View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Elsevier - Publisher Connector

ELSEVIER
Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 659 (2008) 476–482
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Magnetic strings in Einstein–Born–Infeld-dilaton gravity

M.H. Dehghani [a,b], A. Sheykhi [c,*], S.H. Hendi [d]

[a] *Physics Department and Biruni Observatory, Shiraz University, Shiraz 71454, Iran*
[b] *Research Institute for Astrophysics and Astronomy of Maragha (RIAAM), Maragha, Iran*
[c] *Department of Physics, Shahid Bahonar University, P.O. Box 76175-132, Kerman, Iran*
[d] *Department of Physic, College of Science, Yasouj University, Yasouj 75914, Iran*

Received 1 October 2007; received in revised form 5 November 2007; accepted 9 November 2007

Available online 17 November 2007

Editor: M. Cvetič

Abstract

A class of spinning magnetic string in 4-dimensional Einstein-dilaton gravity with Liouville type potential which produces a longitudinal nonlinear electromagnetic field is presented. These solutions have no curvature singularity and no horizon, but have a conic geometry. In these spacetimes, when the rotation parameter does not vanish, there exists an electric field, and therefore the spinning string has a net electric charge which is proportional to the rotation parameter. Although the asymptotic behavior of these solutions are neither flat nor (A)dS, we calculate the conserved quantities of these solutions by using the counterterm method. We also generalize these four-dimensional solutions to the case of $(n+1)$-dimensional rotating solutions with $k \leqslant [n/2]$ rotation parameters, and calculate the conserved quantities and electric charge of them.
© 2007 Elsevier B.V. All rights reserved.

1. Introduction

The Born–Infeld [1] type of generalizations of Abelian and non-Abelian gauge theories have received a lot of interest in recent years. This is due to the fact that such generalizations appear naturally in the context of the superstring theory [2]. The nonlinearity of the electromagnetic field brings remarkable properties to avoid the black hole singularity problem which may contradict the strong version of the Penrose cosmic censorship conjecture in some cases. Actually a new nonlinear electromagnetism was proposed, which produces a nonsingular exact black hole solution satisfying the weak energy condition [3], and has distinct properties from Bardeen black holes [4]. The Born–Infeld action including a dilaton and an axion field, appears in the couplings of an open superstring and an Abelian gauge field. This action, describing a Born–Infeld-dilaton-axion system coupled to Einstein gravity, can be considered as a nonlinear extension of the Abelian field of Einstein–Maxwell-dilaton-axion gravity. Exact static solutions of Einstein–Born–Infeld (EBI) gravity in arbitrary dimensions with positive, zero or negative constant curvature horizons have been constructed [5–7]. Rotating solutions of Einstein (Gauss–Bonnet)–Born–Infeld in various dimensions with flat horizons have also been obtained [8,9]. When a dilaton field is coupled to gravity, it has profound consequences for the black hole/string solutions. Many attempts have been done to construct exact solutions of Einstein–Maxwell-dilaton (EMd) and Einstein–Born–Infeld-dilaton (EBId) gravity. While exact static dilaton black hole solutions of EMd gravity have been constructed in [10–15], exact rotating black holes solutions with curved horizons have been obtained only for some limited values of the coupling constant[16–18]. For general dilaton coupling, the properties of rotating charged dilaton black holes only with infinitesimally small charge [19] or small angular momentum have been investigated [20–22]. When the horizons are flat, rotating solutions of EMd gravity with Liouville-type potential in four [23] and $(n+1)$-dimensions have been constructed [24]. The studies on the black hole solutions of EBId gravity in three and four dimensions have been carried out in [25] and [26–28], respectively. Thermodynamics of $(n+1)$-dimensional EBId solutions with flat [29] and curved horizons have also been explored [30]. The appearance of dilaton changes the asymptotic behavior of

* Corresponding author.
E-mail addresses: mhd@shirazu.ac.ir (M.H. Dehghani), sheykhi@mail.uk.ac.ir (A. Sheykhi), hendi@mail.yu.ac.ir (S.H. Hendi).

0370-2693/$ – see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.physletb.2007.11.015

the solutions to be neither asymptotically flat nor (anti)-de Sitter [(A)dS]. There are two motivations for exploring nonasymptotically flat nor (A)dS solutions of Einstein gravity. First, these solutions can shed some light on the possible extensions of AdS/CFT correspondence. Indeed, it has been speculated that the linear dilaton spacetimes, which arise as near-horizon limits of dilatonic black holes, might exhibit holography [31]. The second motivation comes from the fact that such solutions may be used to extend the range of validity of methods and tools originally developed for, and tested in the case of, asymptotically flat or asymptotically AdS black holes.

On the other hand, there are many papers which are dealing directly with the issue of spacetimes in the context of cosmic string theory [32]. All of these solutions are horizonless and have a conical geometry; they are everywhere flat except at the location of the line source. An extension to include the electromagnetic field has also been done [33]. Asymptotically AdS spacetimes generated by static and spinning magnetic sources in three and four-dimensional Einstein–Maxwell gravity with negative cosmological constant have been investigated in [34,35]. The generalization of these rotating solutions to higher dimensions and higher derivative gravity have also been done in [36] and [37], respectively. In the context of electromagnetic cosmic string, it was shown that there are cosmic strings, known as superconducting cosmic string, that behave as superconductors and have interesting interactions with astrophysical magnetic fields [38]. The properties of these superconducting cosmic strings have been investigated in [39]. Superconducting cosmic strings have also been studied in Brans–Dicke theory [40], and in dilaton gravity [41]. Exact magnetic rotating solutions in three dimensions have been considered in [42] while, two classes of magnetic rotating solutions in four and higher dimensional EMd gravity with Liouville-type potential have been explored in [43] and [44], respectively. These solutions are not black holes, and represent spacetimes with conic singularities. In the absence of a dilaton field, magnetic rotating solutions of $(n+1)$-dimensional EBI theory have also been constructed [45].

