



Generalized uncertainty principle and Hořava–Lifshitz gravity

Yun Soo Myung

Institute of Basic Science and School of Computer Aided Science, Inje University, Gimhae 621-749, Republic of Korea

ARTICLE INFO

Article history:

Received 1 August 2009

Received in revised form 12 August 2009

Accepted 13 August 2009

Available online 15 August 2009

Editor: T. Yanagida

ABSTRACT

We explore a connection between generalized uncertainty principle (GUP) and modified Hořava–Lifshitz (HL) gravity. The GUP density function may be replaced by the cutoff function for the renormalization group of modified Hořava–Lifshitz gravity. We find the GUP-corrected graviton propagators and compare these with tensor propagators in the HL gravity. Two are qualitatively similar, but the p^5 -term arisen from Cotton tensor is missed in the GUP-corrected graviton propagator.

© 2009 Elsevier B.V. Open access under [CC BY license](#).

1. Introduction

Recently Hořava has proposed a renormalizable theory of gravity at a Lifshitz point [1], which may be regarded as a UV complete candidate for general relativity. At short distances the theory of $z = 3$ Hořava–Lifshitz (HL) gravity describes interacting non-relativistic gravitons and is supposed to be power counting renormalizable in $(1 + 3)$ dimensions. Recently, the HL gravity theory has been intensively investigated in [2–27], its cosmological applications in [28,29], and its black hole solutions in [30–32].

It seems that the GUP effect on the Schwarzschild black hole is related to black holes in the deformed Hořava–Lifshitz gravity [33]. We could not confirm a solid connection between the GUP and the black hole of modified Hořava–Lifshitz gravity, although we have obtained partial connections between them.

However, it was known that the generalized uncertainty principle provides naturally a UV cutoff to the local quantum field theory as gravity effects [34,35].

It is known that the UV-propagator for tensor modes t_{ij} take a complicated form Eq. (49) including upto p^6 -term from the Cotton bilinear term $C_{ij}C_{ij}$. At low energies, the UV-propagator may reduce to a conventional IR-propagator as $G_{\text{IR}}(\omega, \vec{p}) = 1/(\omega^2 - c^2\vec{p}^2)$ for $z = 1$ HL gravity. It is very important to understand why the UV-propagator takes a complicated form in the non-relativistic gravity theory.

In this work, we investigate a connection between GUP and modified Hořava–Lifshitz gravity. The GUP density function may be replaced by a cutoff function for the renormalization group study of modified Hořava–Lifshitz gravity. We find GUP-corrected graviton propagators and compare these with UV-tensor propagators in the HL gravity. Two are similar, but the p^5 -term arisen from Cotton tensor is missed in the GUP-corrected graviton propagator. This shows that a power-counting renormalizable theory of the HL gravity is closely related to the GUP.

2. HL gravity

Introducing the ADM formalism where the metric is parameterized [36]

$$ds_{\text{ADM}}^2 = -N^2 dt^2 + g_{ij}(dx^i - N^i dt)(dx^j - N^j dt), \quad (1)$$

the Einstein–Hilbert action can be expressed as

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^4x \sqrt{g} N [K_{ij}K^{ij} - K^2 + R - 2\Lambda], \quad (2)$$

where G is Newton's constant and extrinsic curvature K_{ij} takes the form

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i). \quad (3)$$

E-mail address: ysmyung@inje.ac.kr.

Here, a dot denotes a derivative with respect to t . An action of the non-relativistic renormalizable gravitational theory is given by [1]

$$S_{\text{HL}} = \int dt d^3x [\mathcal{L}_K + \mathcal{L}_V], \quad (4)$$

where the kinetic terms are given by

$$\mathcal{L}_K = \frac{2}{\kappa^2} \sqrt{g} N K_{ij} \mathcal{G}^{ijkl} K_{kl} = \frac{2}{\kappa^2} \sqrt{g} N (K_{ij} K^{ij} - \lambda K^2), \quad (5)$$

with the DeWitt metric

$$\mathcal{G}^{ijkl} = \frac{1}{2} (g^{ik} g^{jl} - g^{il} g^{jk}) - \lambda g^{ij} g^{kl} \quad (6)$$

and its inverse metric

$$\mathcal{G}_{ijkl} = \frac{1}{2} (g_{ik} g_{jl} - g_{il} g_{jk}) - \frac{\lambda}{3\lambda - 1} g_{ij} g_{kl}. \quad (7)$$

The potential terms is determined by the detailed balance condition (DBC) as

$$\begin{aligned} \mathcal{L}_V = -\frac{\kappa^2}{2} \sqrt{g} N E^{ij} \mathcal{G}_{ijkl} E^{kl} = \sqrt{g} N \left\{ \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left(\frac{1-4\lambda}{4} R^2 + \Lambda_W R - 3\Lambda_W^2 \right) \right. \\ \left. - \frac{\kappa^2}{2W^4} \left(C_{ij} - \frac{\mu W^2}{2} R_{ij} \right) \left(C^{ij} - \frac{\mu W^2}{2} R^{ij} \right) \right\}. \end{aligned} \quad (8)$$

