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Tracking Control Scheme for Multiple Autonomous Underwater Vehicles Subject to Union of Boundaries

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Abstract

This paper presents a new region boundary-based tracking control for Multiple Autonomous Underwater Vehicles (MAUVs). The proposed controller enables MAUVs to track a moving target formed by the union of two or more boundaries. In this case, multiplicative potential energy function is used to unite the whole boundaries. Moreover, each underwater vehicle navigates into a specific position on the boundary lines or surfaces while the target itself is moving. A non-negative Lyapunov-like function is presented for stability analysis of the MAUVs. Simulation results on 6 degrees-of-freedom AUVs are presented to illustrate the performance of new tracking control scheme.

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Keywords: multiplicative potential energy function; dynamic region boundary-based control; autonomous underwater vehicles; edge-based segmentation approach.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_1 = [x \ y \ z]^T \in \mathbb{R}^3$</td>
<td>position vector of the vehicle expressed in the inertial-fixed frame</td>
</tr>
<tr>
<td>$\eta_2 = [\phi \ \theta \ \psi]^T \in \mathbb{R}^3$</td>
<td>orientation vector of the vehicle expressed in the inertial-fixed frame</td>
</tr>
<tr>
<td>$v_1 = [u \ v \ w]^T \in \mathbb{R}^3$</td>
<td>linear velocity vector expressed in the body-fixed frame</td>
</tr>
<tr>
<td>$v_2 = [p \ q \ r]^T \in \mathbb{R}^3$</td>
<td>angular velocity vector expressed in the body-fixed frame</td>
</tr>
<tr>
<td>$v = [v_1 \ v_2]^T \in \mathbb{R}^6$</td>
<td>velocity state vector with respect to the body-fixed frame</td>
</tr>
<tr>
<td>$J(\eta_2)$</td>
<td>Jacobian matrix</td>
</tr>
<tr>
<td>$J^{-1}(\eta_2)$</td>
<td>inverse of the Jacobian matrix</td>
</tr>
<tr>
<td>$M$</td>
<td>inertia matrix including the added mass term</td>
</tr>
<tr>
<td>$C(v)$</td>
<td>matrix of the Coriolis and centripetal forces including the added mass term</td>
</tr>
<tr>
<td>$D(v)$</td>
<td>matrix of hydrodynamic damping and lift force</td>
</tr>
<tr>
<td>$g(\eta)$</td>
<td>vector of restoring force</td>
</tr>
<tr>
<td>$\tau$</td>
<td>vector of generalized forces acting on the vehicle</td>
</tr>
<tr>
<td>$\mathbf{\Theta}_d \in \mathbb{R}^{n_p}$</td>
<td>a set of dynamic parameters</td>
</tr>
<tr>
<td>$Y_d(\eta, u, v) \in \mathbb{R}^{6 \times n_p}$</td>
<td>a known regression matrix</td>
</tr>
<tr>
<td>$n_p$</td>
<td>total number of physical parameters</td>
</tr>
<tr>
<td>$\delta \eta_i \in \mathbb{R}^6$</td>
<td>continuous first partial derivatives of $i^{th}$ AUV</td>
</tr>
<tr>
<td>$\Delta \xi_i$</td>
<td>general error term for $i^{th}$ AUV</td>
</tr>
<tr>
<td>$k_{pi}, k_{zi}, k_{ei}$</td>
<td>positive scalar for $i^{th}$ AUV</td>
</tr>
</tbody>
</table>

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alpha \( a \) – positive constant
\( r_i(t) \) – filtered tracking error vector for \( i^{th} \) AUV
\( K_{vi} \) – positive constant matrix for \( i^{th} \) AUV
\( \theta_{di} \) – estimated parameters for \( i^{th} \) AUV
\( L_{di} \) – symmetric positive definite for \( i^{th} \) AUV
\( \tilde{\theta}_{di} = \theta_{di} - \theta_{di} \) – parameter estimation error
\( V \) – non-negative Lyapunov function
\( S_1 \) – scaling matrix of \( \eta_1 \)
\( S_2 \) – scaling matrix of \( \eta_2 \)
\( \kappa_p, \kappa_i \) – time-varying tolerance vector

1. Introduction

The offshore oil and gas fields have been notably the major players for underwater technology, especially in the control technology of Autonomous Underwater Vehicle (AUV) [1]. In many subsea missions, a platoon of AUVs is preferred than single vehicle since it provides greater flexibility and redundancy. One significant application of MAUVs is to inspect subsea oil pipeline where a number of AUVs are required to hover above the pipeline at identical or different depths along parallel paths, and map the pipeline from different viewpoints using multiple copies of the same suite of vision sensors. A specific formation of these AUVs is considered to make the overlap of vision coverage on the pipeline and hence all pieces of pipeline are monitored. It leads to the possibility of detecting any abnormalities in an underwater pipeline. Note that a team of AUVs could accomplish the monitoring task more rapidly and cost-effectively than that could be done by a single AUV [2]. Thus, the coordinated formation control of MAUVs remains the focus of many researchers to improve the performance of existing strategies especially for the mission of pipeline inspection.