Our aim in this Letter is to construct $(n+1)$-dimensional horizonless solutions of EBId gravity. The motivation for studying these kinds of solutions is that they may be interpreted as cosmic strings. Cosmic strings are topological defects that arise from the possible phase transitions in the early universe, and may play an important role in the formation of primordial structures. Besides there are two main reasons for studying higher dimensional solutions of EBId gravity. The first originates from string theory, which is a promising approach to quantum gravity. String theory predicts that spacetime has more than four-dimensions. For a while it was thought that the extra spatial dimensions would be of the order of the Planck scale, making a geometric description unreliable, but it has recently been realized that there is a way to make the extra dimensions relatively large and still be unobservable. This is if we live on a three-dimensional surface (brane) in a higher dimensional spacetime (bulk) [46,47]. In such a scenario, all gravitational objects are higher dimensional. The second reason for studying higher dimensional solutions has nothing to do with string theory. Four-dimensional solutions have a number of remarkable properties. It is natural to ask whether these properties are general features of the solutions or whether they crucially depend on the world being four-dimensional.

The outline of our Letter is as follows: In Section 2, we present the basic field equations and general formalism of calculating the conserved quantities. In Section 3, we obtain the magnetic rotating solutions of Einstein equation in the presence of dilaton and nonlinear electromagnetic fields, and explore their properties. The last section is devoted to summary and conclusions.

2. Field equations and conserved quantities

We consider the $(n+1)$-dimensional action in which gravity is coupled to dilaton and Born–Infeld fields with an action

$$I_G = -\frac{1}{16\pi}\int_{\mathcal{M}} d^{n+1}x\sqrt{-g}\left(\mathcal{R} - \frac{4}{n-1}(\nabla\Phi)^2 - V(\Phi) + L(F,\Phi)\right) - \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^n x\sqrt{-h}\Theta(h), \tag{1}$$

where $\mathcal{R}$ is the Ricci scalar curvature, Φ is the dilaton field, $V(\Phi)$ is a potential for Φ and $F^2 = F_{\mu\nu}F^{\mu\nu}$ ($F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor and A_μ is the electromagnetic potential). The last term in Eq. (1) is the Gibbons–Hawking boundary term which is chosen such that the variational principle is well-defined. The manifold $\mathcal{M}$ has metric $g_{\mu\nu}$ and covariant derivative ∇_μ. Θ is the trace of the extrinsic curvature Θ^{ab} of any boundary(ies) $\partial\mathcal{M}$ of the manifold $\mathcal{M}$, with induced metric(s) h_{ab}. In this Letter, we consider the action (1) with a Liouville type potential,

$$V(\Phi) = 2\Lambda e^{4\alpha\Phi/(n-1)}, \tag{2}$$

where Λ is a constant which may be referred to as the cosmological constant, since in the absence of the dilaton field ($\Phi = 0$) the action (1) reduces to the action of Einstein–Born–Infeld gravity with cosmological constant [6,7]. The Born–Infeld, $L(F,\Phi)$, part of the action is given by

$$L(F,\Phi) = 4\beta^2 e^{4\alpha\Phi/(n-1)}\left(1 - \sqrt{1 + \frac{e^{-8\alpha\Phi/(n-1)}F^2}{2\beta^2}}\right). \tag{3}$$

Here, α is a constant determining the strength of coupling of the scalar and electromagnetic field and β is called the Born–Infeld parameter with dimension of mass. In the limit $\beta \to \infty$, $L(F,\Phi)$ reduces to the standard Maxwell field coupled to a dilaton field

$$L(F,\Phi) = -e^{-4\alpha\Phi/(n-1)}F^2, \tag{4}$$

and $L(F,\Phi) \to 0$ as $\beta \to 0$. It is convenient to set

$$L(F,\Phi) = 4\beta^2 e^{4\alpha\Phi/(n-1)}\mathcal{L}(Y), \tag{5}$$

where

$$\mathcal{L}(Y) = 1 - \sqrt{1+Y}, \tag{6}$$

$$Y = \frac{e^{-8\alpha\Phi/(n-1)}F^2}{2\beta^2}. \tag{7}$$

The equations of motion can be obtained by varying the action (1) with respect to the gravitational field $g_{\mu\nu}$, the dilaton field Φ and the gauge field A_μ which yields the following field equations

$$\begin{aligned}\mathcal{R}_{\mu\nu} = {} & \frac{4}{n-1}\left(\partial_\mu\Phi\partial_\nu\Phi + \frac{1}{4}g_{\mu\nu}V(\Phi)\right) \\ & - 4e^{-4\alpha\Phi/(n-1)}\partial_Y\mathcal{L}(Y)F_{\mu\eta}F_\nu^\eta \\ & + \frac{4\beta^2}{n-1}e^{4\alpha\Phi/(n-1)}\big[2Y\partial_Y\mathcal{L}(Y) - \mathcal{L}(Y)\big]g_{\mu\nu},\end{aligned} \tag{8}$$

$$\begin{aligned}\nabla^2\Phi = {} & \frac{n-1}{8}\frac{\partial V}{\partial\Phi} \\ & + 2\alpha\beta^2 e^{4\alpha\Phi/(n-1)}\big[2\,Y\partial_Y\mathcal{L}(Y) - \mathcal{L}(Y)\big],\end{aligned} \tag{9}$$

$$\nabla_\mu\big(e^{-4\alpha\Phi/(n-1)}\partial_Y\mathcal{L}(Y)F^{\mu\nu}\big) = 0. \tag{10}$$

In particular, in the case of the linear electrodynamics with $\mathcal{L}(Y) = -\frac{1}{2}Y$, the system of Eqs. (8)–(10) reduce to the well-known equations of EMd gravity [14].

