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Probabilistic analysis of the retention time in stormwater detention facilities

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Abstract

Stormwater detention facilities are often used in modern drainage systems to reduce the hydraulic load on existing sewers, due to the increase of impermeable surfaces and to the more frequent extreme rainfalls, consequence of climate changes. Although their design is mainly aimed to limit uncontrolled spills into receiving water bodies, storage capacity for water quality enhancement is often considered, mainly with the purpose of increasing the retention time. Standard analysis is usually based on empirical methods or on continuous simulations. This paper focuses on the probabilistic analysis of retention times aimed to provide guidance to engineers for the design of stormwater detention facilities. In particular, the influence on retention time of the possibility of water mixing from consecutive rainfall events, due to the pre-filling of the storage capacity from previous runoffs has been investigated. Derived expression has been tested by their application to a case study.

Keywords: Probabilistic analysis; retention time; stormwater detention ; pre-filling.

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1. Introduction

Stormwater detention facilities are often used in urban drainage systems as tools for floods control and the improvement of the quality of treated waters. The main objectives in their design are to limit spills into receiving water bodies and to guarantee sufficient retention times for the sedimentation of pollutants contained in stormwaters. The Authors, that have already addressed the first issue in a previous work [1] focus, in this paper, on the analysis of retention times.

This aspect is very important in the design of stormwater detention facilities to evaluate the performance of stormwater treatment in pollutants removal [2]. Generally the retention time should be as long as it is for the sedimentation of particles contained in the stormwaters and the improvement of water quality.

Several authors in the literature have dealt with what could be the best range for retention times and concluded that 24-48 hours can be an optimal interval. Shorter retention times are not sufficient to allow a good sedimentation of most of suspended solids, while longer retention times are useless because most of particles contained in stormwaters sediment in few days [3]. Moreover, long retention times can cause smell problems resulting from the combination of wastewater quality, temperature and time [4].

Studies about which is the optimum retention time, [5] and [6], observed that the retention time also depends on the size of the particles and concluded that a retention time of 24 hours can remove most of the particles less than 10 μm in diameter and all the particles larger than 10 μm; the higher removal efficiency of larger particles is due to a greater settling velocity and a stronger first flush than smaller particles.

The simplest way for the estimation of the distribution of retention times is its measure by a pulse of a non-reactive tracer chemical dissolved into the inlet to the facility. Often, when this is no possible, numerical methods have been proposed [7], [8]. In particular the effects of the hydraulic of the system and the hydrology of the flow rate on the distribution of the retention time have been deeply investigated in the literature [9]. These approaches can have a high computational burden and can be difficult to apply when long-time information about inflows and outflows from the stormwater detention facilities are no available.

Another issue in the analysis of the distribution of retention times is about how calculate the retention time. The common definition of retention time, volume divided by the flow rate is theoretically applicable only when steady state conditions prevail (constant volume and flow rate) [10]. In most case they are not verified and the definition of retention time is more complex and strongly correlated to the management rule of the storage. Moreover, water mixing from the pre-filling of the storage capacity from previous rainfall events is often neglected.

This paper proposed an analytical probabilistic approach to estimate the probability distribution function of retention times in a stormwater detention facility. These kind of approaches have been developed as alternative to simplified methods and continuous simulations in many fields of the practical engineering [11], [12], [13], [14], [15] because are generally simple to implement and reliable when long-term series of data are not available.

The probability distribution function of the retention times has been calculated for different conditions of storage, considering the possibility of spill when the storage capacity is full and the possibility of water mixing from consecutive rainfalls due to the pre-filling of the capacity from previous events.

Finally resultant expressions have been applied to a case study and their results have been compared with those obtained from the continuous simulations of observed data.

2. Modeling of rainfall and storage processes

For the definition of retention time and the estimation of its probability distribution function some simplified assumptions on the hydrology of inflows and on the storage process have been made. An on-line stormwater detention facility has been considered and rainfall-runoff transformation has been neglected, as typical for catchment with short corrivation times. Runoff volume for unit of catchment surface v has been expressed by:

$$v=\varphi\cdot(h-IA) \tag{1}$$

where h: rainfall depth, φ: runoff coefficient, IA: Initial Abstraction.