Besides the regulation control, trajectory tracking is one of the advanced methods for AUV motion control, where the vehicle is required to be a certain point at a desired time. In the scenario of underwater pipeline inspection, there are three parallel paths followed by three AUVs in coordinated space, which are elevated from the seabed and offset from the underwater oil pipeline, and the speeds of vehicles along the pipeline should be the same as that determined by the end-user. Therefore, in the point of view of control design, the challenging of underwater oil pipeline inspection falls into the category of formation tracking control.

In recent studies, Li et al. [3] presented an adaptive region tracking control for an AUV where a region is used rather than a point due to minimal the control effort to track the region. Note that the total potential energy of the desired region is a summation of the potential energy associated with each region. Inspired from [3], Hou and Cheah [4] proposed a multiplicative potential function for a swarm robot. Within this control method, the desired shapes such as star shape region and N-shape region can be formed based on the union of all regions defined by corresponding inequality functions.

In this research work, a new tracking control scheme based on adaptive region boundary approach is proposed for MAUVs. The control formulation utilizes a boundary as desired target rather than a region or a point. Note that a target is specified by at least two sub-regions intersecting at the same point [5]. If these sub-regions are defined to be arbitrary small, then the concept of region boundary control is a generalization of the conventional tracking control problem. When adopting the multiplicative function, the total potential energy of the desired boundary is a union of the potential energy associated with each boundary. In addition, an edge-based segmentation approach is used to allow the MAUVs to navigate into a desired location on the boundary lines or surfaces, whilst the target itself is moving. Lyapunov-like function is presented for the stability analysis and simulation results on 6 degrees-of-freedom AUVs are presented to demonstrate the performance of the proposed controller. The rest of the paper is organized as follow: Section 2 describes kinematic and dynamic properties of a single AUV. Section 3 presents the proposed controller along with its stability analysis in the Lyapunov function. Simulation results are shown in Section 4. Finally, the summary and conclusions are presented in Section 5.

2. Kinematic and Dynamic Model of an AUV

2.1. Kinematic Model

The relationship between inertial and body-fixed vehicle velocity can be described using the Jacobian matrix \( J(\eta_2) \) in the following form

\[
\begin{bmatrix}
\dot{\eta}_1 \\
\dot{\eta}_2
\end{bmatrix} =
\begin{bmatrix}
J_1(\eta_2) & 0_{3x3} \\
0_{3x3} & J_2(\eta_2)
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
\iff \dot{\eta} = J(\eta_2)v
\]
where $\eta_1 = [x \ y \ z]^T \in \mathbb{R}^3$ and $\eta_2 = [\phi \ \theta \ \psi]^T \in \mathbb{R}^3$ denote the position and the orientation of the vehicle, respectively, expressed in the inertial-fixed frame. $J_1$ and $J_2$ are the transformation matrices expressed in terms of the Euler angles. The linear and angular velocity vectors, $v_1 = [u \ v \ w]^T \in \mathbb{R}^3$ and $v_2 = [p \ q \ r]^T \in \mathbb{R}^3$, respectively, are described in terms of the body-fixed frame.

2.2. Dynamic Model

The dynamic equation of motion for an underwater vehicle has been previously investigated in detail [6]. Due to hydrodynamic effects acting on the system such as added mass, drag, lift and buoyancy forces, the equation becomes highly nonlinear and coupled. Let the velocity state vector with respect to the body-fixed frame be defined by $v \in \mathbb{R}^6$, the underwater vehicle dynamic equation can be expressed in closed form as

\[
M \dot{v} + C(v) v + D(v) v + g(\eta) = \tau
\]

(2)

where $M$ is the inertia matrix including the added mass term, $C(v)$ represents the matrix of the Coriolis and centripetal forces including the added mass term, $D(v)$ denotes the hydrodynamic damping and lift force, $g(\eta)$ is the restoring force and $\tau$ is the vector of generalized forces acting on the vehicle. The dynamic equation in (2) preserves the following properties [6]:

Property 1: The inertia matrix $M$ is symmetric and positive definite such that $M = M^T > 0$.