The conserved mass and angular momentum of the solutions of the above field equations can be calculated through the use of the substraction method of Brown and York [48]. Such a procedure causes the resulting physical quantities to depend on the choice of reference background. For asymptotically (A)dS solutions, the way that one deals with these divergences is through the use of counterterm method inspired by (A)dS/CFT correspondence [49]. However, in the presence of a non-trivial dilaton field, the spacetime may not behave as either dS ($\Lambda > 0$) or AdS ($\Lambda < 0$). In fact, it has been shown that with the exception of a pure cosmological constant potential where $\alpha = 0$, no AdS or dS static spherically symmetric solution exist for Liouville-type potential [13]. But, as in the case of asymptotically AdS spacetimes, according to the domain-wall/QFT (quantum field theory) correspondence [50], there may be a suitable counterterm for the stress energy tensor which removes the divergences. In this paper, we deal with the spacetimes with zero curvature boundary [$R_{abcd}(h) = 0$], and therefore the counterterm for the stress energy tensor should be proportional to h^{ab}. Thus, the finite stress-energy tensor in $(n+1)$-dimensional Einstein-dilaton gravity with Liouville-type potential may be written as

$$T^{ab} = \frac{1}{8\pi}\left[\Theta^{ab} - \Theta h^{ab} + \frac{n-1}{l_{\mathrm{eff}}}h^{ab}\right], \tag{11}$$

where l_{eff} is given by

$$l_{\mathrm{eff}}^2 = \frac{(n-1)(\alpha^2 - n)}{2\Lambda}e^{-4\alpha\Phi/(n-1)}. \tag{12}$$

In the particular case $\alpha = 0$, the effective l_{eff}^2 of Eq. (12) reduces to $l^2 = -n(n-1)/2\Lambda$ of the AdS spacetimes. The first two terms in Eq. (11) is the variation of the action (1) with respect to h_{ab}, and the last term is the counterterm which removes the divergences. One may note that the counterterm has the same form as in the case of asymptotically AdS solutions with zero curvature boundary, where l is replaced by l_{eff}. To compute the conserved charges of the spacetime, one should choose a spacelike surface $\mathcal{B}$ in $\partial\mathcal{M}$ with metric σ_{ij}, and write the boundary metric in ADM (Arnowitt–Deser–Misner) form:

$$\begin{aligned}h_{ab}\,dx^a\,dx^a = {} & -N^2\,dt^2 \\ & + \sigma_{ij}\big(d\varphi^i + V^i\,dt\big)\big(d\varphi^j + V^j\,dt\big),\end{aligned} \tag{13}$$

where the coordinates φ^i are the angular variables parameterizing the hypersurface of constant r around the origin, and N and V^i are the lapse and shift functions, respectively. When there is a Killing vector field ξ on the boundary, then the quasilocal conserved quantities associated with the stress tensors of Eq. (11) can be written as

$$Q(\xi) = \int_{\mathcal{B}} d^{n-1}x\,\sqrt{\sigma}T_{ab}n^a\xi^b, \tag{14}$$

where σ is the determinant of the metric σ_{ij}, ξ and n^a are the Killing vector field and the unit normal vector on the boundary $\mathcal{B}$. For boundaries with timelike ($\xi = \partial/\partial t$) and rotational Killing vector field ($\varsigma_i = \partial/\partial\phi^i$), one obtains the quasilocal mass and components of the total angular momentum as

$$M = \int_{\mathcal{B}} d^{n-1}x\,\sqrt{\sigma}T_{ab}n^a\xi^b, \tag{15}$$

$$J_i = \int_{\mathcal{B}} d^{n-1}x\,\sqrt{\sigma}T_{ab}n^a\varsigma_i^b. \tag{16}$$

Note that these quantities depend on the location of the boundary $\mathcal{B}$ in the spacetime, although each is independent of the particular choice of foliation $\mathcal{B}$ within the surface $\partial\mathcal{M}$.

3. Magnetic rotating solutions

In this section we are going to obtain rotating horizonless solutions of the field equations (8)–(10). First, we construct the rotating 4-dimensional spacetimes generated by a magnetic source which produces a longitudinal magnetic field. Second, we generalize these 4-dimensional solutions to the case of $(n+1)$-dimensional solutions.

3.1. Longitudinal magnetic field solutions

Here we want to obtain the 4-dimensional solution of Eqs. (8)–(10) which produces a longitudinal magnetic fields along the z direction. We assume the following form for the metric

$$\begin{aligned}ds^2 = {} & -\frac{\rho^2}{l^2}R^2(\rho)(\Xi\,dt - a\,d\phi)^2 + f(\rho)\left(\frac{a}{l}\,dt - \Xi l\,d\phi\right)^2 \\ & + \frac{d\rho^2}{f(\rho)} + \frac{\rho^2}{l^2}R^2(\rho)\,dz^2,\end{aligned} \tag{17}$$

where a is the rotation parameter and $\Xi = \sqrt{1 + a^2/l^2}$. The functions $f(\rho)$ and $R(\rho)$ should be determined and l has the dimension of length which is related to the cosmological constant Λ for the case of Liouville-type potential with constant Φ.

The angular coordinate ϕ is dimensionless as usual and ranges in $[0, 2\pi]$, while ρ and z have dimension of length.

The electromagnetic field equation (10) can be integrated immediately to give

$$F_{\phi\rho} = \frac{q\Xi l e^{2\alpha\Phi}}{(\rho R)^2\sqrt{1-\frac{q^2}{\beta^2(\rho R)^4}}}, \qquad F_{t\rho} = -\frac{a}{\Xi l^2}F_{\phi\rho}, \tag{18}$$

where q is the charge parameter of the string. To solve the system of Eqs. (8) and (9) for three unknown functions $f(\rho)$, $R(\rho)$ and $\Phi(\rho)$, we make the ansatz

$$R(\rho) = e^{\alpha\Phi}. \tag{19}$$

Using (19), the electromagnetic fields (18) and the metric (17), one can show that Eqs. (8) and (9) have solutions of the form

$$f(\rho) = \frac{(\Lambda - 2\beta^2)(\alpha^2+1)^2 b^{2\gamma}}{\alpha^2-3}\rho^{2(1-\gamma)} + \frac{m}{\rho^{1-2\gamma}} \tag{20}$$