Here the E tensor is defined by

$$E^{ij} = \frac{1}{W^2} C^{ij} - \frac{\mu}{2} \left(R^{ij} - \frac{R}{2} g^{ij} + \Lambda_W g^{ij} \right) \quad (9)$$

with the Cotton tensor C_{ij}

$$C^{ij} = \frac{\epsilon^{ik\ell}}{\sqrt{g}} \nabla_k (R^j{}_{\ell} - \frac{1}{4} R \delta^j_{\ell}). \quad (10)$$

Explicitly, E_{ij} could be derived from the Euclidean topologically massive gravity

$$E^{ij} = \frac{1}{\sqrt{g}} \frac{\delta W_{\text{TMG}}}{\delta g_{ij}} \quad (11)$$

with

$$W_{\text{TMG}} = \frac{1}{W^2} \int d^3x \epsilon^{ikl} \left(\Gamma_{il}^m \partial_j \Gamma_{km}^l + \frac{2}{3} \Gamma_{il}^n \Gamma_{jm}^l \Gamma_{kn}^m \right) - \mu \int d^3x \sqrt{g} (R - 2\Lambda_W), \quad (12)$$

where ϵ^{ikl} is a tensor density with $\epsilon^{123} = 1$.

In the IR limit, comparing \mathcal{L}_0 with Eq. (2) of general relativity, the speed of light, Newton's constant and the cosmological constant are given by

$$c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1-3\lambda}}, \quad G = \frac{\kappa^2}{32\pi c}, \quad \Lambda_{\text{cc}} = \frac{3}{2} \Lambda_W. \quad (13)$$

The equations of motion were derived in [28] and [30]. We would like to mention that the IR vacuum of this theory is anti-de Sitter (AdS_4) spacetimes. Hence, it is interesting to take a limit of the theory, which may lead to a Minkowski vacuum in the IR sector. To this end, one may deform the theory by introducing “ $\mu^4 R$ ” ($\tilde{\mathcal{L}}_V = \mathcal{L}_V + \sqrt{g} N \mu^4 R$) and then, take the $\Lambda_W \rightarrow 0$ limit [31]. This does not alter the UV properties of the theory, while it changes the IR properties. That is, there exists a Minkowski vacuum, instead of an AdS vacuum. In the IR limit, the speed of light and Newton's constant are given by

$$c^2 = \frac{\kappa^2 \mu^4}{2}, \quad G = \frac{\kappa^2}{32\pi c}, \quad \lambda = 1. \quad (14)$$

3. GUP

A meaningful prediction of various theories of quantum gravity (string theory) and black holes is the presence of a minimum measurable length or a maximum observable momentum. This has provided the generalized uncertainty principle which modifies commutation relations between position coordinates and momenta. Also the black hole solution of modified HL gravity reminds us the Schwarzschild black hole modified with GUP [34]. Hence, we make a close connection between GUP and Hořava–Lifshitz gravity. A commutation relation of

$$[\vec{x}, \vec{p}] = i\hbar(1 + \beta^2 \vec{p}^2) \quad (15)$$

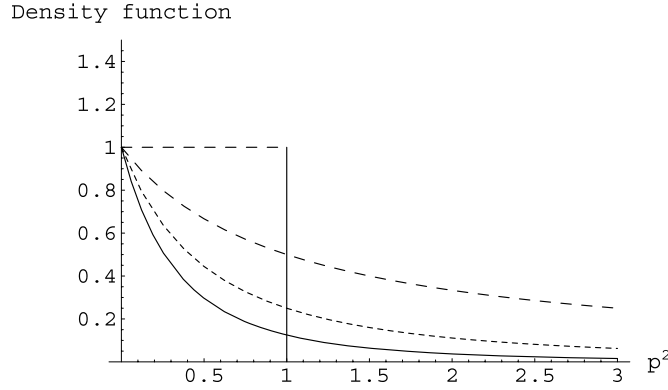


Fig. 1. Density functions for regularization of a LQFT as function of p^2 with $\beta = 1$. The dashed line denotes “arbitrary uniform density function” as the UV cutoff $1/\sqrt{\beta} = 1$ for $p^2 \in [0, 1]$ required by hand, while three curves represent the GUP density function in Eq. (20) for $D = 1, 2,$ and 3 from top to bottom. These curves cut effectively off the integral beyond $p = 1/\sqrt{\beta}$.

