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"Non-renormalization" without supersymmetry

João P. Rodrigues

School of Physics and Centre for Theoretical Physics, University of the Witwatersrand, Johannesburg, Wits 2050, South Africa [1]

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Abstract

The g_{YM} perturbed, non-supersymmetric extension of the dual single matrix description of $1/2$ BPS states, within the Hilbert space reduction to the oscillator subsector associated with chiral primaries is considered. This matrix model is described in terms of a single Hermitean matrix. It is found that, apart from a trivial shift in the energy, the large N background, spectrum and interaction of invariant states *are independent* of g_{YM}. This property applies to more general D terms.

1. Background and motivation

Recent studies of giant gravitons in AdS backgrounds [1–8] have led to a dual matrix model description of $1/2$ BPS states in terms of the large N limit of a single complex matrix [8] in a harmonic potential [9].

This can be motivated as follows: starting with the leading Kaluza–Klein compactification of the bosonic sector of $\mathcal{N}=4$ SYM on $S^3 \times R$, one chooses the plane defined by two adjoint scalars ($N \times N$ Hermitean matrices) X_1 and X_2. The corresponding bosonic sector of the Hamiltonian is

$$\hat{H} \equiv \frac{1}{2}\operatorname{Tr}\left(P_1^2\right) + \frac{1}{2}\operatorname{Tr}\left(P_2^2\right) + \frac{w^2}{2}\operatorname{Tr}\left(X_1^2\right) + \frac{w^2}{2}\operatorname{Tr}\left(X_2^2\right) - g_{\mathrm{YM}}^2 \operatorname{Tr}[X_1, X_2][X_1, X_2] \quad (1)$$

with P_1 (P_2) canonical conjugate to X_1 (X_2, respectively). The harmonic potential arises as a result of the coupling to the curvature of the manifold.

In terms of the complex matrix $Z = X_1 + iX_2$ and canonical momentum $\Pi = 1/2(P_1 - iP_2)$,

$$\hat{H} = 2\operatorname{Tr}\Pi^\dagger\Pi + \frac{w^2}{2}\operatorname{Tr}\left(Z^\dagger Z\right) + \frac{g_{\mathrm{YM}}^2}{4}\operatorname{Tr}\left[Z, Z^\dagger\right]\left[Z, Z^\dagger\right]. \quad (2)$$

Introducing matrix valued creation and annihilation operators

$$Z = \frac{1}{\sqrt{w}}\left(A + B^\dagger\right), \qquad \Pi = -i\frac{\sqrt{w}}{2}\left(A^\dagger - B\right),$$

motion in this plane is then characterized by the energy E and the two-dimensional angular momentum:

$$\hat{J} = \operatorname{Tr}\left(A^\dagger A\right) - \operatorname{Tr}\left(B^\dagger B\right). \quad (3)$$

A (and B) quanta carry well-defined charge 1 (-1, respectively).

When imbedded in $\mathcal{N}=4$ SYM, the g_{YM} dependent interaction in (2) is one of the so-called D terms, and is subject to non-renormalization theorems. One may then consider the free theory. In this case, the (invariant) eigenstates are $\operatorname{Tr}((A^\dagger)^n)\operatorname{Tr}((B^\dagger)^m)|0\rangle$.

$1/2$ BPS states then correspond to a restriction of the theory to the (chiral) sector with no B excitations, for which $E = E_0 = wJ$ (E_0 is the free theory energy).[2]

When restricted to correlators of these chiral primary operators, the dynamics of the system is fully described by free fermions in the harmonic oscillator potential. It actually turns out, as shown in [11], that the gravity description of $1/2$ BPS states is completely determined by a phase space density function associated with a general fermionic droplet configuration.

E-mail address: joao.rodrigues@wits.ac.za.

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[2] The B sector can also be projected out by taking a pp wave limit [10]. This requires J to become large.