The three random variables that mainly drive the storage process in a stormwater detention facility, rainfall depth h, rainfall duration θ and interevent time d, have been considered independent and exponentially distributed; their probability distribution function can be expressed as:

$$f_h = \xi \cdot e^{-\xi \cdot h} \tag{2}$$

$$f_\theta = \lambda \cdot e^{-\lambda \cdot \theta} \tag{3}$$

$$f_d = \psi \cdot e^{-\psi \cdot (d - IETD)} \tag{4}$$

where $\xi = 1/\mu_h$, $\lambda = 1/\mu_\theta$ and $\psi = 1/(\mu_d - IETD)$ with μ_h: average rainfall depth, μ_θ: average rainfall duration, μ_d: average interevent time.

IETD (InterEvent Time Definition) is the minimum dry time used to identify independent rainfall events; if the dry time between two consecutive rainfall events is smaller than IETD, the two events are joined together into a single event, otherwise are considered independent.

Although the Gamma and the Weibull probability distribution functions best fitted the frequency function of the main rainfall variables [16], the hypothesis of exponential probability distribution is often used due to the easiness of its integration [17]. For highly urbanized catchment where $IA \to 0$ and $\varphi \to 1$, runoff volume equals to rainfall volume $v = h$ and its probability distribution function can be considered exponential distributed.

For the modeling of the storage process two consecutive rainfall events have been considered (Fig. 1).

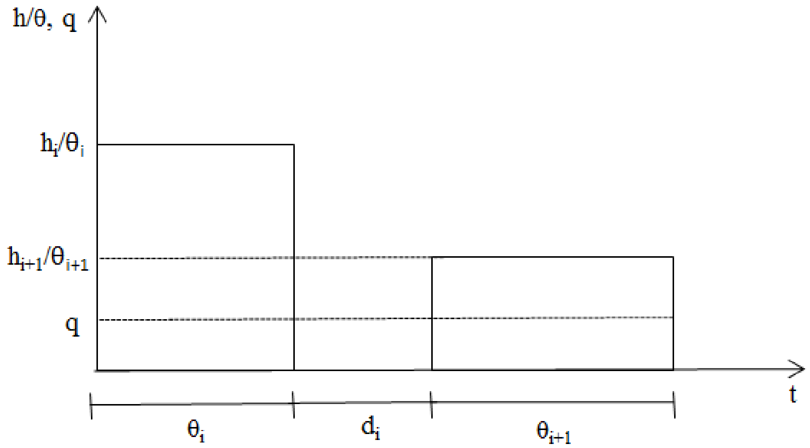

Fig. 1. Schematization of a couple of runoff events.

Rainfall intensity h/θ for each event has been considered constant as well as the outflow rate q.

The definition of retention time depends on the management rule of discharges. Typically, for on-line stormwater detention facilities, they start emptying as soon as they begin to fill. Considering rectangular events with inflow rates greater than outflow rates, this means soon after the beginning of each event. As usual when analysing the distribution of retention times in stormwater detention facilities, the average retention time has been considered; it has been calculated as half of the average emptying time.

3. Estimation of the probability distribution function of retention times

The probability distribution function of retention times has been calculated with reference to the couple of rainfall events *i* and *i+1* of Fig. 1.

The distribution of retention times is primarily influenced by the hydraulic of the system, that is the flow patterns that develop in the storage during an event, and by the hydrology that is the temporal distribution of inflows.

Two different conditions of storage have been analyzed:

- Single runoff: the possibility of pre-filling from the event *i* at the beginning of the event *i+1* is excluded;
- Possibility of pre-filling from the event *i* at the beginning of the event *i+1*.

which correspond respectively to the conditions $w_0/q \leq IETD$ and $w_0/q > IETD$ with w_0: volume of the stormwater detention facility.

Moreover, the possibility of spill when the capacity is full at the end of each event has been considered.