leads to the generalized uncertainty relation

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left[1 + \alpha^2 l_p^2 \frac{(\Delta p)^2}{\hbar^2} \right], \tag{16}$$

with $l_p = \sqrt{G\hbar/c^3}$ the Planck length. Here a parameter $\alpha = \hbar\sqrt{\beta}/l_p$ is introduced to indicate the GUP effect. The Planck mass is given by $m_p = \sqrt{\hbar c/G}$. The above implies a lower bound on the length scale

$$\Delta x \geq (\Delta x)_{\text{min}} \approx \hbar\sqrt{\beta} = \alpha l_p, \tag{17}$$

which means that the Planck length plays the role of a fundamental scale. On the other hand, Eq. (16) implies the upper bound on the momentum as

$$\Delta p \leq (\Delta p)_{\text{max}} \approx \frac{1}{\sqrt{\beta}} = \frac{m_p c}{\alpha}. \tag{18}$$

Importantly, it was known that the generalized uncertainty principle provides naturally a UV cutoff to the local quantum field theory (LQFT) as gravity effects [34,35]. The GUP relation of Eq. (15) has an effect on the density of states in D -dimensional momentum space as

$$d^D \vec{p} \mathcal{D}_D(\beta \vec{p}^2), \tag{19}$$

where a density function (weight factor) $\mathcal{D}_D(\beta \vec{p}^2)$ is defined by

$$\mathcal{D}_D(\beta \vec{p}^2) = \frac{1}{(1 + \beta \vec{p}^2)^D}. \tag{20}$$

As is depicted in Fig. 1, this function cuts effectively off the integral beyond $p = 1/\sqrt{\beta}$. Due to strong suppression of density of states at high momenta, a relevant quantity will be rendered finite with $1/\sqrt{\beta}$ acting effectively as a UV cutoff. We wish to mention that this function may be related to the Cotton-term of $C_{ij}C^{ij}$ in Eq. (8) because the latter contains a sixth order derivative. We note that the arbitrary uniform density function is introduced by hand and thus, the physics beyond the cutoff ($\vec{p}^2 > 1/\sqrt{\beta} = 1$) never contributes to a relevant quantity.

The right-hand side of Eq. (15) includes a \vec{p} -dependent term and thus affect the cell size in phase space as “being \vec{p} -dependent”. Making use of the Liouville theorem, one could show that the invariant weighted-phase space volume under time evolution is given by [34]

$$\frac{d^D \vec{x} d^D \vec{p}}{(1 + \beta \vec{p}^2)^D}, \tag{21}$$

where the classical commutation relations corresponding to the quantum commutation relation of Eq. (15) are given via $[A, B]/i\hbar \rightarrow \{A, B\}$ by

$$\{x_i, p_j\} = (1 + \beta p^2) \delta_{ij}, \quad \{p_i, p_j\} = 0, \quad \{x_i, x_j\} = 2\beta(p_i x_j - p_j x_i). \tag{22}$$

Actually, $1/\sqrt{\beta}$ plays the role of a UV cutoff Λ of the momentum integration as [35]

$$\frac{1}{\sqrt{\beta}} \rightarrow \Lambda. \tag{23}$$

As a concrete example, by assuming that the zero-point energy of each oscillator is of $\hbar\omega/2 = \hbar\sqrt{\vec{p}^2 + m^2}/2$, the cosmological constant is calculated to be

$$\Lambda_{\text{cc}}(m) = \int \frac{d^3 \vec{p}}{(1 + \beta \vec{p}^2)^3} \left[\frac{\sqrt{\vec{p}^2 + m^2}}{2} \right] = 2\pi \int_0^\infty \frac{p^2 dp}{(1 + \beta p^2)^3} \sqrt{p^2 + m^2} = \frac{\pi}{2\beta^2} f(\beta m^2), \tag{24}$$

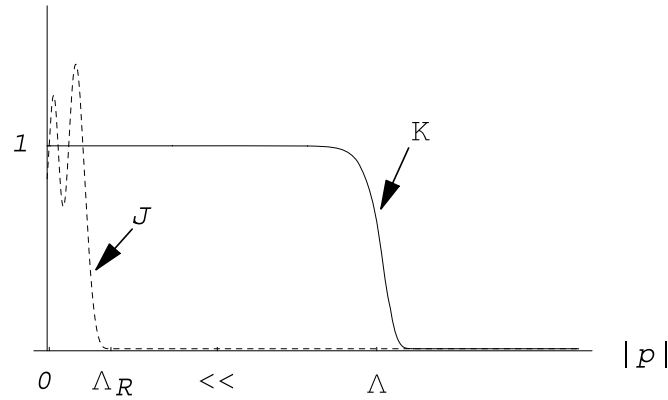


Fig. 2. The linear source term is constrained to $J(p) = 0$ for $p > \Lambda_R$ so as to only excite Green functions with low energy. The quadratic term contains a cutoff function $\mathcal{K}(\frac{p^2}{\Lambda^2})$ with the property that $\mathcal{K} = 1$ for $p < \Lambda$ and then falls off smoothly to zero for $p \geq \Lambda$.

