j\big)^2\\&\qquad+\frac{1}{3}\big(\beta^i\big(1\delta_i^j\big)\beta_j+\gamma^i\big(-2\delta_i^j\big)\gamma_j\big)^2\\&\qquad+\frac{5}{7}\beta^i\beta^j d_{ijk}d^{klm}\beta_l\beta_m\\&\qquad+4\frac{5}{7}\beta^i\gamma^j d_{ijk}d^{klm}\beta_l\gamma_m.\end{aligned}\tag{26}$$

From the potential (26) we can determine the F-, D- and the scalar soft terms which break softly the supersymmetric theory obtained by CSDR over $Sp(4)/(SU(2)\times U(1))_{\text{non-max}}$. Specifically we find that the F-term contributions to the potential (26) come from the superpotential

$$\mathcal{W}\big(B^i,C^j\big)=\sqrt{\frac{5}{7}}d_{ijk}B^iB^jC^k.\tag{27}$$

Similarly the D-term contributions to the potential (26) are given by the sum

$$\frac{1}{2}D^\alpha D^\alpha+\frac{1}{2}DD,\tag{28}$$

where

$$D^\alpha=\sqrt{12}\big(\beta^i\big(G^\alpha\big)_i^j\beta_j+\gamma^i\big(G^\alpha\big)_i^j\gamma_j\big)$$

and

$$D=\sqrt{\frac{2}{3}}\big(\beta^i\big(1\delta_i^j\big)\beta_j+\gamma^i\big(-2\delta_i^j\big)\gamma_j\big)$$

corresponding to the vector supermultiplets of $E_6\times U(1)$. The remaining terms in the potential (26) are the soft breaking mass and trilinear terms and they form the scalar SSB part of the Lagrangian,

$$\begin{aligned}&\mathcal{L}_{\text{scalar SSB}}=\\&\quad-\frac{6}{R_1^2}\beta^i\beta_i-\frac{4}{R_2^2}\gamma^i\gamma_i\\&\quad+\left[4\sqrt{\frac{10}{7}}R_2\left(\frac{1}{R_2^2}+\frac{1}{2R_1^2}\right)d_{ijk}\beta^i\beta^j\gamma^k+\text{h.c.}\right].\end{aligned}\tag{29}$$

The gaugino mass has been calculated in Ref. [20] to be

$$M=(1+3\tau)\frac{R_2^2+2R_1^2}{8R_1^2R_2}. \tag{30}$$

We note that the chosen embedding of $R=SU(2)\times U(1)$ in E_8 satisfies the condition (15) which guarantees the renormalizability of the four-dimensional theory, while the absence of any other term that does not belong to the supersymmetric $E_6\times U(1)$ theory or to its SSB sector guarantees the improved ultraviolet behaviour of the theory. Finally note the contribution of the torsion in the gaugino mass (30).

3.2. Supersymmetry breaking by reduction over $SU(3)/(U(1)\times U(1))$

In this model the only difference as compared to the previous one is that the chosen coset space to reduce the same theory is the non-symmetric $B=SU(3)/U(1)\times U(1)$. The decompositions to be used are

$$\begin{aligned}E_8&\supset SU(2)\times U(1)\times E_6\\&\supset U(1)_1\times U(1)_2\times E_6.\end{aligned}$$

The 248 of E_8 is decomposed under $SU(2)\times U(1)$ according to (25) whereas the decomposition under $U(1)_1\times U(1)_2$ is the following:

$$\begin{aligned}248={}&(0,0;1)+(0,0;1)+\left(3,\frac{1}{2};1\right)+\left(-3,\frac{1}{2};1\right)\\&+(0,-1;1)+(0,1;1)+\left(-3,-\frac{1}{2};1\right)\\&+\left(-3,-\frac{1}{2};1\right)+(0,0;78)+\left(3,\frac{1}{2};27\right)\\&+\left(-3,\frac{1}{2};27\right)+(0,-1;27)\\&+\left(-3,-\frac{1}{2};\overline{27}\right)+\left(3,-\frac{1}{2};\overline{27}\right)+(0,1;\overline{27}).\end{aligned} \tag{31}$$