The retention time, considering a couple of runoff events and the management rule discussed at the end of the paragraph 2 can range from a minimum of zero to a maximum of $\theta_i + d_i + \theta_{i+1} + w_0/q$. This last condition corresponds to the emptying time of a stormwater detention facility pre-filled at the beginning of the event *i+1* and full at its end. All other possible values of retention times fall within the interval defined by these two limits.

If the possibility of pre-filling is excluded ($w_0/q \leq IETD$), the average retention time t_R only depends on the single runoff event. The stored volume at the end of the event *i* results:

$$w_i = \begin{cases} w_0 & \text{case A1} \\ h_i - q \cdot \theta_i & \text{case A2} \end{cases} \tag{5}$$

distinguishing the two cases in which the event spills or not (*case A1* and *case A2*):
case A1 = event i without spill: $0 < h_i - q \cdot \theta_i < w_0$;
case A2 = event i with spill: $h_i - q \cdot \theta_i \geq w_0$;

The average retention time t_R can be expressed by:

$$t_R = \frac{1}{2} \cdot \begin{cases} h_i/q & \text{case A1} \\ \theta + w_0/q & \text{case A2} \end{cases} \tag{6}$$

It has been assumed that the probability distribution functions of rainfall depth, duration and interevent time of the event *i* coincide with those of the event *i+1*: $f_{h,i}=f_{h,i+i}=f_h$, $f_{\theta,i}=f_{\theta,i+1}=f_\theta$ and $f_d=f_{d,i}=f_{d,i+1}$.

The probability distribution function of the average retention time can be expressed by:

$$P(t_R > t_0) = \int_{\theta=2\cdot t_0}^{\infty} f_\vartheta \cdot d\vartheta \cdot \int_{q\cdot\theta}^{w_0+q\cdot\theta} f_h \cdot dh +$$

$$+ \begin{cases} \int_{\theta=0}^{2\cdot t_0} f_\theta \cdot d\theta \int_{h=2\cdot q\cdot t_0}^{w_0+q\cdot\theta} f_h \cdot dh + \int_{\theta=0}^{\infty} f_\theta \cdot d\theta \int_{h=w_0+q\cdot\theta}^{\infty} f_h \cdot dh & \frac{w_0}{2\cdot q} \geq t_0 \\ \int_{\theta=2\cdot t_0-\frac{w_0}{q}}^{2\cdot t_0} f_\theta \cdot d\theta \int_{h=2\cdot q\cdot t_0}^{w_0+q\cdot\theta_8} f_h \cdot dh + \int_{\theta=2\cdot t_0-\frac{w_0}{q}}^{\infty} f_\theta \cdot d\theta \int_{h=w_0+q\cdot\theta}^{\infty} f_h \cdot dh & \frac{w_0}{2\cdot q} < t_0 \end{cases} \tag{7}$$

With t_0: fixed retention time. Integrating, the solution is:

$$P(t_R > t_0) = \frac{e^{-2\cdot t_0\cdot(q\cdot\xi+\lambda)}}{1+q^*} \cdot \left(1 - e^{-\xi\cdot w_0}\right) +$$

$$+e^{-2\cdot\xi\cdot q\cdot t_0}\cdot\left[1-e^{-2\cdot\lambda\cdot t_0}+\frac{e^{-(\xi\cdot w_0+2\cdot\lambda\cdot t_0)}}{1+q^*}\right] \qquad \frac{w_0}{2\cdot q}\geq t_0$$

$$+e^{-2\cdot t_0\cdot(q\cdot\xi+\lambda)}\cdot\left\{\frac{1}{1+q^*}\cdot\left[e^{\frac{\lambda\cdot w_0}{q}}\cdot(1+q^*)+e^{-\xi\cdot w_0}\right]-1\right\} \qquad \frac{w_0}{2\cdot q}<t_0$$

Where: $q^* = q\cdot\xi/\lambda$.