with $f(0) = 1$ and $\hbar = 1$. Then, one obtains the cosmological constant for the massless case

$$\Lambda_{cc}(0) = \frac{\pi}{2\beta^2} \rightarrow \frac{\pi}{2}\Lambda^4. \tag{25}$$

Finally, the GUP commutation relation in Eq. (15) can be extended into [37]

$$[\vec{x}, \vec{p}] = i\hbar e^{\beta^2 \vec{p}^2}, \tag{26}$$

which includes all order corrections to the Heisenberg uncertainty principle. In this case, the density function is given by an exponential function [38]

$$\mathcal{D}_D^{\text{all}}(\beta \vec{p}^2) = \frac{1}{e^{\beta^2 \vec{p}^2}}. \tag{27}$$

4. Cutoff function for a relativistic theory

It is well known that even the simplest local quantum field theories (LQFT) are useless because the answer to any loop calculation is infinite. A standard example is the 1-loop correction to the mass in scalar $\tilde{\lambda}\phi^3$ [39]

$$\Delta m^2 = \frac{\tilde{\lambda}^2}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 + m^2)((p + q)^2 + m^2)} \tag{28}$$

$$= \text{finite} + \mathcal{C} \int \frac{dp}{p} = \infty, \tag{29}$$

with $p^2 = p_\mu p^\mu$ and a constant \mathcal{C} . The reason why we have this meaningless result is clear because of integrating all the way to infinity in momentum space. One way to avoid infinity is to introduce a UV cutoff Λ . However, we run into trouble with LQFT, because a regularized theory is no longer unitary since we “arbitrarily” removed part of the phase space (by hand) to which there was associated a non-zero amplitude. In order to find an appropriate situation, we wish to probe the system at some energy scale Λ_R namely, incoming momenta in Feynman graphs obey $p \leq \Lambda_R$, while keeping $\Lambda \gg \Lambda_R$. If we can make all physical observables at Λ_R independent of Λ , then we can safely take $\Lambda \rightarrow \infty$. It is convenient to parameterize the energy scale using an RG “time” parameter flowing towards lower and lower energies

$$\Lambda(t) = \Lambda(0)e^{-t}. \tag{30}$$

Let us demand that changing the cutoff Λ leaves the partition function invariant as

$$\partial_t Z[J] = 0. \tag{31}$$

Then we could define the partition function of a LQFT by

$$Z[J] = \frac{I[J]}{I[0]}, \tag{32}$$

where

$$I[J] = \int [d\phi] e^{-(S_0 + S_1 + S_J)}. \tag{33}$$

The action is composed of linear source term, quadratic kinetic term and polynomial interaction term as

$$S_J = \int \frac{d^4 p}{(2\pi)^4} J(p, \Lambda_R) \phi(-p), \quad (34)$$

$$S_0 = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \phi(p) \phi(-p) \frac{\tilde{\Delta}(p^2)}{\mathcal{K}(p^2/\Lambda^2)}, \quad (35)$$

$$S_I = \sum_{n=3}^{\infty} \int \frac{d^4 p_1}{(2\pi)^4} \cdots \frac{d^4 p_n}{(2\pi)^4} \delta^4\left(\sum p_i\right) g_n(p_1, \dots, p_n; \Lambda) \phi(p_1) \cdots \phi(p_n), \quad (36)$$

where $\tilde{\Delta}(p^2)$ is the inverse propagator in four-momentum space. That is, $\tilde{\Delta}(p^2) = p^2 + m^2$ for a massive scalar field. Here we include a source function $J(p)$ and a smooth cutoff function $\mathcal{K}(p^2/\Lambda^2)$ with specific properties as was described in Fig. 2. From Eq. (35), a relativistic propagator could be derived to take the form

$$\tilde{\Delta}(p^2)^{-1} \times \mathcal{K}\left(\frac{p^2}{\Lambda^2}\right). \quad (37)$$

We mention two important properties: $\partial_t \mathcal{K} \cdot J = 0$ because they have disjoint support and $\partial_t J(p) = 0$ because J depends only on Λ_R . Finally, using these properties, one arrives at

$$\partial_t (e^{-S_I}) = -\frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{\partial_t \mathcal{K}}{\tilde{\Delta}(p^2)} \frac{\delta}{\delta \phi(p)} \frac{\delta}{\delta \phi(-p)} e^{-S_I}. \quad (38)$$

Eq. (38) describes the infinitesimal change of the interaction Lagrangian upon changing the UV cutoff Λ . This dependence of the coupling constants on the cutoff Λ is called the ‘‘RG flow’’. A procedure of decreasing the cutoff on $|p|$ infinitesimally from Λ to $\Lambda - \delta\Lambda$ is called integrating out a momentum shell. We note that no infinities are encountered here because all the momentum integrals are done in an infinitesimally finite range.