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As suggested in [12], the dynamics of the A, $A^\dagger$ system can be described in terms of a Hermitean matrix M

$$M \equiv \frac{1}{\sqrt{2w}}\left(A + A^\dagger\right), \qquad P_M = -i\sqrt{\frac{w}{2}}\left(A - A^\dagger\right), \tag{4}$$

while retaining B, $B^\dagger$ creation and annihilation operators. The change of variables is:

$$\begin{aligned} X_1 &= \frac{1}{\sqrt{2}}M + \frac{1}{2\sqrt{w}}\left(B + B^\dagger\right), \\ X_2 &= \frac{1}{\sqrt{2w}}P_M + \frac{i}{2\sqrt{w}}\left(B - B^\dagger\right), \\ P_1 &= \frac{1}{\sqrt{2}}P_M - \frac{i\sqrt{w}}{2}\left(B - B^\dagger\right), \\ P_2 &= -\frac{w}{\sqrt{2}}M + \frac{\sqrt{w}}{2}\left(B + B^\dagger\right). \end{aligned} \tag{5}$$

As shown in [12], in the 1/2 BPS sector, the gravitational degrees of freedom emerge in a natural way from the density of eigenvalues description of the large N dynamics of the Hermitean matrix M. The energy and flux of the 1/2 BPS states obtained in [11] are exactly those of the leading large N configuration of the Hermitean matrix M in a harmonic potential, in a bosonic phase space density description.[3] This will be briefly reviewed in the next section. Fluctuations about the large N matrix configuration were shown to agree with fluctuations [12,14] about the gravity background of [11].

The purpose of this Letter is to investigate, in the absence of supersymmetry, the non-perturbative consequences of the quartic $g_{\rm YM}$ interaction in (1) while still restricting the theory to the sector with no B excitations. In other words, the system of interest is

$$\hat{H} = w\,\mathrm{Tr}\left(A^\dagger A\right) + \frac{g_{\rm YM}^2}{4w^2}\,\mathrm{Tr}\left[A^\dagger, A\right]\left[A^\dagger, A\right]. \tag{6}$$

In order to study its large N limit, I will use the variables (5), in terms of which the Hamiltonian (6) becomes:

$$\hat{H}_A = \frac{1}{2}\,\mathrm{Tr}\,P_M^2 + \frac{w^2}{2}\,\mathrm{Tr}\,M^2 - \frac{g_{\rm YM}^2}{4w^2}\,\mathrm{Tr}[M, P_M][M, P_M]. \tag{7}$$

In the absence of supersymmetry, the argument for the decoupling of B excitations is potentially weakened. It should always be a good approximation for external states with large J charges [13] and for large w. However, the Hamiltonian (6) (or (7)) is of great interest per se as it contains an interaction with the structure typical of a Yang–Mills interaction.

Remarkably, we will find that, apart from a trivial shift in the energy, the large N background, spectrum and interaction of gauge invariant states *are independent* of $g_{\rm YM}$, *even in the absence of supersymmetry*.

This Letter is organized as follows: within the collective field theory approach [15], a general argument is presented in Section 2 for the "non-renormalization" properties of the theory, and an explicit non-perturbative argument is developed in Section 3. A simple diagrammatic check is carried out Section 4, and a generalization presented in Section 5. Section 6 is reserved for a brief conclusion.

2. A general argument

In order to obtain the large N limit of (7), we will make use of collective field theory approach [15]. The starting point of this approach is to consider the action of the Hamiltonian on wave functionals of gauge invariant operators, i.e., operators invariant under the transformation:

$$M \to U^\dagger M U, \quad U \text{ unitary}. \tag{8}$$

For a single matrix model such as (7), these can be chosen as the density of eigenvalues λ_i, $i = 1, \ldots, N$, of the matrix M:

$$\phi(x) = \int \frac{dk}{2\pi} e^{-ikx}\,\mathrm{Tr}\left(e^{ikM}\right) = \sum_{i=1}^{N} \delta(x - \lambda_i), \tag{9}$$

or its Fourier transform

$$\phi_k = \mathrm{Tr}\left(e^{ikM}\right). \tag{10}$$

Let us analyze the structure of the $g_{\rm YM}$ dependent operator in (7) in more detail. It can be written as

$$-\frac{g_{\rm YM}^2}{4w^2}\,\mathrm{Tr}[M, P_M][M, P_M] = -\frac{g_{\rm YM}^2}{4w^2}\hat{G}_{ij}\hat{G}_{ji} + \frac{g_{\rm YM}^2 N^3}{4w^2}, \tag{11}$$

where

$$\hat{G}_{ij} \equiv M_{ik}\hat{P}_{kj} - M_{kj}\hat{P}_{ik}. \tag{12}$$

We recognize the generators of the transformation (8). Therefore, when restricted to the gauge invariant subspace, the $g_{\rm YM}$ term in (7) does not contribute, except for the trivial constant shift in energy in (11).[4]

3. Explicit non-perturbative argument

Because, as a result of the use of the chain rule in the kinetic energy operator, the interactions in the theory organize themselves with different powers of N, it is important to provide an explicit verification of the general argument presented in the previous section.