$$+ 2\beta^2(\alpha^2+1) b^{2\gamma}\rho^{2\gamma-1} \times \int \rho^{2(1-2\gamma)}\sqrt{(1-\zeta)}\,d\rho, \tag{21}$$

$$\Phi(\rho) = \frac{\alpha}{1+\alpha^2}\ln\left(\frac{b}{\rho}\right), \tag{22}$$

where $\gamma = \alpha^2/(1+\alpha^2)$ and

$$\zeta \equiv \frac{q^2}{\beta^2 b^{4\gamma}\rho^{4(1-\gamma)}}. \tag{23}$$

Eq. (18) shows that ρ should be greater than $\rho_0 = (q/\beta b^{2\gamma})^{1/(2-2\gamma)}$ in order to have a real nonlinear electromagnetic field and consequently a real spacetime. Indeed, as we will see below, we may remove the region $\rho < \rho_0$ by a transformation. The integral can be done in terms of hypergeometric function and can be written in a compact form. The result is

$$f(\rho) = \frac{(\alpha^2+1)^2 b^{2\gamma}\rho^{2(1-\gamma)}}{\alpha^2-3}\left[\Lambda + 2\beta^2\left(1 - {}_2F_1\left(\left[-\frac{1}{2}, \frac{\alpha^2-3}{4}\right], \left[\frac{\alpha^2+1}{4}\right], \zeta\right)\right)\right] + \frac{m}{\rho^{1-2\gamma}}, \tag{24}$$

where b and m are arbitrary constants. One may note that as $\beta \to \infty$ this solution reduces to the 4-dimensional magnetic strings given in Ref. [43]. In the absence of dilaton field ($\alpha = \gamma = 0$), the above solutions reduce to the 4-dimensional horizonless rotating solutions of Einstein–Born–Infeld gravity presented in [45]. One can easily show that the gauge potential A_μ corresponding to the electromagnetic tensor (18) can be written as

$$A_\mu = \frac{q}{\rho} \times {}_2F_1\left(\left[\frac{1}{2}, \frac{\alpha^2+1}{4}\right], \left[\frac{\alpha^2+5}{4}\right], \zeta\right) \times \left(\frac{a}{l}\delta^t_\mu - \Xi l\delta^\phi_\mu\right). \tag{25}$$

Now we study the properties of these solutions. To do this, we first look for the curvature singularities in the presence of dilaton field. It is easy to show that the Kretschmann invariant $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverges at $\rho = \rho_0$, it is finite for $\rho > \rho_0$ and goes to zero as $\rho \to \infty$. Therefore one might think that there is a curvature singularity located at $\rho = \rho_0$. Two cases happen. In the first case the function $f(\rho)$ has one or more real root(s) larger than ρ_0. In this case the function $f(\rho)$ is negative for $\rho < r_+$, and positive for $\rho > r_+$ where r_+ is the largest real root of $f(\rho) = 0$. Indeed, $g_{\rho\rho}$ and $g_{\phi\phi}$ are related by $f(\rho) = g_{\rho\rho}^{-1} = l^{-2}g_{\phi\phi}$, and therefore when $g_{\rho\rho}$ becomes negative (which occurs for $\rho_0 < \rho < r_+$) so does $g_{\phi\phi}$. This leads to an apparent change of signature of the metric from $+2$ to $+1$, and therefore indicates that we are using an incorrect extension. To get rid of this incorrect extension, we introduce the new radial coordinate r as

$$r^2 = \rho^2 - r_+^2 \Rightarrow d\rho^2 = \frac{r^2}{r^2+r_+^2}dr^2. \tag{26}$$

With this new coordinate, the metric (17) becomes

$$ds^2 = -\frac{b^{2\gamma}(r^2+r_+^2)^{(1-\gamma)}}{l^2}(\Xi\,dt - a\,d\phi)^2 + f(r)\left(\frac{a}{l}dt - \Xi l\,d\phi\right)^2 + \frac{r^2}{(r^2+r_+^2)f(r)}dr^2 + \frac{b^{2\gamma}(r^2+r_+^2)^{(1-\gamma)}}{l^2}dz^2, \tag{27}$$

where the coordinates r and z assume the values $0 \leqslant r < \infty$ and $-\infty \leqslant z < \infty$. The function $f(r)$ is now given as

$$f(r) = \frac{(\alpha^2+1)^2 b^{2\gamma}}{\alpha^2-3}(r^2+r_+^2)^{(1-\gamma)}\left[\Lambda + 2\beta^2\left(1 - {}_2F_1\left(\left[-\frac{1}{2}, \frac{\alpha^2-3}{4}\right], \left[\frac{\alpha^2+1}{4}\right], \eta\right)\right)\right] + \frac{m}{(r^2+r_+^2)^{(1-2\gamma)/2}}, \tag{28}$$

where

$$\eta \equiv \frac{q^2}{\beta^2 b^{4\gamma}(r^2+r_+^2)^{2(1-\gamma)}}. \tag{29}$$

The gauge potential in the new coordinate is

$$A_\mu = \frac{q}{(r^2+r_+^2)^{1/2}} \times {}_2F_1\left(\left[\frac{1}{2}, \frac{\alpha^2+1}{4}\right], \left[\frac{\alpha^2+5}{4}\right], \eta\right) \times \left(\frac{a}{l}\delta^t_\mu - \Xi l\delta^\phi_\mu\right). \tag{30}$$