If the possibility of pre-filling is considered ($w_0/q > IETD$), the pre-filling volume $w_{pr,i}$ at the beginning of the event *i+1* can be expressed as:

$$w_{pr,i}=\begin{cases} h_i-q\cdot(d_i+\theta_i) & \text{case B1}\\ w_0-q\cdot d_i & \text{case B2}\end{cases} \tag{8}$$

distinguishing the two cases of event *i* with spills or not (*case B1* and *case B2*):
case B1 = case A1 + pre-filling at the beginning of the event $_{i+1}$: $h_i-q\cdot(d_i+\theta_i)>0$;
case B2 = case A2 + pre-filling at the beginning of the event $_{i+1}$: $w_0-q\cdot d_i>0$;

The stored volume at the end of the event *i+1* can be expressed as:

$$w_{i+1}=\begin{cases} w_0 & \text{case C2;case D2}\\ h_i-q\cdot\theta_i-q\cdot d_i+h_{i+1}-q\cdot\theta_{i+1} & \text{case C1}\\ w_0-q\cdot d_i+h_{i+1}-q\cdot\theta_{i+1} & \text{case D1}\\ h_i/q & \text{case A1}\\ \theta_i+w_0/q & \text{case A2}\end{cases} \tag{9}$$

where *case C1*, *case C2*, *case D1* and *case D2* consider all the possible combinations of spills from events *i* and *i+1*:
case C1 = case B1 + event $_{i+1}$ *without spills*: $0<h_i-q\cdot\theta_i-q\cdot d_i+h_{i+1}-q\cdot\theta_{i+1}<w_0$;
case C2 = case B1 + event $_{i+1}$ *with spills*: $h_i-q\cdot\theta_i-q\cdot d_i+h_{i+1}-q\cdot\theta_{i+1}\geq w_0$;
case D1 = *case B2 + event* $_{i+1}$ *without spills*: $0<w_0-q\cdot d_i+h_{i+1}-q\cdot\theta_{i+1}<w_0$;
case D2 = case B2 + *event* $_{i+1}$ *with spills*: $w_0-q\cdot d_i+h_{i+1}-q\cdot\theta_{i+1}\geq w_0$;

The average retention time results:

$$t_R=\frac{1}{2}\cdot\begin{cases} \theta_i+d_i+\theta_{i+1}+w_0/q & \text{case C2;case D2}\\ (h_i+h_{i+1})/q & \text{case C1}\\ w_0/q+\theta_i+h_{i+1}/q & \text{case D1}\\ h_i/q & \text{case A1}\\ \theta_i+w_0/q & \text{case A2}\end{cases} \tag{10}$$

For the assumption that the event *i* and the event *i+1* are equal in probability *case D1* cannot occur.

If *case C1* occurs, this condition can by expressed by:

$$\int_{d=IETD}^{t_0} f_d\cdot dd\cdot\int_{\theta=t_0-d}^{\infty} f_\theta\cdot d\theta\cdot\int_{h=q\cdot(\theta+d)}^{\frac{w_0+2\cdot q\cdot\theta+q\cdot d}{2}} f_h\cdot dh \qquad IETD<t_0<\frac{w_0}{q} \tag{11}$$

$$\int_{d=IETD}^{w_0/q} f_d\cdot dd\cdot\int_{\theta=t_0-d}^{\infty} f_\theta\cdot d\theta\cdot\int_{h=q\cdot(\theta+d)}^{\frac{w_0+2\cdot q\cdot\theta+q\cdot d}{2}} f_h\cdot dh \qquad IETD<\frac{w_0}{q}<t_0 \tag{12}$$

$$\int_{d=IETD}^{2\cdot t_0-w_0/q} f_d\cdot dd\cdot\int_{\theta=t_0-\frac{w_0}{2\cdot q}-\frac{d}{2}}^{t_0-d} f_\theta\cdot d\theta\cdot\int_{h=q\cdot t_0}^{\frac{w_0+2\cdot q\cdot\theta+q\cdot d}{2}} f_h\cdot dh \qquad IETD<2\cdot t_0-\frac{w_0}{q}<\frac{w_0}{q} \tag{13}$$

$$\int_{d=IETD}^{w_0/q} f_d \cdot dd \cdot \int_{\theta=t_0-\frac{w_0}{2\cdot q}-\frac{d}{2}}^{t_0-d} f_\theta \cdot d\theta \cdot \int_{h=q\cdot t_0}^{\frac{w_0+2\cdot q\cdot\theta+q\cdot d}{2}} f_h \cdot dh \qquad IETD<\frac{w_0}{q}<2\cdot t_0-\frac{w_0}{q} \tag{14}$$