What is different from previous applications of the collective field theory to the large N limit of the single matrix Hamiltonian (7), is the sigma model nature of the kinetic energy operator and the presence of terms linear in momentum:

$$\begin{aligned} T &= \frac{1}{2}g_{i_1,i_2,i_3,i_4}(M)P_{i_1,i_2}P_{i_3,i_4} - \frac{i}{2}s_{i_1,i_2}(M)P_{i_1,i_2} \\ &\equiv \frac{1}{2}g_{AB}(X)P_A P_B - \frac{i}{2}s_A(X)P_A, \\ P_{i_1,i_2} &\equiv -i\frac{\partial}{\partial M_{i_2,i_1}}, \end{aligned}$$

[3] This is already suggested in (5), as $X_1 = 1/\sqrt{2}M + \cdots$ and $X_2 = 1/\sqrt{2}wP_M + \cdots$.

[4] One may also think of the system as gauged, in which case the invariance under (8) results from Gauss' law, and restriction to wave functionals of gauge invariant operators explicitly satisfies Gauss' law.

$$g_{i_1,i_2,i_3,i_4}(M)=\delta_{i_1,i_4}\delta_{i_2,i_3}+\frac{g_{\rm YM}^2}{w^2}\big((M^2)_{i_4,i_1}\delta_{i_2,i_3}-M_{i_4,i_1}M_{i_2,i_3}\big),$$
$$s_{i_1,i_2}(M)=-\frac{g_{\rm YM}^2}{w^2}\big(NM_{i_2,i_1}-{\rm Tr}(M)\delta_{i_1,i_2}\big). \tag{13}$$

Denoting by ϕ_α a generic gauge invariant variable, the kinetic energy operator (13) takes the form, when acting on functionals of ϕ_α,

$$T=-\frac{1}{2}g_{AB}(X)\frac{\partial}{\partial X_A}\frac{\partial}{\partial X_B}-\frac{1}{2}s_A(X)\frac{\partial}{\partial X_A}$$
$$=-\frac{1}{2}\left(\bar\omega_\alpha\frac{\partial}{\partial\phi_\alpha}+\Omega_{\alpha,\beta}\frac{\partial}{\partial\phi_\alpha}\frac{\partial}{\partial\phi_\beta}\right),$$
$$\bar\omega_\alpha=g_{AB}(X)\frac{\partial^2\phi_\alpha}{\partial X_A\partial X_B}+s_A(X)\frac{\partial\phi_\alpha}{\partial X_A},$$
$$\Omega_{\alpha,\beta}=g_{AB}(X)\frac{\partial\phi_\alpha}{\partial X_A}\frac{\partial\phi_\beta}{\partial X_B}. \tag{14}$$

For $\Omega_{\alpha,\beta}$, I obtain

$$\Omega_{k,k'}=-kk'\phi_{k+k'}. \tag{15}$$

Due to the antisymmetric nature of the Yang–Mills interaction reflected in the sigma model nature of the kinetic energy (13), the result above for $\Omega_{k,k'}$ is independent of $g_{\rm YM}$ and is the same as that for the standard kinetic energy.

For $\bar\omega_\alpha$, I obtain

$$\bar\omega(x)=-2\partial_x\left\{\phi(x)\left[\int dy\frac{\phi(y)}{x-y}+\frac{g_{\rm YM}^2}{2w^2}\int dy\,(x-y)\phi(y)+\frac{g_{\rm YM}^2}{2w^2}\left(\int dy\,y\phi(y)-Nx\right)\right]\right\}$$
$$=-2\partial_x\left\{\phi(x)\left[\int dy\frac{\phi(y)}{x-y}+\frac{g_{\rm YM}^2}{2w^2}x\left(\int dy\,\phi(y)-N\right)\right]\right\}. \tag{16}$$

Interpreting $N={\rm Tr}(1)=\int dx\,\phi(x)$, one would immediately conclude that both (15) and (16) are independent of $g_{\rm YM}$, proving our result. However, we follow the more rigorous approach of imposing this constraint via a Lagrange multiplier, and choose to enforce the constraint after variation.