Finally, we propose that the cutoff function $\mathcal{K}(p^2/\Lambda^2)$ for a relativistic theory can be replaced by the density function $\mathcal{D}_D(\beta\vec{p}^2)$ for a non-relativistic gravity theory:

$$\mathcal{K}(p^2/\Lambda^2) \rightarrow \mathcal{D}_D(\beta\vec{p}^2). \quad (39)$$

This is quite reasonable because two functions play the similar role in suppressing high momenta (UV region).

5. Propagators of HL gravity

We wish to consider perturbations of the metric around Minkowski spacetimes, which is a solution to the $z = 3$ HL gravity (4)

$$g_{ij} = \delta_{ij} + w h_{ij}, \quad N = 1 + w n, \quad N_i = w n_i. \quad (40)$$

In order to have tensor propagator, it is convenient to use the cosmological decomposition in terms of scalar, vector, and tensor modes under spatial rotations $SO(3)$ [40]

$$n = -\frac{1}{2} A, \quad n_i = \partial_i B + V_i, \quad h_{ij} = \psi \delta_{ij} + \partial_i \partial_j E + 2\partial_{(i} F_{j)} + t_{ij}, \quad (41)$$

where $\partial^i F_i = \partial^i V_i = \partial^i t_{ij} = t_{ii} = 0$. The last two conditions mean that t_{ij} is a transverse and traceless tensor in three dimensions. Using this decomposition, the scalar modes (A, B, ψ, E), the vector modes (V_i, F_i), and the tensor modes (t_{ij}) decouple completely from each other. These all amount to 10 degrees of freedom for a symmetric tensor in four dimensions. Hereafter we consider tensor modes only.

5.1. Tensor modes

The field equation for tensor modes is given by [23]

$$\ddot{t}_{ij} - \frac{\mu^4 \kappa^2}{2} \Delta t_{ij} + \frac{\mu^2 \kappa^4}{16} \Delta^2 t_{ij} - \frac{\mu \kappa^4 \gamma^2}{4w^2} \epsilon_{ilm} \partial^l \Delta^2 t_j^m - \frac{\kappa^4}{4w^4} \Delta^3 t_{ij} = T_{ij}, \quad (42)$$

with external source T_{ij} and the Laplacian $\Delta = \partial_i^2 \rightarrow -\vec{p}^2$. We could not obtain the covariant propagator because of the presence of ϵ -term. Assuming a massless graviton propagation along the x^3 -direction with $\vec{p} = (0, 0, p_3)$, then the t_{ij} can be expressed in terms of polarization components as [27]

$$t_{ij} = \begin{pmatrix} t_+ & t_\times & 0 \\ t_\times & -t_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (43)$$

Using this parametrization, we find two coupled equations for different polarizations

$$\ddot{t}_+ - \frac{\mu^4 \kappa^2}{2} \Delta t_+ + \frac{\kappa^4 \mu^2}{16} \Delta^2 t_+ + \frac{\kappa^4 \mu}{4w^2} \partial_3 \Delta^2 t_\times - \frac{\kappa^4}{4w^4} \Delta^3 t_+ = T_+, \quad (44)$$

$$\ddot{t}_\times - \frac{\mu^4 \kappa^2}{2} \Delta t_\times + \frac{\kappa^4 \mu^2}{16} \Delta^2 t_\times - \frac{\kappa^4 \mu}{4w^2} \partial_3 \Delta^2 t_+ - \frac{\kappa^4}{4w^4} \Delta^3 t_\times = T_\times. \quad (45)$$

In order to find two independent components, we introduce the left-right base defined by

$$h_{L/R} = \frac{1}{\sqrt{2}}(h_+ \pm ih_\times), \quad (46)$$

where $h_L(h_R)$ represent the left (right)-handed modes. After Fourier-transformation, we find two decoupled equations

$$-\omega^2 t_L + c^2 \vec{p}^2 t_L + \frac{\kappa^4 \mu^2}{16} (\vec{p}^2)^2 t_L - \frac{\kappa^4 \mu}{4w^2} p_3 (\vec{p}^2)^2 t_L + \frac{\kappa^4}{4w^4} (\vec{p}^2)^3 t_L = T_L, \quad (47)$$

$$-\omega^2 t_R + c^2 \vec{p}^2 t_R + \frac{\kappa^4 \mu^2}{16} (\vec{p}^2)^2 t_R + \frac{\kappa^4 \mu}{4w^2} p_3 (\vec{p}^2)^2 t_R + \frac{\kappa^4}{4w^4} (\vec{p}^2)^3 t_R = T_R. \quad (48)$$