In the present case R is chosen to be identified with the $U(1)_1\times U(1)_2$ of the latter decomposition. Therefore the resulting four-dimensional gauge group is

$$H=C_{E_8}\big(U(1)_1\times U(1)_2\big)=U(1)_1\times U(1)_2\times E_6.$$

Again the two $U(1)$'s appear because R $(=U(1)_1\times U(1)_2)$ centralizes itself. The $R=U(1)\times U(1)$ content of $SU(3)/U(1)\times U(1)$ vector and spinor are $(3,\frac{1}{2})+(-3,\frac{1}{2})+(0,-1)+(-3,-\frac{1}{2})+(3,-\frac{1}{2})+(0,1)$ and $(0,0)+(3,\frac{1}{2})+(-3,\frac{1}{2})+(0,-1)$, respectively. Thus applying the CSDR rules (9)–(12) we find that the surviving fields in four dimensions are three $\mathcal{N}=1$ vector multiplets V^α, $V_{(1)}$, $V_{(2)}$ (where α is an E_6, 78 index and the other two refer to the two $U(1)$'s) containing the gauge fields of $U(1)_1\times U(1)_2\times E_6$. The matter content consists of three $\mathcal{N}=1$ chiral multiplets (A^i, B^i, C^i) with i an E_6, 27 index and three $\mathcal{N}=1$ chiral multiplets (A, B, C) which are E_6 singlets and carry $U(1)_1\times U(1)_2$ charges.

We find that the unconstrained scalar fields transform under $U(1)_1\times U(1)_2\times E_6$ as

$$\begin{aligned}&\alpha_i\sim\left(3,\frac{1}{2};27\right),\qquad \alpha\sim\left(3,\frac{1}{2};1\right),\\&\beta_i\sim\left(-3,\frac{1}{2};27\right),\qquad \beta\sim\left(-3,\frac{1}{2};1\right),\\&\gamma_i\sim(0,-1;27),\qquad \gamma\sim(0,-1;1).\end{aligned} \tag{32}$$

The potential of the four-dimensional theory in terms of the unconstrained fields given in (32) is the following

$$\begin{aligned}&V\big(\alpha^i,\beta^j,\gamma^k,\alpha,\beta,\gamma\big)\\&\quad=\text{const.}+\left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\alpha^i\alpha_i\\&\qquad+\left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\bar{\alpha}\alpha\\&\qquad+\left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\beta^i\beta_i+\left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\bar{\beta}\beta\\&\qquad+\left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\gamma^i\gamma_i+\left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\bar{\gamma}\gamma\\&\qquad+\left[\sqrt{2}\,80\left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)d_{ijk}\alpha^i\beta^j\gamma^k\right.\\&\qquad\left.+\sqrt{2}\,80\left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)\alpha\beta\gamma+\text{h.c.}\right]\\&\qquad+\frac{1}{6}\Big(\alpha^i\big(G^\alpha\big)_i^j\alpha_j+\beta^i\big(G^\alpha\big)_i^j\beta_j+\gamma^i\big(G^\alpha\big)_i^j\gamma_j\Big)^2\\&\qquad+\frac{10}{6}\Big(\alpha^i\big(3\delta_i^j\big)\alpha_j+\bar{\alpha}(3)\alpha+\beta^i\big(-3\delta_i^j\big)\beta_j\\&\qquad\qquad+\bar{\beta}(-3)\beta\Big)^2\end{aligned}$$

$$
\begin{aligned}
&+\frac{40}{6}\left(\alpha^i\left(\frac{1}{2}\delta_i^j\right)\alpha_j+\bar{\alpha}\left(\frac{1}{2}\right)\alpha+\beta^i\left(\frac{1}{2}\delta_i^j\right)\beta_j\right.\\
&\left.+\bar{\beta}\left(\frac{1}{2}\right)\beta+\gamma^i\left(-1\delta_i^j\right)\gamma_j+\bar{\gamma}(-1)\gamma\right)^2\\
&+40\alpha^i\beta^j d_{ijk}d^{klm}\alpha_l\beta_m+40\beta^i\gamma^j d_{ijk}d^{klm}\beta_l\gamma_m\\
&+40\alpha^i\gamma^j d_{ijk}d^{klm}\alpha_l\gamma_m\\
&+40(\bar{\alpha}\bar{\beta})(\alpha\beta)+40(\bar{\beta}\bar{\gamma})(\beta\gamma)+40(\bar{\gamma}\bar{\alpha})(\gamma\alpha).
\end{aligned}
\tag{33}
$$