Due to the change of variables $X_A\to\phi_\alpha$, one performs the similarity transformation [15] induced by the Jacobian J:

$$\frac{\partial}{\partial\phi_\alpha}\to J^{\frac12}\frac{\partial}{\partial\phi_\alpha}J^{-\frac12},\qquad \Omega_{\alpha,\beta}\frac{\partial\ln J}{\partial\phi_\beta}=\omega_\alpha, \tag{17}$$

where only the leading (in N) expression for $\ln J$ is described. One obtains the form of the collective field Hamiltonian sufficient for the description of the leading large N background and fluctuations:

$$\hat H_{\rm eff}=\frac12\frac{\partial}{\partial\phi_\alpha}\Omega_{\alpha,\beta}\frac{\partial}{\partial\phi_\beta}+\frac18\bar\omega_\alpha\Omega^{-1}_{\alpha,\beta}\bar\omega_\beta+\int dx\frac12 w^2x^2\phi(x)-\mu\left(\int dx\,\phi(x)-N\right)$$
$$=-\frac12\int dx\,\partial_x\frac{\partial}{\partial\phi(x)}\phi(x)\partial_x\frac{\partial}{\partial\phi(x)}+\frac12\int dx\,\phi(x)\left[\int\frac{dy\,\phi(y)}{x-y}+\frac{g_{\rm YM}^2}{2w^2}x\left(\int dy\,\phi(y)-N\right)\right]^2$$
$$+\int dx\frac12 w^2x^2\phi(x)-\mu\left(\int dx\,\phi(x)-N\right). \tag{18}$$

The Lagrange multiplier μ enforces the constraint

$$\int dx\,\phi(x)=N. \tag{19}$$

To exhibit explicitly the N dependence, we rescale

$$x\to\sqrt{N}x,\qquad \phi(x)\to\sqrt{N}\phi(x),$$
$$\mu\to N\mu,\qquad -i\frac{\partial}{\partial\phi(x)}\equiv\Pi(x)\to\frac1N\Pi(x). \tag{20}$$

Using the identity

$$\int dx\,\phi(x)\left[\int\frac{dy\,\phi(y)}{x-y}\right]^2=\frac{\pi^2}{3}\int dx\,\phi^3(x), \tag{21}$$

we obtain

$$H_{\rm eff}=\frac{1}{2N^2}\int dx\,\partial_x\Pi(x)\phi(x)\partial_x\Pi(x)+N^2\left[\int dx\frac{\pi^2}{6}\phi^3(x)+\frac{\lambda^2}{8w^4}\left(\int dx\,x^2\phi(x)\right)\left(\int dy\,\phi(y)-1\right)^2\right.$$
$$+\frac{\lambda}{4w^2}\left(\int dx\,\phi(x)\right)^2\left(\int dy\,\phi(y)-1\right)$$
$$\left.+\int dx\,\phi(x)\frac{w^2x^2}{2}-\mu\left(\int dx\,\phi(x)-1\right)\right], \tag{22}$$

where $\lambda=g_{\rm YM}^2N$ is 't Hooft's coupling.

The large N configuration is the semiclassical background corresponding to the minimum of the effective potential in (22), and satisfies

$$\frac{\pi^2}{2}\phi_0^2+\frac{\lambda}{4w^2}+\frac12 w^2x^2-\mu=0, \tag{23}$$

where the constraint $\int dx\,\phi_0(x)=1$ has been applied after variation. This constraint fixes $\mu=w+\frac{\lambda}{4w^2}$, and we arrive at the Wigner distribution background

$$\pi\phi_0(x)=\sqrt{2w-w^2x^2}. \tag{24}$$

All dependence on μ (and λ) has disappeared, and the large N background is identical to the free case.

It is useful to review here the emergence of the droplet picture when $\lambda=0$. In this case, if we let [16] $p_\pm\equiv\partial_x\Pi(x)/N^2\pm\pi\phi(x)$, then (22) has a very natural phase space representation as

$$H^0_{\text{eff}} = \frac{N^2}{2\pi} \int_{p_-}^{p_+} \int dp\, dx \left(\frac{p^2}{2} + \frac{x^2}{2} - \mu \right).$$

As $N \to \infty$, the boundary of the droplet is given by $p_\pm^2 + x^2 = 2\mu = 2w$, since $p_\pm \to \pm\pi\phi_0(x) = \pm\sqrt{2w - w^2x^2}$. This is in agreement with the energy of the gravity solutions considered in [11], with $x_1 \to x, x_2 \to p$.[5]

When $\lambda \neq 0$, all possible λ dependence is absorbed in the definition of μ, and since then the large N background is independent of μ (and λ), it has a similar fermionic description to that of the free case.