If *case C2* occurs, this condition can by expressed by:

$$\int_{d=IETD}^{2\cdot t_0-w_0/q} f_d \cdot dd \cdot \int_{\theta=t_0-\frac{w_0}{2\cdot q}-\frac{d}{2}}^{\infty} f_\theta \cdot d\theta \cdot \int_{h=\frac{w_0+2\cdot q\cdot\theta+q\cdot d}{2}}^{w_0+q\cdot\theta} f_h \cdot dh \qquad IETD<2\cdot t_0-\frac{w_0}{q}<\frac{w_0}{q} \tag{15}$$

$$\int_{d=IETD}^{w_0/q} f_d \cdot dd \cdot \int_{\theta=t_0-\frac{w_0}{2\cdot q}-\frac{d}{2}}^{\infty} f_\theta \cdot d\theta \cdot \int_{h=\frac{w_0+2\cdot q\cdot\theta+q\cdot d}{2}}^{w_0+q\cdot\theta} f_h \cdot dh \qquad IETD<\frac{w_0}{q}<2\cdot t_0-\frac{w_0}{q} \tag{16}$$

If *case D2* occurs, this condition can by expressed by:

$$\int_{d=IETD}^{2\cdot t_0-w_0/q} f_d \cdot dd \cdot \int_{\theta=t_0-\frac{w_0}{2\cdot q}-\frac{d}{2}}^{\infty} f_\theta \cdot d\theta \cdot \int_{h=w_0+q\cdot\theta}^{\infty} f_h \cdot dh \qquad IETD<2\cdot t_0-\frac{w_0}{q}<\frac{w_0}{q} \tag{17}$$

$$\int_{d=IETD}^{w_0/q} f_d \cdot dd \cdot \int_{\theta=t_0-\frac{w_0}{2\cdot q}-\frac{d}{2}}^{\infty} f_\theta \cdot d\theta \cdot \int_{h=w_0+q\cdot\theta}^{\infty} f_h \cdot dh \qquad IETD<\frac{w_0}{q}<2\cdot t_0-\frac{w_0}{q} \tag{18}$$

From the merge of equation (15) with equation (17) and of equation (16) with equation (18) it results:

$$\int_{d=IETD}^{2\cdot t_0-w_0/q} f_d \cdot dd \cdot \int_{\theta=t_0-\frac{w_0}{2\cdot q}-\frac{d}{2}}^{\infty} f_\theta \cdot d\theta \cdot \int_{h=\frac{w_0+2\cdot q\cdot\theta+q\cdot d}{2}}^{\infty} f_h \cdot dh \qquad IETD<2\cdot t_0-\frac{w_0}{q}<\frac{w_0}{q} \tag{19}$$

$$\int_{d=IETD}^{w_0/q} f_d \cdot dd \cdot \int_{\theta=t_0-\frac{w_0}{2\cdot q}-\frac{d}{2}}^{\infty} f_\theta \cdot d\theta \cdot \int_{h=\frac{w_0+2\cdot q\cdot\theta+q\cdot d}{2}}^{\infty} f_h \cdot dh \qquad IETD<\frac{w_0}{q}<2\cdot t_0-\frac{w_0}{q} \tag{20}$$

Summarizing conditions above the probability distribution functions of the average retention time can be calculated.