From the potential (33) we read the F-, D- and scalar soft terms. The F-terms are obtained from the superpotential

$$
\begin{aligned}
&\mathcal{W}\left(A^i,B^j,C^k,A,B,C\right)\\
&\quad=\sqrt{40}\,d_{ijk}A^iB^jC^k+\sqrt{40}\,ABC.
\end{aligned}
\tag{34}
$$

The D-terms have the structure

$$\frac{1}{2}D^\alpha D^\alpha+\frac{1}{2}D_1D_1+\frac{1}{2}D_2D_2, \tag{35}$$

where

$$D^\alpha=\frac{1}{\sqrt{3}}\left(\alpha^i\left(G^\alpha\right)_i^j\alpha_j+\beta^i\left(G^\alpha\right)_i^j\beta_j+\gamma^i\left(G^\alpha\right)_i^j\gamma_j\right),$$

$$
\begin{aligned}
D_1=\sqrt{\frac{10}{3}}&\left(\alpha^i\left(3\delta_i^j\right)\alpha_j+\bar{\alpha}(3)\alpha\right.\\
&\left.+\beta^i\left(-3\delta_i^j\right)\beta_j+\bar{\beta}(-3)\beta\right)
\end{aligned}
$$

and

$$
\begin{aligned}
D_2=\sqrt{\frac{40}{3}}&\left(\alpha^i\left(\frac{1}{2}\delta_i^j\right)\alpha_j+\bar{\alpha}\left(\frac{1}{2}\right)\alpha+\beta^i\left(\frac{1}{2}\delta_i^j\right)\beta_j\right.\\
&+\bar{\beta}\left(\frac{1}{2}\right)\beta+\gamma^i\left(-1\delta_i^j\right)\gamma_j\\
&\left.+\bar{\gamma}(-1)\gamma\right),
\end{aligned}
$$

which correspond to the $U(1)_1\times U(1)_2\times E_6$ vector supermultiplet content of the four-dimensional theory. The rest terms are the trilinear and mass terms which break supersymmetry softly and they form the scalar SSB part of the Lagrangian,

$$
\begin{aligned}
&\mathcal{L}_{\text{scalar SSB}}\\
&\quad=\left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\alpha^i\alpha_i+\left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\bar{\alpha}\alpha\\
&\quad+\left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\beta^i\beta_i+\left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\bar{\beta}\beta\\
&\quad+\left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\gamma^i\gamma_i+\left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\bar{\gamma}\gamma\\
&\quad+\left[\sqrt{2}\,80\left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)d_{ijk}\alpha^i\beta^j\gamma^k\right.\\
&\quad\quad+\sqrt{2}\,80\left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)\alpha\beta\gamma\\
&\quad\quad\left.+\text{h.c.}\right].
\end{aligned}
\tag{36}
$$

Finally in order to determine the gaugino mass we calculate the V operator appearing in Eq. (19). We find that the gauginos acquire a geometrical mass

$$M=(1+3\tau)\frac{(R_1^2+R_2^2+R_3^2)}{8\sqrt{R_1^2R_2^2R_3^2}}. \tag{37}$$

We note again that the chosen embedding satisfies the condition (15) and the absence in the four-dimensional theory of any other term that does not belong to the supersymmetric $U(1)_1\times U(1)_2\times E_6$ gauge theory or to its SSB sector. The gaugino mass (37) has a contribution from the torsion of the coset space similarly to the reduction over the other non-symmetric spaces. Contrary to the gaugino mass term the soft scalar terms of the SSB do not receive contributions from the torsion in all models. This is due to the fact that gauge fields, contrary to fermions, do not couple to torsion.