We have UV-tensor propagators

$$t_{L/R} = - \frac{T_{L/R}}{\omega^2 - c^2 \vec{p}^2 - \frac{c^2 \kappa^2}{8\mu^2} (\vec{p}^2)^2 \pm \frac{c^2 \kappa^2}{2w^2 \mu^3} p_3 (\vec{p}^2)^2 - \frac{c^2 \kappa^2}{2w^4 \mu^4} (\vec{p}^2)^3}. \quad (49)$$

We note that the left-handed mode is not allowed because it may give rise to ghost ($-\frac{c^2 \kappa^2}{2w^2 \mu^3} p_3 (\vec{p}^2)^2$), while the right-handed mode is allowed because there is no ghost ($\frac{c^2 \kappa^2}{2w^2 \mu^3} p_3 (\vec{p}^2)^2$). Finally, we have UV-propagators in the Lorentz-frame with $p^\mu = (\omega, 0, 0, p_3)$ as

$$t_{L/R} = - \frac{T_{L/R}}{\omega^2 - c^2 p_3^2 - \frac{c^2 \kappa^2}{8\mu^2} p_3^4 \pm \frac{c^2 \kappa^2}{2w^2 \mu^3} p_3^5 - \frac{c^2 \kappa^2}{2w^4 \mu^4} p_3^6}. \quad (50)$$

5.2. GUP-corrected propagators

Here we propose that the GUP-corrected tensor propagators may take the form

$$G_{\text{IR}}(\omega, \vec{p}) \times \mathcal{D}_D(\beta \vec{p}^2), \quad (51)$$

where the IR-propagator $G_{\text{IR}}(\omega, \vec{p})$ is defined by

$$G_{\text{IR}}(\omega, \vec{p}) = \frac{1}{\omega^2 - c^2 \vec{p}^2}. \quad (52)$$

For $D = 1$, its form takes

$$t_{ij}^{1\text{DGUP}} = -G_{\text{IR}}(\omega, \vec{p}) \times \mathcal{D}_1(\beta \vec{p}^2) T_{ij} = - \frac{T_{ij}}{(\omega^2 - c^2 \vec{p}^2)(1 + \beta \vec{p}^2)} = - \frac{T_{ij}}{\omega^2 - c^2(1 - \frac{\beta \omega^2}{c^2})\vec{p}^2 - c^2 \beta^2 (\vec{p}^2)^2} \quad (53)$$

which may be related to the propagator of $z = 2$ HL gravity defined by the Einstein gravity

$$W_{\text{EG}} = \mu \int d^3x \sqrt{g} [R - 2\Lambda_W]. \quad (54)$$

The $D = 2$ GUP-corrected tensor propagator is given by

$$\begin{aligned} t_{ij}^{2\text{DGUP}} &= -G_{\text{IR}}(\omega, \vec{p}) \times \mathcal{D}_2(\beta \vec{p}^2) T_{ij} = - \frac{T_{ij}}{(\omega^2 - c^2 \vec{p}^2)(1 + \beta \vec{p}^2)^2} \\ &= - \frac{T_{ij}}{\omega^2 - c^2(1 - \frac{2\beta \omega^2}{c^2})\vec{p}^2 - c^2(2\beta - \frac{\beta^2 \omega^2}{c^2})(\vec{p}^2)^2 - c^2 \beta^2 (\vec{p}^2)^3}, \end{aligned} \quad (55)$$

where scaling dimensions are given by $[\beta] = -2$, $[\omega] = 3$, and $[c] = 2$. This may be related to UV-tensor propagator (49) for $z = 3$ HL gravity because the highest space derivative is sixth order. At this stage, it is not clear why the $D = 2$ GUP-corrected tensor propagator take a qualitatively similar form like UV-tensor propagator of $z = 3$ HL gravity except the p^5 -term. We conjecture that this may be possible because the $z = 3$ HL gravity originates from the detailed balance condition. Finally, the $D = 3$ GUP-corrected tensor propagator is given by

$$\begin{aligned} t_{ij}^{3\text{DGUP}} &= -G_{\text{IR}}(\omega, \vec{p}) \times \mathcal{D}_3(\beta \vec{p}^2) T_{ij} = - \frac{T_{ij}}{(\omega^2 - c^2 \vec{p}^2)(1 + \beta \vec{p}^2)^3} \\ &= - \frac{T_{ij}}{\omega^2 - c^2(1 - \frac{3\beta \omega^2}{c^2})\vec{p}^2 - 3c^2(\beta - \frac{\beta^2 \omega^2}{c^2})(\vec{p}^2)^2 - c^2(3\beta^2 - \frac{\beta^3 \omega^2}{c^2})(\vec{p}^2)^3 - c^2 \beta^3 (\vec{p}^2)^4}, \end{aligned} \quad (56)$$

which may be related to the UV-tensor propagator in $z = 4$ HL gravity because the highest space derivative has eighth order. The $z = 4$ HL gravity was constructed, through the detailed balance condition, from the new massive gravity [41]

$$W_{\text{NMG}} = \int d^3x \sqrt{g} \left[-\mu(R - 2\Lambda_W) + \frac{1}{M} \left(R_{\mu\nu} R^{\mu\nu} - \frac{3}{8} R^2 \right) \right]. \quad (57)$$