- For $IETD<t_0<\frac{w_0}{q}$:

$$P[t_R>t_0]=\int_{d_1}^{d_2} f_d \cdot dd \cdot \int_{\theta_1}^{\theta_2} f_\theta \cdot d\theta \cdot \int_{h_1}^{h_2} f_h \cdot dh \tag{21}$$

$d_1 = IETD$; $d_2 = t_0$;
$\theta_1 = t_0 - d$; $\theta_2 = \infty$;
$h_1 = q\cdot(\theta + d)$; $h_2 = (w_0 + 2\cdot q\cdot\theta + q\cdot d) / 2$

Integrating, it results:

$$P[t_R > t_0] = \frac{e^{-t_0\cdot(\xi\cdot q+\lambda)}}{1+q^*}\cdot\left\{\gamma\cdot e^{IETD\cdot\left(\frac{q\cdot\xi}{2}+\lambda\right)-\frac{\xi\cdot w_0}{2}} + \delta\cdot e^{\lambda\cdot IETD} - e^{\psi\cdot IETD}\cdot\left[\gamma\cdot e^{-\frac{\xi\cdot w_0}{2}+\frac{\xi\cdot q\cdot t_0}{2}+t_0\cdot(\lambda-\psi)} + \delta\cdot e^{t_0\cdot(\lambda-\psi)}\right]\right\}$$

with: $\gamma = 2\cdot\psi/(q\cdot\xi+2\cdot\lambda-2\cdot\psi)$; $\delta = \psi/(\psi-\lambda)$.

- For $IETD<\frac{w_0}{q}<t_0$:

$$P[t_R>t_0]=\int_{d_3}^{d_4} f_d \cdot dd \cdot \int_{\theta_3}^{\theta_4} f_\theta \cdot d\theta \cdot \int_{h_3}^{h_4} f_h \cdot dh \tag{22}$$

$d_3 = IETD$; $d_4 = w_0/q$;
$\theta_3 = t_0 - d$; $\theta_4 = \infty$;
$h_3 = q\cdot(\theta + d)$; $h_4 = (w_0 + 2\cdot q\cdot\theta + q\cdot d) / 2$.

Integrating, it results:

$$P[t_R>t_0]=\frac{e^{-t_0\cdot(\xi\cdot q+\lambda)}}{1+q^*}\cdot\left\{\gamma\cdot e^{IETD\cdot\left(\frac{q\cdot\xi}{2}+\lambda\right)-\frac{\xi\cdot w_0}{2}}+\delta\cdot e^{\lambda\cdot IETD}-\delta\cdot n\cdot e^{\psi\cdot IETD+\frac{w_0}{q}\cdot(\lambda-\psi)}\right\}$$

with: $n = q\cdot\xi/(q\cdot\xi+2\cdot\lambda-2\cdot\psi)$.

- For $IETD<2\cdot t_0-\frac{w_0}{q}<\frac{w_0}{q}$:

$$P[t_R > t_0] = \int_{d_5}^{d_6} f_d \cdot dd \cdot \int_{\theta_5}^{\theta_6} f_\theta \cdot d\theta \cdot \int_{h_5}^{h_6} f_h \cdot dh + \int_{d_7}^{d_8} f_d \cdot dd \cdot \int_{\theta_7}^{\theta_8} f_\theta \cdot d\theta \cdot \int_{h_7}^{h_8} f_h \cdot dh \qquad (23)$$

$d_5 = d_7 = IETD$; $d_6 = d_8 = 2\cdot t_0 - w_0/q$;
$\theta_5 = \theta_7 = t_0 - w_0/(2\cdot q) - d/2$; $\theta_6 = t_0 - d$; $\theta_8 = \infty$;
$h_5 = q\cdot t_0$; $h_6 = h_7= (w_0 + 2\cdot q\cdot\theta + q\cdot d) / 2$; $h_8 = \infty$.