One can easily show that the Kretschmann scalar does not diverge in the range $0 \leqslant r < \infty$. However, the spacetime has a conic geometry and has a conical singularity at $r = 0$, since:

$$\lim_{r\to 0}\frac{1}{r}\sqrt{\frac{g_{\phi\phi}}{g_{rr}}} \neq 1. \tag{31}$$

That is, as the radius r tends to zero, the limit of the ratio "circumference/radius" is not 2π and therefore the spacetime has a conical singularity at $r = 0$. The canonical singularity can be removed if one identifies the coordinate ϕ with the period

$$\text{Period}_\phi = 2\pi\left(\lim_{r\to 0}\frac{1}{r}\sqrt{\frac{g_{\phi\phi}}{g_{rr}}}\right)^{-1} = 2\pi(1-4\mu), \tag{32}$$

where μ is given by

$$\mu \equiv \frac{1}{4}\Bigg[1-\Bigg(-\frac{(2\beta^2+\Lambda)lb^{2\gamma}(\alpha^2+1)}{2}{r_+}^{1-2\gamma} + l\beta^2 b^{2\gamma}(\alpha^2+1){r_+}^{1-2\gamma} \times {}_2F_1\left(\left[-\frac{1}{2},\frac{\alpha^2-3}{4}\right],\left[\frac{\alpha^2+1}{4}\right],\eta_{+0}\right) - \frac{2lq^2b^{-2\gamma}(\alpha^2+1)}{(\alpha^2+1){r_+}^{(3-2\gamma)}} \times {}_2F_1\left(\left[\frac{1}{2},\frac{\alpha^2+1}{4}\right],\left[\frac{\alpha^2+5}{4}\right],\eta_{+0}\right)\Bigg)^{-1}\Bigg]. \tag{33}$$

Here we have defined $\eta_{+0} = \eta(r=0)$ and we have expressed the mass parameter m in terms of q and Λ by using equation $f(r=0)=0$. From Eqs. (31)–(33), one concludes that near the origin $r=0$, the metric (27) describes a spacetime which is locally flat but has a conical singularity at $r=0$ with a deficit angle $\delta\phi = 8\pi\mu$. Since near the origin the metric (27) is identical to the spacetime generated by a cosmic string, by use of Vilenkin procedure [52] μ of Eq. (33) can be interpreted as the mass per unit length of the string. In the second case, the function $f(\rho)$ is positive for $\rho > \rho_0$, and the transformation $r^2 = \rho^2 - \rho_0^2$ removes the imaginary part of the metric. In this case, we have a spacetime with naked singularity which we are not interested in it. In the rest of the Letter we consider only the first case.

Next we investigate the casual structure of the spacetime given in Eq. (27). The metric (27) and the other metric that we will present in this Letter are neither asymptotically flat nor (A)dS. As one can see from Eq. (28), there is no solution for $\alpha = \sqrt{3}$. The cases with $\alpha > \sqrt{3}$ and $\alpha < \sqrt{3}$ should be considered separately. For $\alpha > \sqrt{3}$, as r goes to infinity the dominant term in Eq. (28) is the last term, and therefore the function $f(r)$ is positive in the whole spacetime, despite the sign of the cosmological constant Λ, and is zero at $r=0$. Thus, the solution given by Eqs. (27)–(28) exhibits a spacetime with conic singularity at $r=0$. For $\alpha < \sqrt{3}$, the dominant term for large values of r is the first term, and therefore the function $f(r)$ given in Eq. (28) is positive in the whole spacetime only for negative values of Λ. In this case the solution describes a spacetime with conic singularity at $r=0$. The solution is not acceptable for $\alpha < \sqrt{3}$ with positive values of Λ, since the function $f(r)$ is negative for large values of r.

Of course, one may ask for the completeness of the spacetime with $r \geqslant 0$ (or $\rho \geqslant r_+$) [35,51]. It is easy to see that the spacetime described by metric (27) is both null and timelike geodesically complete. In fact, we can show that every null or timelike geodesic starting from an arbitrary point can either extend to infinite values of the affine parameter along the geodesic or end on a singularity at $r=0$. To do this, first, we perform the rotation boost

$$(\Xi t - a\phi) \mapsto t, \qquad (at - \Xi l^2\phi) \mapsto l^2\phi, \tag{34}$$

in the t–ϕ plane. Then the metric (27) becomes

$$ds^2 = -\frac{b^{2\gamma}(r^2+r_+^2)^{(1-\gamma)}}{l^2}dt^2 + \frac{r^2}{(r^2+r_+^2)f(r)}dr^2 \tag{35}$$

$$+ l^2 f(r)\,d\phi^2 + \frac{b^{2\gamma}(r^2+r_+^2)^{(1-\gamma)}}{l^2}dz^2. \tag{36}$$

Using the geodesic equation, one obtains

$$\dot{t} = \frac{l^2}{b^{2\gamma}(r^2+r_+^2)^{1-\gamma}}E, \qquad \dot{\phi} = \frac{1}{l^2 f(r)}L,$$
$$\dot{z} = \frac{l^2}{b^{2\gamma}(r^2+r_+^2)^{1-\gamma}}P, \tag{37}$$

$$r^2\dot{r}^2 = (r^2+r_+^2)f(r)\left[\frac{l^2(E^2-P^2)}{b^{2\gamma}(r^2+r_+^2)^{1-\gamma}} - \xi\right] - \frac{r^2+r_+^2}{l^2}L^2, \tag{38}$$

where the dot denotes the derivative with respect to an affine parameter and ξ is zero for null geodesics and $+1$ for timelike geodesics. E, L and P are the conserved quantities associated with the coordinates t, ϕ and z respectively. Notice that $f(r)$ is always positive for $r > 0$ and zero for $r = 0$. First we consider the null geodesics ($\xi = 0$). (i) If $E^2 > P^2$, the spiraling particles ($L > 0$) coming from infinity have a turning point at $r_{tp} > 0$, while the nonspiraling particles ($L = 0$) have a turning point at $r_{tp} = 0$. (ii) If $E^2 = P^2$ and $L = 0$, then the velocities $\dot{r}$ and $\dot{\phi}$ vanish for any value of r, and therefore the null particles moves on the z-axis. (iii) For $E^2 = P^2$ and $L \neq 0$, and also for $E^2 < P^2$ and any values of L, there is no possible null geodesic. Second, we analyze the timelike geodesics ($\xi = +1$). Timelike geodesics are possible only if $l^2(E^2-P^2) > b^{2\gamma}r_+^{2(1-\gamma)}$. In this case the turning points for the nonspiraling particles ($L = 0$) are $r^1_{tp} = 0$ and r^2_{tp} given as

$$r^2_{tp} = \sqrt{\left[b^{-2\gamma}l^2(E^2-P^2)\right]^{1/(1-\gamma)} - r_+^2}, \tag{39}$$

while the spiraling ($L \neq 0$) timelike particles are bound between r^a_{tp} and r^b_{tp} given by

$$0 < r^a_{tp} \leqslant r^b_{tp} < r^2_{tp}. \tag{40}$$

Therefore, we have confirmed that the spacetime described by Eq. (27) is both null and timelike geodesically complete.