For the small fluctuation spectrum, one shifts

$$\phi(x) = \phi_0 + \frac{1}{\sqrt{\pi}N} \partial_x \eta, \qquad \partial_x \Pi(x) = -\sqrt{\pi} N P(x)$$

to find the quadratic operator

$$\begin{aligned} H_2^0 = {}& \frac{1}{2} \int dx\, (\pi\phi_0) P^2(x) + \frac{1}{2} \int dx\, (\pi\phi_0)(\partial_x\eta)^2 \\ &+ \frac{\lambda^2}{8w^4\pi} \left(\int dx\, x^2\phi_0 \right) \left(\int dx\, \partial_x\eta \right)^2 \\ &+ \frac{\lambda}{2w^2\pi} \left(\int dx\, \partial_x\eta \right)^2. \end{aligned} \tag{25}$$

The variable with a gravity interpretation as the angular variable in the plane of the droplet [11] is the classical "time" of flight ϕ

$$\begin{aligned} &\frac{dx}{d\phi} = \pi\phi_0, \quad x(\phi) = -\sqrt{\frac{2}{w}} \cos(w\phi), \\ &\pi\phi_0 = \sqrt{2w} \sin(w\phi),\ 0 \leqslant \phi \leqslant \frac{\pi}{w}, \end{aligned} \tag{26}$$

in terms of which the quadratic Hamiltonian takes the form:

$$\begin{aligned} H_2^0 = {}& \frac{1}{2} \int d\phi\, P^2(\phi) + \frac{1}{2} \int d\phi\, (\partial_\phi\eta)^2 \\ &+ \frac{\lambda^2}{8w^4} \left(\int d\phi\, x^2(\phi)\phi_0^2(\phi) \right) \left(\int d\phi\, \partial_\phi\eta \right)^2 \\ &+ \frac{\lambda}{2w^2\pi} \left(\int d\phi\, \partial_\phi\eta \right)^2. \end{aligned} \tag{27}$$

Except for the last two terms, one recognizes the Hamiltonian of a $1+1$ massless boson. For consistency of the time evolution of the constraint (19), we impose Dirichlet boundary conditions [17] at the classical turning points:

$$\psi_j(\phi) = \sqrt{\frac{2w}{\pi}} \sin(jw\phi), \quad j = 1, 2, \ldots. \tag{28}$$

Since then

$$\int d\phi\, \partial_\phi\eta = 0 \tag{29}$$

the last two terms in (27) vanish, and we obtain a $1+1$ massless boson Hamiltonian with spectrum

$$\epsilon_j = wj, \quad j = 1, 2, 3, \ldots, \tag{30}$$

[5] In the notation of [11], our solution is restricted to $y \to 0$.

again independent of λ.

It is not difficult to see that as a result of both (19) and (29) the collective field cubic interaction is independent of λ.

In addition to (18), there is in the collective field theory an extra subleading contribution proportional to $\int dx\, \partial w(x)/\partial\phi(x)$, responsible for the one loop contribution to the energy and a tadpole interaction term [18]. The additional λ dependent contribution is proportional to a total derivative of $x\phi(x)$.

4. A simple diagrammatic test

Let us consider, for instance, the g^2_{YM} contribution to the two point function

$$\frac{1}{N^2} \langle \text{Tr}\big(A^2(t_2)\big) \text{Tr}\big(A^{\dagger 2}(t_1)\big) \rangle, \quad t_2 > t_1, \tag{31}$$

resulting from the quartic interaction in (6). This can be done using the propagator

$$\begin{aligned} &\langle 0 | T\big(\text{Tr}\big(A_{i_1 j_1}(t_2) A^\dagger_{i_2 j_2}(t_1)\big)\big) | 0 \rangle \\ &\quad = \theta(t_2 - t_1) e^{-iw(t_2 - t_1)} \delta i_1 j_2 \delta i_2 j_1. \end{aligned} \tag{32}$$

Consider first the $\text{Tr}(A^\dagger A^\dagger A A)$ interaction. There are two types of connected planar diagrams, one the usual connected diagram without self contractions and the other involving the "dressing" of an internal leg. For the first, one obtains

$$\begin{aligned} &\frac{g^2_{\text{YM}}}{4w^2} \,.\, 4N \,.\, \int_{t_1}^{t_2} dt\, e^{-i2w(t_2 - t)} e^{-i2w(t - t_1)} \\ &\quad = \frac{\lambda}{4w^2} \,.\, 4 \,.\, (t_2 - t_1) e^{-i2w(t_2 - t_1)}. \end{aligned} \tag{33}$$