Integrating, it results:

$$\begin{aligned}P[t_R>t_0]=&\frac{\gamma}{1+q^*}\cdot e^{-\frac{\xi\cdot w_0}{2}}\cdot\left[e^{-\frac{w_0}{q}\cdot(\psi+\lambda)+\psi\cdot IETD-\frac{\xi\cdot w_0}{2}-t_0\cdot(2\cdot\psi-\lambda)}-e^{-t_0\cdot(\xi\cdot q+\lambda)}\right]+\\&+\delta\cdot e^{-t_0\cdot(\xi\cdot q+\lambda)}\cdot\left[e^{\psi\cdot IETD-\frac{w_0}{q}\cdot(\lambda-\psi)-2\cdot t_0\cdot(\psi-\lambda)}-e^{\lambda\cdot IETD}\right]+\\&+\frac{2\cdot m\cdot q^*}{1+q^*}\cdot e^{-\xi\cdot q\cdot t_0}\cdot\left[e^{\psi\cdot\left(IETD-2\cdot t_0-\frac{w_0}{q}\right)}-e^{\frac{\lambda}{2}\cdot\left(IETD+\frac{w_0}{q}-2\cdot t_0\right)}\right]+\\&+\frac{\alpha}{1+q^*}\cdot e^{-t_0\cdot(\xi\cdot q+\lambda)}\cdot\left[e^{\frac{\lambda}{2}\cdot\left(\frac{w_0}{q}-IETD\right)}-e^{\psi\cdot\left(\frac{w_0}{q}+IETD-2\cdot t_0\right)-\lambda\cdot\left(t_0-\frac{w_0}{q}\right)}\right]+\end{aligned}$$

with: $\alpha = 2\cdot\psi/(2\cdot\psi+\lambda)$; $m = \psi/(\lambda-2\cdot\psi)$.

- For $IETD<\frac{w_0}{q}<2\cdot t_0-\frac{w_0}{q}$:

$$P[t_R > t_0] = \int_{d_9}^{d_{10}} f_d \cdot dd \cdot \int_{\theta_9}^{\theta_{10}} f_\theta \cdot d\theta \cdot \int_{h_9}^{h_{10}} f_h \cdot dh + \int_{d_{11}}^{d_{12}} f_d \cdot dd \cdot \int_{\theta_{11}}^{\theta_{12}} f_\theta \cdot d\theta \cdot \int_{h_{11}}^{h_{12}} f_h \cdot dh \qquad (24)$$

$d_9 = d_{11} = IETD$; $d_{10} = d_{12} = w_0/q$;
$\theta_9 = \theta_{11} = t_0 - w_0/(2\cdot q) - d/2$; $\theta_{10} = t_0 - d$; $\theta_{12} = \infty$;
$h_9 = q\cdot t_0$; $h_{10} = h_{11}= (w_0 + 2\cdot q\cdot\theta + q\cdot d) / 2$; $h_{12} = \infty$.

Integrating, it results:

$$P[t_R>t_0]=e^{-t_0\cdot(\xi\cdot q+\lambda)}\cdot\left[-\frac{\gamma}{1+q^*}\cdot e^{IETD\cdot\left(\frac{q\cdot\xi}{2}+\lambda\right)-\frac{\xi\cdot w_0}{2}}-\delta\cdot e^{\lambda\cdot IETD}-\frac{2\cdot m\cdot q^*}{1+q^*}\cdot e^{\frac{\lambda}{2}\cdot\left(IETD+\frac{w_0}{q}\right)}+m\cdot z\cdot n\cdot e^{\psi\cdot IETD-\frac{w_0}{q}\cdot(\psi-\lambda)}\right]+$$

$$+\frac{\alpha}{1+q^*}\cdot e^{-t_0\cdot(\xi\cdot q+\lambda)}\cdot\left[e^{\frac{\lambda}{2}\cdot\left(\frac{w_0}{q}-IETD\right)}-e^{-\psi\cdot\left(\frac{w_0}{q}-IETD\right)}\right]$$

with: $z = \lambda/(\lambda-\psi)$.

4. Case study

Final equations for the estimation of the probability distribution function of average retention times have been validated by their application to a series of recorded rainfall data and results have been compared to those obtained from the continuous simulation of the same series of data. The rainfall series recorded at Milano-Monviso gauge station in the period 1991-2005 has been used and an IETD = 10 hours for the identification of independent rainfall events has been considered. Table 1 shows average values, variation coefficients and correlations among rainfall depth, rainfall duration and interevent time.