Finally, we calculate the conserved quantities of these solutions. The mass and angular momentum per unit length of the strings ($\alpha < \sqrt{3}$) can be calculated through the use of Eqs. (15) and (16). We find

$$M = \frac{b^{2\gamma}}{4}\left(\frac{(3-\alpha^2)\Xi^2 - 2}{1+\alpha^2}\right)m, \tag{41}$$

$$J = \frac{b^{2\gamma}}{4}\left(\frac{3-\alpha^2}{1+\alpha^2}\right)\Xi ma. \tag{42}$$

For $a = 0$ ($\Xi = 1$), the angular momentum per unit volume vanishes, and therefore a is the rotational parameter of the spacetime. Of course, one may note that these conserved charges are similar to the conserved charges of the magnetic rotating dilaton string obtained in Ref. [43]. In the absence of dilaton field ($\alpha = \gamma = 0$) they reduce to that obtained in [45].

3.2. $(n+1)$-dimensional rotating solutions with all rotation parameters

Our aim here is to construct the $(n+1)$-dimensional longitudinal magnetic field solutions with a complete set of rotation parameters. The rotation group in $n+1$ dimensions is $SO(n)$ and therefore the number of independent rotation parameters is $[n/2]$, where $[x]$ denotes the integer part of x. We now generalize the above solution given in Eq. (27) with $k \leqslant [n/2]$ rotation parameters. This generalized solution can be written as

$$\begin{aligned} ds^2 = &-\frac{b^{2\gamma}(r^2+r_+^2)^{(1-\gamma)}}{l^2}\left(\Xi\, dt - \sum_{i=1}^{k} a_i\, d\phi^i\right)^2 \\ &+ f(r)\left(\sqrt{\Xi^2-1}\, dt - \frac{\Xi}{\sqrt{\Xi^2-1}}\sum_{i=1}^{k} a_i\, d\phi^i\right)^2 \\ &+ b^{2\gamma}\left(r^2+r_+^2\right)^{(1-\gamma)}\sum_{i=1}^{n-k-2}\left(d\psi^i\right)^2 \\ &+ \frac{b^{2\gamma}(r^2+r_+^2)^{(1-\gamma)}}{l^2(\Xi^2-1)}\sum_{i<j}^{k}(a_i\, d\phi_j - a_j\, d\phi_i)^2 \\ &+ \frac{r^2\, dr^2}{(r^2+r_+^2)f(r)} + \frac{b^{2\gamma}(r^2+r_+^2)^{(1-\gamma)}}{l^2}dz^2, \end{aligned} \tag{43}$$

where $\Xi = \sqrt{1+\sum_i^k a_i^2/l^2}$ and $f(r)$ is given as

$$\begin{aligned} f(r) = &\frac{2(\alpha^2+1)^2 b^{2\gamma}(r^2+r_+^2)^{(1-\gamma)}}{(n-1)(\alpha^2-n)}\left[\Lambda + 2\beta^2\left(1 \right.\right. \\ &\left.\left. - {}_2F_1\left(\left[-\frac{1}{2}, \frac{\alpha^2-n}{2n-2}\right], \left[\frac{\alpha^2+n-2}{2n-2}\right], \eta\right)\right)\right] \\ &+ \frac{m}{(r^2+r_+^2)^{[(n-1)(1-\gamma)-1]/2}}. \end{aligned} \tag{44}$$

The gauge potential is

$$\begin{aligned} A_\mu = &\frac{q b^{(3-n)\gamma}}{\lambda(r^2+r_+^2)^{\lambda/2}}\, {}_2F_1\left(\left[\frac{1}{2}, \frac{n+\alpha^2-2}{2(n-1)}\right], \left[\frac{3n+\alpha^2-4}{2(n-1)}\right], \eta\right) \\ &\times\left(\sqrt{\Xi^2-1}\,\delta_\mu^t - \frac{\Xi}{\sqrt{\Xi^2-1}}a_i\delta_\mu^i\right) \\ &\text{(no sum on } i), \end{aligned} \tag{45}$$

where $\lambda = (n-3)(1-\gamma)+1$. Again this spacetime has no horizon and curvature singularity. However, it has a conical singularity at $r=0$. One should note that these solutions reduce to those discussed in [45], in the absence of dilaton field ($\alpha = \gamma = 0$) and those presented in [44] as $\beta \to \infty$.

Next we calculate the conserved quantities of the $(n+1)$-dimensional solutions. The mass and angular momentum per unit length of the strings ($\alpha < \sqrt{n}$) can be calculated through the use of Eqs. (15) and (16). We find

$$M = \frac{(2\pi)^{n-3} b^{(n-1)\gamma}}{4}\left(\frac{(n-\alpha^2)\Xi^2-(n-1)}{1+\alpha^2}\right)m, \tag{46}$$

$$J_i = \frac{(2\pi)^{n-3} b^{(n-1)\gamma}}{4}\left(\frac{n-\alpha^2}{1+\alpha^2}\right)\Xi m a_i. \tag{47}$$

For $a_i = 0$ ($\Xi = 1$), the angular momentum per unit volume vanishes, and therefore a_i's are the rotational parameters of the spacetime. One may note that these conserved quantities are similar to the conserved quantities of the $(n+1)$-dimensional magnetic rotating dilaton string obtained in Ref. [44].