For the other type of diagram, one obtains

$$\begin{aligned} &\frac{g^2_{\text{YM}}}{4w^2} \,.\, 8N \,.\, e^{-iw(t_2 - t_1)} \int_{t_1}^{t_2} dt\, e^{-iw(t_2 - t)} e^{-iw(t - t_1)} \\ &\quad = \frac{\lambda}{4w^2} \,.\, 8 \,.\, (t_2 - t_1) e^{-i2w(t_2 - t_1)}. \end{aligned} \tag{34}$$

Turning now to the $\text{Tr}(A^\dagger A A^\dagger A)$ interaction, one establishes that the only type of planar diagram that is generated is the diagram (34) with a self-contraction associated with the "dressing" of an internal leg. This diagram now has a symmetry factor of 12, and since the interaction has the opposite sign, it exactly cancels the sum of (33) and (34).

The arguments of the previous sections show that this generalizes to general invariant external states and to arbitrary orders of perturbation theory.

Other families of states, that are not protected by BPS arguments but with energies independent of g_{YM} for different diagrammatic reasons, have also been reported in the literature [19].

5. General D terms

Consider now the general g^2_{YM} potential

$$-g^2_{\text{YM}} \sum_{i<j} \text{Tr}\big([X_i, X_j][X_i, X_j]\big), \tag{35}$$

written in terms of complex fields $Z = X_1 + iX_2$, $\phi = X_3 + iX_4$ and $\psi = X_5 + iX_6$. The D terms take the form

$$g_{\mathrm{YM}}^2 \operatorname{Tr}\biggl(\frac{1}{4}[Z^\dagger, Z][Z^\dagger, Z] + \frac{1}{4}[\phi^\dagger, \phi][\phi^\dagger, \phi] + \frac{1}{4}[\psi^\dagger, \psi][\psi^\dagger, \psi] + \frac{1}{2}[\phi^\dagger, \phi][\psi^\dagger, \psi] + \frac{1}{2}[\phi^\dagger, \phi][Z^\dagger, Z] + \frac{1}{2}[\psi^\dagger, \psi][Z^\dagger, Z]\biggr). \tag{36}$$

They can be equivalently written as

$$\frac{g_{\mathrm{YM}}^2}{4} \operatorname{Tr}\bigl(\bigl([Z^\dagger, Z] + [\phi^\dagger, \phi] + [\psi^\dagger, \psi]\bigr) \times \bigl([Z^\dagger, Z] + [\phi^\dagger, \phi] + [\psi^\dagger, \psi]\bigr)\bigr). \tag{37}$$

One introduces, as before, matrix valued creation and annihilation operators

$$Z = \frac{1}{\sqrt{w}}\bigl(A + B^\dagger\bigr), \qquad \phi = \frac{1}{\sqrt{w}}\bigl(C + D^\dagger\bigr), \qquad \psi = \frac{1}{\sqrt{w}}\bigl(E + F^\dagger\bigr). \tag{38}$$

If one is interested only in correlators of chiral primaries such as $\operatorname{Tr}(Z^{\dagger m_1} \phi^{\dagger q_1} \psi^{\dagger s_1} \cdots)$, these can be excited from the vacuum as $\operatorname{Tr}(A^{\dagger m_1} C^{\dagger q_1} E^{\dagger s_1} \cdots)$ and described in terms of Hermitean matrices M, Q and S. Using an argument similar to the general argument of Section 2, and taking into account the special form of (37), we conclude that the terms in (36) involving only A, C and E oscillators (and their conjugates) do not contribute to these amplitudes.

6. Conclusion

In this Letter, the large N limit of the system

$$\hat{H} = w \operatorname{Tr}\bigl(A^\dagger A\bigr) + \frac{g_{\mathrm{YM}}^2}{4w^2} \operatorname{Tr}\bigl[A^\dagger, A\bigr]\bigl[A^\dagger, A\bigr], \tag{39}$$

when restricted to the subsector of chiral primary states $\operatorname{Tr}(A^{\dagger n})$ has been shown to be independent of g_{YM}. In order to explicitly confirm this result, the collective field theory method has been generalized to include sigma model type kinetic energy operators. This "non-renormalization" result applies to more general D terms.

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