Table 1. Average values μ, variation coefficients V and correlations ρ of rainfall variables

μ_h [mm]	18,49	**V_h [-]**	1,15	**$\rho_{h,\theta}$ [-]**	0,62
μ_θ [hour]	14,37	**V_θ [-]**	1,03	**$\rho_{\theta,d}$ [-]**	0,11
μ_d [hour]	172,81	**V_d [-]**	1,30	**$\rho_{d,h}$ [-]**	0,11

Constant outflow rates q = 0,5 mm/hour and q = 1 mm/hour and a retention time t_0 = 24 hours have been used in the calculation. Figure 2 compares results from the application of the final equations (7), (21), (22), (23) and (24) with the analysis of frequency of the simulated data.

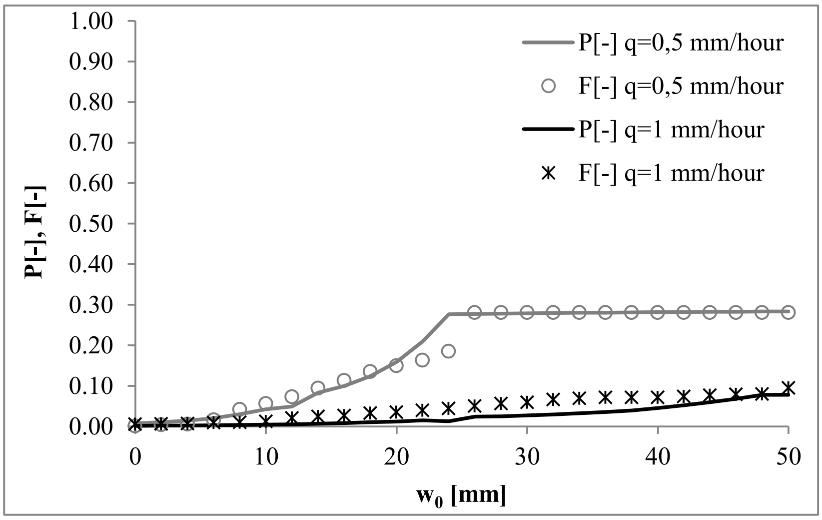

Fig 2. Probability distribution function P[-] and frequency F[-] of average retention times.

The probability distribution function of average retention times calculated by the application of the proposed method, well-fitted to frequencies calculated from the continuous simulation of observed data. The little differences can be due to the simplifying assumptions of the modeling:

- Exponential distribution of rainfall characteristics: this hypothesis is not completely satisfied because, as shown in Table 1, variation coefficients are little different from one, especially for rainfall depth and interevent time;

- Independence of rainfall variables: the correlation between rainfall depth and rainfall duration is not negligible (see Table 1);
- Pre-filling from only a previous rainfall event: as discussed by [18] this is valid in most cases but water carryover from more than one previous rainfall event can sometimes occur;
- Equality in probability of each rainfall event: as consequence of this hypothesis, *case D1* cannot occur and the possibility of spill from the first event only is neglected.

5. Conclusion

The analytical probabilistic method proposed in this paper allows estimating the probability distribution function of averages retention times in a stormwater detention facility. It can be a valid aid for designers when long-term series of rainfall data are no available and only average values of the main rainfall variables are known, as often happens in the practice. The knowledge of the probability that the average retention time exceeds a fixed value can give valuable suggestions about the efficiency of the detention facility in the sedimentation of different pollutants contained in stomwaters. On the other hand, resulting formulas can also be used in the design of a stormwater detention facility to estimate the storage capacity that guarantees to have a sufficient average retention time for pollutants removal and to understand if an increase in storage volume corresponds to an effective increase of retention times. The proposed approach needs to be tested by its application to different rainfall series to better study its reliability and the influence on results of the different simplifying assumptions.

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