4. Conclusions

The CSDR was originally introduced as a scheme which, making use of higher dimensions, incorporates in a unified manner the gauge and the ad hoc Higgs sector of the spontaneously broken gauge theories in four dimensions [7]. Next fermions were introduced in the scheme and the ad hoc Yukawa interactions have also been included in the unified description [14].

Considerable progress has also been made in attempts to describe the observed low-energy world within the CSDR framework. Among the new possibilities emerged from the subsequent studies of the CSDR scheme are the following: (a) the possibility to obtain chiral fermions in four dimensions resulting from vector-like representations of the higher-dimensional gauge theory [8,13]. This possibility can be realized due the presence of non-trivial background gauge configurations which are introduced by the CSDR constructions [29], (b) the possibility to deform the metric of certain non-symmetric coset spaces and thereby obtain more than one scales [8,19,30], (c) the possibility to use coset spaces, which are multiply connected. This can be achieved by exploiting the discrete symmetries of the S/R [8,31]. Then one might introduce topologically non-trivial gauge field [32] configurations with vanishing field strength and induce additional breaking of the gauge symmetry. It is the Hosotani mechanism [33] applied in the CSDR.

In the above list recently has been added the interesting possibility that the popular softly broken supersymmetric four-dimensional chiral gauge theories might have their origin in a higher-dimensional supersymmetric theory with only vector supermultiplet [6], which is dimensionally reduced over non-symmetric coset spaces. In the present work we have extended the previous observations [8,13,20] and the concrete proposal of Ref. [6] in the remaining six-dimensional non-symmetric coset spaces, demonstrating in this way that the claim of Ref. [6] holds more generally and it is not just a peculiarity of the coset space that was used.

Given the recent interest on the Scherk–Schwarz mechanism [34], it is worth adding few comments concerning the relation among the Scherk–Schwarz and our mechanism. Without making any attempt to cover the many aspects of the subject discussed over years it seems that Scherk and Schwarz [5], were influenced by the work of Forgacs and Manton [7], which was done few months earlier and used the generalized reduction on which the CSDR is based on, i.e., they also allowed dependence of various fields on the compact space coordinates corresponding to a gauge transformation. Moreover in Ref. [5], among others, they have examined the reduction of supersymmetric Yang–Mills theories in the above sense as we do. The real difference is that they did the reduction on a group manifold instead of coset space, which is a limiting case of coset space with $R = I$ and has the obvious problem that the resulting four-dimensional theory has no chiral fermions. They claimed without going in the details that supersymmetry was broken.

Schwarz more than twenty years later, in Ref. [35], was describing the basic idea of the Scherk–Schwarz mechanism as follows: "the idea is that in a theory with extra dimensions and global symmetries that do not commute with supersymmetry (R symmetries and $(-1)^F$ are examples), one could arrange for a twisted compactification, and that this would break supersymmetry". In case of ordinary reduction of a ten-dimensional supersymmetric Yang–Mills theory one obtains $\mathcal{N} = 4$ supersymmetric Yang–Mills theory in four dimensions. This has a global $SU(4)$ R symmetry which is identified with the tangent space $SO(6)$. In the CSDR in order to solve the constraints imposed on the fermions one has to embed R (of S/R) into $SO(6)$. Moreover the four-dimensional Lagrangian resulting from CSDR has an a global symmetry R (of the S/R). Therefore the CSDR satisfies automatically the criterion stated by Schwarz above that could lead to supersymmetry breaking.

Acknowledgements

We would like to thank L. Alvarez-Gaume, C. Bachas, G.L. Cardoso, P. Forgacs, A. Kehagias, C. Kounnas, G. Koutsoumbas, D. Luest and D. Suematsu for useful discussions.