Finally, we calculate the electric charge of the solutions (27) and (43) obtained in this section. To determine the electric field we should consider the projections of the electromagnetic field tensor on special hypersurfaces. The normal to such hypersurfaces is

$$u^0 = \frac{1}{N}, \qquad u^r = 0, \qquad u^i = -\frac{V^i}{N}, \tag{48}$$

and the electric field is $E^\mu = g^{\mu\rho}e^{-4\alpha\phi/(n-1)}F_{\rho\nu}u^\nu$. Then the electric charge per unit length can be found by calculating the flux of the electric field at infinity, yielding

$$Q = \frac{(2\pi)^{n-3}}{4}\sqrt{\Xi^2-1}\,q. \tag{49}$$

It is worth noticing that the electric charge of the system per unit volume is proportional to the rotation parameter, and is zero for the case of a static solution. Again, in the absence of a nontrivial dilaton ($\alpha = \gamma = 0$), these conserved charges reduce to the conserved charges of $(n+1)$-dimensional horizonless rotating solutions of Einstein–Born–Infeld gravity presented in [45].

4. Summary and conclusions

To sum up, the Einstein–Born–Infeld action including a dilaton field, appears in the couplings of an open superstring and an Abelian gauge field. This action can be considered as a nonlinear extension of the Abelian field of Einstein–Maxwell-dilaton theory. It is worth finding exact solutions of Einstein–Born–Infeld-dilaton gravity for an arbitrary value of the dilaton coupling constant, and investigate how the properties of the solutions are modified when a dilaton is present. In this Letter, we constructed a new analytic solution of the 4-dimensional Einstein–Born–Infeld-dilaton theory in the presence of Liouville-type potential for the dilaton field. These solutions describe 4-dimensional rotating strings with a Longitudinal magnetic field and they have conic singularity at $r=0$. Besides, they are horizonless without curvature singularity. Because of the presence of the dilaton field, these solutions are neither asymptotically flat nor (A)dS. In the presence of Liouville-type potential, we obtained exact solutions provided $\alpha \neq \sqrt{3}$. In the absence of a dilaton field ($\alpha = \gamma = 0$), these solutions reduce to the 4-dimensional horizonless rotating solutions of Einstein–Born–Infeld gravity presented in [45], while in the case $\beta \to \infty$ they reduce to the 4-dimensional magnetic rotating dilaton string given in Ref. [43]. We confirmed that these solutions are both null and timelike geodesically complete by showing that in this spacetime every null or timelike geodesic starting from an arbitrary point can either extend to infinite values of the affine parameter along the geodesic or end on a singularity at $r = 0$. For the rotating string, when the rotation parameter is nonzero, the string has a net electric charge density which is proportional to the magnitude of the rotation parameter. We also generalized these four-dimensional solutions to the case of $(n + 1)$-dimensional magnetic rotating solutions with $k \leqslant [n/2]$ rotation parameters. Finally, we calculated the conserved quantities of these solutions by using the counterterm method inspired by the AdS/CFT correspondence.

Acknowledgements

This work was partially supported by Research Institute for Astrophysics and Astronomy of Maragha, Iran, and also by Shahid Bahonar University of Kerman.