6. Discussions

We have explored a connection between the GUP commutator of (15) and the Hořava–Lifshitz gravity (a candidate of quantum gravity). Explicitly, we have replaced a relativistic cutoff function $\mathcal{K}(\frac{p^2}{\Lambda^2})$ by a non-relativistic density function $\mathcal{D}_D(\beta\vec{p}^2)$ to derive GUP-corrected graviton propagators. These were compared to a UV-tensor graviton propagator in the HL gravity. We point out that two are qualitatively similar, but the p^5 -term arisen from the crossed operation of Cotton and Ricci tensors did not appear in the GUP-corrected propagators. Also, it is unclear why the $D = 2$ GUP-corrected tensor propagator (not the $D = 3$ GUP-corrected propagator) takes a similar form from the $z = 3$ HL gravity. We conjecture that it may be related to the detailed balance condition. Even though our GUP-corrected propagator does not lead to a precise graviton propagator, this approach will provide a hint to understand quantum aspects of the HL gravity.

A key point to understand a connection between two seemingly different approaches is to recognize “effects of quantum gravity”. The GUP provides naturally a UV cutoff $1/\sqrt{\beta}$ to the LQFT as effects of quantum gravity through the density function $\mathcal{D}_D(\beta\vec{p}^2)$. The modified HL gravity action is composed of higher space derivatives terms from the detailed balance condition like R^2 , R_{ij}^2 , $R_{ij}C_{ij}$ and C_{ij}^2 in addition to $\mu^4 R$, to become a power-counting renormalizable quantum gravity theory. All these higher derivative terms modify the tensor propagator into the UV-tensor propagator in Eq. (49) without ghost. We need a further study to justify whether there exists an exact connection between GUP and HL gravity.

Consequently, we have shown that effects of quantum gravity are imprinted on the GUP, which may explain the UV-tensor propagator of the modified HL gravity.

Acknowledgements

The author was supported by the SRC Program of the KOSEF through the Center for Quantum Spacetime (CQUeST) of Sogang University with grant number R11-2005-021-03001-0.

References

- [1] P. Horava, Phys. Rev. D 79 (2009) 084008, arXiv:0901.3775 [hep-th].
- [2] P. Horava, JHEP 0903 (2009) 020, arXiv:0812.4287 [hep-th].
- [3] M. Visser, Phys. Rev. D 80 (2009) 025011, arXiv:0902.0590 [hep-th].
- [4] P. Hořava, arXiv:0902.3657 [hep-th].
- [5] A. Volovich, C. Wen, arXiv:0903.2455 [hep-th].
- [6] J. Kluson, arXiv:0904.1343 [hep-th].
- [7] H. Nikolic, arXiv:0904.3412 [hep-th].
- [8] H. Nastase, arXiv:0904.3604 [hep-th].
- [9] K.I. Izawa, arXiv:0904.3593 [hep-th].
- [10] G.E. Volovik, arXiv:0904.4113 [gr-qc].
- [11] T.P. Sotiriou, M. Visser, S. Weinfurtner, arXiv:0904.4464 [hep-th].
- [12] B. Chen, Q.G. Huang, arXiv:0904.4565 [hep-th].
- [13] R.G. Cai, B. Hu, H.B. Zhang, arXiv:0905.0255 [hep-th].
- [14] T. Nishioka, arXiv:0905.0473 [hep-th].
- [15] D. Orlando, S. Reffert, arXiv:0905.0301 [hep-th].
- [16] Y.W. Kim, H.W. Lee, Y.S. Myung, arXiv:0905.3423 [hep-th].
- [17] R.A. Konoplya, arXiv:0905.1523 [hep-th].
- [18] M. Li, Y. Pang, arXiv:0905.2751 [hep-th].
- [19] C. Charmousis, G. Niz, A. Padilla, P.M. Saffin, arXiv:0905.2579 [hep-th].
- [20] T.P. Sotiriou, M. Visser, S. Weinfurtner, arXiv:0905.2798 [hep-th].
- [21] G. Calcagni, arXiv:0905.3740 [hep-th].
- [22] M. Sakamoto, arXiv:0905.4326 [hep-th].
- [23] Y.S. Myung, arXiv:0906.0848 [hep-th].
- [24] C. Germani, A. Kehagias, K. Sfetsos, arXiv:0906.1201 [hep-th].
- [25] D. Blas, O. Pujolas, S. Sibiryakov, arXiv:0906.3046 [hep-th].
- [26] A. Kobakhidze, arXiv:0906.5401 [hep-th].
- [27] C. Bogdanos, E.N. Saridakis, arXiv:0907.1636 [hep-th].
- [28] E. Kiritsis, G. Kofinas, arXiv:0904.1334 [hep-th].
- [29] G. Calcagni, arXiv:0904.0829 [hep-th];
T. Takahashi, J. Soda, arXiv:0904.0554 [hep-th];
S. Mukohyama, JCAP 0906 (2009) 001, arXiv:0904.2190 [hep-th];
R. Brandenberger, arXiv:0904.2835 [hep-th];
Y.S. Piao, arXiv:0904.4117 [hep-th];
X. Gao, arXiv:0904.4187 [hep-th];
B. Chen, S. Pi, J.Z. Tang, arXiv:0905.2300 [hep-th];
E.N. Saridakis, arXiv:0905.3532 [hep-th];
S. Mukohyama, arXiv:0905.3563 [hep-th];
S. Mukohyama, M. Minamitsuji, arXiv:0905.3892 [astro-ph.CO];
A. Wang, Y. Wu, arXiv:0905.4117 [hep-th];
S. Nojiri, S.D. Odintsov, arXiv:0905.4213 [hep-th];
X. Gao, Y. Wang, R. Brandenberger, A. Riotto, arXiv:0905.3821 [hep-th];
M.i. Park, arXiv:0906.4275 [hep-th];
S. Mukohyama, arXiv:0906.5069 [hep-th];
A. Wang, R. Maartens, arXiv:0907.1748 [hep-th];
S.S. Kim, T. Kim, Y. Kim, arXiv:0907.3093 [hep-th];
Y. Lu, Y.S. Piao, arXiv:0907.3982 [hep-th].
- [30] H. Lu, J. Mei, C.N. Pope, arXiv:0904.1595 [hep-th].
- [31] A. Kehagias, K. Sfetsos, Phys. Lett. B 678 (2009) 123, arXiv:0905.0477 [hep-th].