References

[1] J. Wess, B. Zumino, Phys. Lett. B 49 (1974) 52;
J. Iliopoulos, B. Zumino, Nucl. Phys. 76 (1974) 310;
S. Ferrara, J. Iliopoulos, B. Zumino, Nucl. Phys. 77 (1974) 413;
K. Fujikawa, W. Lang, Nucl. Phys. 88 (1975) 61.
[2] P. Fayet, J. Iliopoulos, Phys. Lett. B 51 (1974) 461.
[3] P. Fayet, Phys. Lett. B 58 (1975) 67;
L. O'Raifeartaigh, Nucl. Phys. 96 (1975) 331.
[4] E. Cremmer, Phys. Lett. B 116 (1982) 231;
A. Chamseddine, R. Arnowitt, P. Nath, Phys. Rev. Lett. 49 (1982) 970;
R. Barbieri, S. Ferrara, C. Savoy, Phys. Lett. B 110 (1982) 343;
L.J. Hall, J. Lykken, S. Weinberg, Phys. Rev. D 27 (1983) 2359.
[5] J. Scherk, J.H. Schwarz, Nucl. Phys. B 153 (1979) 61.
[6] P. Manousselis, G. Zoupanos, Phys. Lett. B 504 (2001) 122.
[7] P. Forgacs, N.S. Manton, Commun. Math. Phys. 72 (1980) 15;
E. Witten, Phys. Rev. Lett. 38 (1977) 121.
[8] D. Kapetanakis, G. Zoupanos, Phys. Rep. C 219 (1992) 1, and references therein.
[9] Y. Kubyshin, J.M. Mourao, G. Rudolph, I.P. Volobujev, Lecture Notes Phys., Vol. 349, Springer-Verlag, Heidelberg, 1989.
[10] T.R. Taylor, G. Veneziano, Phys. Lett. B 212 (1988) 147.
[11] K.R. Dienes, E. Dudas, T. Gherghetta, Nucl. Phys. B 537 (1999) 47.