References

[1] M. Born, L. Infeld, Proc. R. Soc. A 144 (1934) 425.
[2] R.G. Leigh, Mod. Phys. Lett. A 4 (1989) 2767.
[3] H. Salazar, A. García, J. Plebański, J. Math. Phys. (N.Y.) 28 (1987) 2171; E. Ayón-Beato, A. García, Phys. Rev. Lett. 80 (1998) 5056; E. Ayón-Beato, A. García, Gen. Relativ. Gravit. 31 (1999) 629; E. Ayón-Beato, A. García, Phys. Lett. B 464 (1999) 25.
[4] A. Borde, Phys. Rev. D 55 (1997) 7615.
[5] D.L. Wiltshire, Phys. Rev. D 38 (1988) 2445; D.A. Rasheed, hep-th/9702087; N. Breton, Phys. Rev. D 67 (2003) 124004; M. Aiello, R. Ferraro, G. Giribet, gr-qc/0408078; M. Cataldo, A. García, Phys. Lett. B 456 (1999) 28; S. Fernando, D. Krug, Gen. Relativ. Gravit. 35 (2003) 129; H. d Oliveira, Class. Quantum Grav. 11 (1994) 1469.
[6] T.K. Dey, Phys. Lett. B 595 (2004) 484.
[7] R.-G. Cai, D.-W. Pang, A. Wang, Phys. Rev. D 70 (2004) 124034.
[8] M.H. Dehghani, H. Rastegar Sedehi, Phys. Rev. D 74 (2006) 124018.
[9] M.H. Dehghani, S.H. Hendi, in preparation.
[10] G.W. Gibbons, K. Maeda, Nucl. Phys. B 298 (1988) 741; T. Koikawa, M. Yoshimura, Phys. Lett. B 189 (1987) 29; D. Brill, J. Horowitz, Phys. Lett. B 262 (1991) 437.
[11] D. Garfinkle, G.T. Horowitz, A. Strominger, Phys. Rev. D 43 (1991) 3140; R. Gregory, J.A. Harvey, Phys. Rev. D 47 (1993) 2411; M. Rakhmanov, Phys. Rev. D 50 (1994) 5155.
[12] S. Mignemi, D. Wiltshire, Class. Quantum Grav. 6 (1989) 987; D. Wiltshire, Phys. Rev. D 44 (1991) 1100; S. Mignemi, D. Wiltshire, Phys. Rev. D 46 (1992) 1475.
[13] S. Poletti, D. Wiltshire, Phys. Rev. D 50 (1994) 7260; S. Poletti, D. Wiltshire, Phys. Rev. D 52 (1995) 3753.
[14] K.C.K. Chan, J.H. Horne, R.B. Mann, Nucl. Phys. B 447 (1995) 441.
[15] R.G. Cai, J.Y. Ji, K.S. Soh, Phys. Rev. D 57 (1998) 6547; R.G. Cai, Y.Z. Zhang, Phys. Rev. D 64 (2001) 104015.
[16] V.P. Frolov, A.I. Zelnikov, U. Bleyer, Ann. Phys. 44 (1987) 371; V. Belinsky, R. Ruffini, Phys. Lett. B 89 (1980) 195.
[17] H. Kunduri, J. Lucietti, Phys. Lett. B 609 (2005) 143; S.S. Yazadjiev, Phys. Rev. D 72 (2005) 104014.
[18] J. Kunz, D. Maison, F.N. Lerida, J. Viebahn, hep-th/0606005.
[19] R. Casadio, B. Harms, Y. Leblanc, P.H. Cox, Phys. Rev. D 55 (1997) 814.
[20] J.H. Horne, G.T. Horowitz, Phys. Rev. D 46 (1992) 1340; K. Shiraishi, Phys. Lett. A 166 (1992) 298; T. Ghosh, P. Mitra, Class. Quantum Grav. 20 (2003) 1403.
[21] A. Sheykhi, N. Riazi, Int. J. Theor. Phys. 45 (2006) 2453.
[22] A. Sheykhi, N. Riazi, Int. J. Mod. Phys. A, in press.
[23] M.H. Dehghani, N. Farhangkhah, Phys. Rev. D 71 (2005) 044008.
[24] A. Sheykhi, M.H. Dehghani, N. Riazi, J. Pakravan, Phys. Rev. D 74 (2006) 084016.
[25] R. Yamazaki, D. Ida, Phys. Rev. D 64 (2001) 024009.
[26] S.S. Yazadjiev, Phys. Rev. D 72 (2005) 044006.
[27] A. Sheykhi, N. Riazi, M.H. Mahzoon, Phys. Rev. D 74 (2006) 044025.
[28] T. Tamaki, T. Torii, Phys. Rev. D 62 (2000) 061501; G. Celement, D. Gal'tsov, Phys. Rev. D 62 (2000) 124013; S.S. Yazadjiev, P.P. Fiziev, T.L. Boyadjiev, M.D. Todorov, Mod. Phys. Lett. A 16 (2001) 2143.
[29] M. H Dehghani, S.H. Hendi, A. Sheykhi, H. Rastegar Sedehi, JCAP 0702 (2007) 020.
[30] A. Sheykhi, N. Riazi, Phys. Rev. D 75 (2007) 024021.
[31] O. Aharony, M. Berkooz, D. Kutasov, N. Seiberg, J. High Energy Phys. 10 (1998) 004.
[32] A. Vilenkin, Phys. Rev. D 23 (1981) 852; W.A. Hiscock, Phys. Rev. D 31 (1985) 3288; D. Harari, P. Sikivie, Phys. Rev. D 37 (1988) 3438; J.R. Gott, Astrophys. J. 288 (1985) 422; A.D. Cohen, D.B. Kaplan, Phys. Lett. B 215 (1988) 65; R. Gregory, Phys. Lett. B 215 (1988) 663; A. Banerjee, N. Banerjee, A.A. Sen, Phys. Rev. D 53 (1996) 5508; M.H. Dehghani, T. Jalali, Phys. Rev. D 66 (2002) 124014; M.H. Dehghani, A. Khodam-Mohammadi, Can. J. Phys. 83 (2005) 229.
[33] B.C. Mukherji, Bull. Calcutta Math. Soc. 30 (1938) 95; W.B. Bonnor, Proc. R. Soc. London A 67 (1954) 225; J.L. Safko, Phys. Rev. D 16 (1977) 1678.
[34] O.J.C. Dias, J.P.S. Lemos, J. High Energy Phys. 01 (2002) 006.
[35] O.J.C. Dias, J.P.S. Lemos, Class. Quantum Grav. 19 (2002) 2265.
[36] M.H. Dehghani, Phys. Rev. D 69 (2004) 044024; M.H. Dehghani, Phys. Rev. D 70 (2004) 064019.
[37] M.H. Dehghani, Phys. Rev. D 69 (2004) 064024.
[38] E. Witten, Nucl. Phys. B 249 (1985) 557; P. Peter, Phys. Rev. D 49 (1994) 5052.
[39] I. Moss, S. Poletti, Phys. Lett. B 199 (1987) 34.
[40] A.A. Sen, Phys. Rev. D 60 (1999) 067501.
[41] C.N. Ferreira, M.E.X. Guimaraes, J.A. Helayel-Neto, Nucl. Phys. B 581 (2000) 165.
[42] O.J.C. Dias, J.P.S. Lemos, Phys. Rev. D 66 (2002) 024034.
[43] M.H. Dehghani, Phys. Rev. D 71 (2005) 064010.
[44] A. Sheykhi, M.H. Dehghani, N. Riazi, Phys. Rev. D 75 (2007) 044020.
[45] H. Safarzadeh, MS thesis, Magnetic Strings in Einstein–Born–Infeld Gravity, Shiraz University, 2007.
[46] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370; L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690.
[47] G. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. B 485 (2000) 208; G. Dvali, G. Gabadadze, Phys. Rev. D 63 (2001) 065007.
[48] J.D. Brown, J.W. York, Phys. Rev. D 47 (1993) 1407.
[49] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231; E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253; O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri, Y. Oz, Phys. Rep. 323 (2000) 183; V. Balasubramanian, P. Kraus, Commun. Math. Phys. 208 (1999) 413.
[50] H.J. Boonstra, K. Skenderis, P.K. Townsend, J. High Energy Phys. 01 (1999) 003; K. Behrndt, E. Bergshoeff, R. Hallbersma, J.P. Van der Scharr, Class. Quantum Grav. 16 (1999) 3517; R.G. Cai, N. Ohta, Phys. Rev. D 62 (2000) 024006.
[51] J.H. Horne, G.T. Horowitz, Nucl. Phys. B 368 (1992) 444.
[52] A. Vilenkin, Phys. Rep. 121 (1985) 263.