- [32] R.G. Cai, L.M. Cao, N. Ohta, arXiv:0904.3670 [hep-th];
R.G. Cai, Y. Liu, Y.W. Sun, JHEP 0906 (2009) 010, arXiv:0904.4104 [hep-th];
E.O. Colgain, H. Yavartanoo, arXiv:0904.4357 [hep-th];
Y.S. Myung, Y.W. Kim, arXiv:0905.0179 [hep-th];
R.G. Cai, L.M. Cao, N. Ohta, arXiv:0905.0751 [hep-th];
A. Ghodsi, arXiv:0905.0836 [hep-th];
S. Chen, J. Jing, arXiv:0905.1409 [gr-qc];
S. Chen, J. Jing, arXiv:0905.2055 [gr-qc];
J. Chen, Y. Wang, arXiv:0905.2786 [gr-qc];
M. Botta-Cantcheff, N. Grandi, M. Sturla, arXiv:0906.0582 [hep-th];
M.i. Park, arXiv:0905.4480 [hep-th];
A. Ghodsi, E. Hatefi, arXiv:0906.1237 [hep-th];
A. Castillo, A. Larranaga, arXiv:0906.4380 [gr-qc];
J.J. Peng, S.Q. Wu, arXiv:0906.5121 [hep-th];
H.W. Lee, Y.W. Kim, Y.S. Myung, arXiv:0907.3568 [hep-th].
- [33] Y.S. Myung, Phys. Lett. B 678 (2009) 127, arXiv:0905.0957 [hep-th].
- [34] L.N. Chang, D. Minic, N. Okamura, T. Takeuchi, Phys. Rev. D 65 (2002) 125028, arXiv:hep-th/0201017.
- [35] Y.W. Kim, H.W. Lee, Y.S. Myung, Phys. Lett. B 673 (2009) 293, arXiv:0811.2279 [gr-qc].
- [36] R.L. Arnowitt, S. Deser, C.W. Misner, The dynamics of general relativity, in: L. Witten (Ed.), Gravitation: An Introduction to Current Research, Wiley, 1962, pp. 227–265, arXiv:gr-qc/0405109, Chapter 7.
- [37] K. Nouicer, Phys. Lett. B 646 (2007) 63, arXiv:0704.1261 [gr-qc].
- [38] Y.-W. Kim, Y.-J. Park, Phys. Lett. B 655 (2007) 172, arXiv:0707.2128 [gr-qc].
- [39] A. Shomer, arXiv:0709.3555 [hep-th].
- [40] V.F. Mukhanov, H.A. Feldman, R.H. Brandenberger, Phys. Rep. 215 (1992) 203.
- [41] E.A. Bergshoeff, O. Hohm, P.K. Townsend, Phys. Rev. Lett. 102 (2009) 201301, arXiv:0901.1766 [hep-th].