[12] T. Kobayashi et al., Nucl. Phys. B 550 (1999) 99;
J. Kubo, H. Terao, G. Zoupanos, Nucl. Phys. B 374 (2000) 495;
J. Kubo, H. Terao, G. Zoupanos, hep-ph/0010069.
[13] G. Chapline, R. Slansky, Nucl. Phys. B 204 (1982) 461.
[14] N.S. Manton, Nucl. Phys. B 193 (1981) 502.
[15] C. Wetterich, Nucl. Phys. B 222 (1985) 20;
L. Palla, Z. Phys. C 24 (1983) 345;
K. Pilch, A.N. Schellekens, J. Math. Phys. 25 (1984) 3455;
P. Forgacs, Z. Horvath, L. Palla, Z. Phys. C 30 (1986) 261;
K.J. Barnes et al., Z. Phys. C 33 (1987) 427.
[16] E. Witten, Phys. Lett. B 144 (1984) 351.
[17] K. Pilch, A.N. Schellekens, Nucl. Phys. B 259 (1985) 673;
D. Luest, Nucl. Phys. B 276 (1985) 220;
D. Kapetanakis, G. Zoupanos, Phys. Lett. B 249 (1990) 66.
[18] G. Chapline, N.S. Manton, Nucl. Phys. B 184 (1981) 391;
Y.A. Kubyshin, J.M. Mourao, I.P. Volobujev, Int. J. Mod. Phys. A 4 (1989) 151;
F.A. Bais et al., Nucl. Phys. B 263 (1986) 557.
[19] K. Farakos et al., Nucl. Phys. B 291 (1987) 128;
K. Farakos et al., Phys. Lett. B 191 (1987) 135.
[20] D. Kapetanakis, G. Zoupanos, Phys. Lett. B 249 (1990) 73;
D. Kapetanakis, G. Zoupanos, Z. Phys. C 56 (1992) 91.
[21] See, e.g., I. Antoniadis, K. Benakli, Int. J. Mod. Phys. A 15 (2000) 4237, and references therein;
N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B 249 (1998) 263;
G. Dvali, S. Randjbar-Daemi, R. Tabbash, The origin of spontaneous symmetry breaking in theories with large extra dimensions, hep-ph/0102307.
[22] S. Dimopoulos, H. Georgi, Phys. Lett. B 117 (1982) 287;
N. Sakai, Z. Phys. C 11 (1981) 155.
[23] E. Fahri, L. Susskind, Phys. Rep. C 74 (1981) 277;
C.T. Hill, in: Proc. of Nagoya Int. Workshop, 1996, p. 54, and references therein.
[24] W. Marciano, Phys. Rev. D 21 (1980) 315;
G. Zoupanos, Phys. Lett. B 129 (1983) 315;
P. Forgacs, G. Zoupanos, Phys. Lett. B 148 (1984) 99;
D. Luest et al., Nucl. Phys. B 268 (1986) 49.
[25] G. Triantaphyllou, J. Phys. G 26 (2000) 99;
G. Triantaphyllou, G. Zoupanos, Phys. Lett. B 489 (2000) 420;
G. Triantaphyllou, Mod. Phys. Lett. A 16 (2001) 53.
[26] M. Green, J.H. Schwarz, Phys. Lett. B 149 (1984) 117;
L. Alvarez-Gaume, E. Witten, Nucl. Phys. B 234 (1983) 269.
[27] See, e.g., M.B. Green, J.H. Schwarz, E. Witten, Superstring Theory, Cambridge Univ. Press, 1987;
D. Luest, S. Theisen, Lecture Notes Phys., Vol. 346, Springer-Verlag, Heidelberg, 1989.
[28] P. Horava, E. Witten, Nucl. Phys. B 460 (1996) 506;
E. Witten, Nucl. Phys. B 471 (1996) 135;
For a review, see C. Munoz, in: Proc. of Corfu Summer Institute on EPP, 1998, hep-th/9906152.
[29] P. Forgacs, Z. Horvath, L. Palla, Phys. Lett. B 147 (1984) 311;
P. Forgacs, Z. Horvath, L. Palla, Phys. Lett. B 148 (1984) 99;
R. Coquereaux, A. Jadczyk, Commun. Math. Phys. 98 (1985) 79;
A.N. Schellekens, Nucl. Phys. B 248 (1984) 706;
S. Randjbar-Daemi, A. Salam, J. Strathdee, Nucl. Phys. B 242 (1983) 447;
S. Randjbar-Daemi, A. Salam, J. Strathdee, Phys. Lett. B 124 (1983) 345.
[30] B.E. Hanlon, G.C. Joshi, Phys. Lett. B 298 (1993) 312.
[31] N. Kozimirov, V.A. Kuzmin, I.I. Tkachev, Sov. J. Nucl. Phys. 49 (1989) 164;
D. Kapetanakis, G. Zoupanos, Phys. Lett. B 232 (1989) 104.
[32] G. Zoupanos, Phys. Lett. B 201 (1988) 301.
[33] Y. Hosotani, Phys. Lett. B 126 (1983) 309;
Y. Hosotani, Phys. Lett. B 129 (1983) 193;
E. Witten, Nucl. Phys. B 258 (1985) 75;
J.D. Breit, B.A. Ovrut, G.C. Segre, Phys. Lett. B 158 (1985) 33;
B. Greene et al., Nucl. Phys. B 278 (1986) 667.
[34] See, e.g., Y. Kawamura, Prog. Theor. Phys. 105 (2001) 691;
D.M. Ghilencea, H. Nilles, Phys. Lett. B 507 (2001) 327;
G. Altarelli, F. Feruglio, Phys. Lett. B 511 (2001) 257;
T. Asaka, W. Buchmuller, L. Covi, Gauge unification in six dimensions, hep-ph/0108021;
J. Bagger, F. Feruglio, F. Zwirner, Brane-induced supersymmetry breaking, hep-th/0108010;
R. Barbieri, L.J. Hall, Y. Nomura, Models of Scherk–Schwarz symmetry breaking in 5D: classification and calculability, hep-th/0107004;
I. Antoniadis, S. Dimopoulos, A. Pomarol, M. Quiros, Nucl. Phys. B 544 (1999) 503;
A. Delgado, A. Pomarol, M. Quiros, Phys. Rev. D 60 (1999) 095008.
[35] J. Schwarz, Reminiscences of collaborations with Joel Scherk, hep-th/0007117.