Property 2: $C(v)$ is the skew-symmetric matrix such that $C(v) = -C^T(v)$.

Property 3: The hydrodynamic damping matrix $D(v)$ is positive definite, i.e.: $D(v) = D^T(v) > 0$.

Property 4: The dynamic model as described in (2) is linear in a set of dynamic parameters $\theta_d \in \mathbb{R}^{n_p}$ and can be written as

\[
\dot{v} + C(v) v + D(v) v + g(\eta) = Y_d(\eta, v) \theta_d
\]

(3)

where $Y_d(\eta, v, \dot{v}) \in \mathbb{R}^{n_p \times n_p}$ is a known regression matrix; $n_p$ is the total number of physical parameters. It is assumed that if the arguments of $Y_d(\cdot)$ are bounded then $Y_d(\cdot)$ is bounded.

3. Tracking Control Scheme Subject to the Union of Boundaries

In region boundary-based control, the desired moving target is specified by at least two sub-regions intersecting at the same point. The inner sub-region acts a repulsive region while the outer sub-region acts as an attractive region. The regulation control concept that has been presented in [5] is extended for coordination control of multiple AUVs [7]. A new proposed tracking control for multiple AUVs subject to the union of boundaries is formulated as follows:

First, a dynamic region of specific shape is defined so that all the vehicles are inside the region. This can be viewed as a global objective of all vehicles. The global objective function for outer sub-region and inner sub-region of MAUVs are described in terms of the body-fixed frame. The corresponding potential energy function for the desired sub-region describes in (4) can be specified as

\[
f_{\text{out}}(\delta \eta_i) \leq 0
\]

(4)

where $\delta \eta_i = \eta_i - \eta_d \in \mathbb{R}^6$ is the continuous first partial derivatives; $\eta_d$ is the time-varying reference point inside the region. The following inequality function can be used for the inner sub-region

\[
f_{\text{in}}(\delta \eta_i) \geq 0
\]

(5)

where the primary and secondary sub-regions share the same reference point, $\eta_d$. Note that, (4) and (5) are defined arbitrarily close to each other, such that

\[
f_{\text{out}}(\delta \eta_i) \approx f_{\text{in}}(\delta \eta_i)
\]

(6)

The corresponding potential energy function for the desired sub-region describes in (4) can be specified as

\[
P_{\text{in}}(\delta \eta_i) = \frac{k_{pi}}{2} \left[ \max(0, f_{\text{out}}(\delta \eta_i)) \right]^2 \leq \begin{cases} 0, & f_{\text{out}}(\delta \eta_i) \leq 0 \\ k_{pi}^2 f_{\text{out}}^2(\delta \eta_i), & f_{\text{out}}(\delta \eta_i) > 0 \end{cases}
\]

(7)

where $k_{pi}$ is a positive scalar for $i^{th}$ AUV. Similarly, the potential energy function for the inner sub-regions in (5) can defined as follows

\[
P_{\text{in}}(\delta \eta_i) = \frac{k_{si}}{2} \left[ \max(0, f_{\text{in}}(\delta \eta_i)) \right]^2 \leq \begin{cases} 0, & f_{\text{in}}(\delta \eta_i) \geq 0 \\ k_{si}^2 f_{\text{in}}^2(\delta \eta_i), & f_{\text{in}}(\delta \eta_i) < 0 \end{cases}
\]

(8)

where $k_{si}$ is a positive scalar for $i^{th}$ AUV. Differentiating (7) and (8) with respect to $\delta \eta_i$ gives

\[
\frac{\partial P_{\text{in}}(\delta \eta_i)}{\partial \eta_i}^T = k_{pi} \max(0, f_{\text{out}}(\delta \eta_i)) \left( \frac{\partial f_{\text{out}}(\delta \eta_i)}{\partial \eta_i} \right)^T
\]

(9)

\[
\frac{\partial P_{\text{in}}(\delta \eta_i)}{\partial \eta_i}^T = k_{si} \max(0, f_{\text{in}}(\delta \eta_i)) \left( \frac{\partial f_{\text{in}}(\delta \eta_i)}{\partial \eta_i} \right)^T
\]

(10)
Now, let (9) and (10) be represented as the primary region error $\tilde{e}_{pi}$ and secondary region error $\tilde{e}_{si}$ respectively in the following form

$$\tilde{e}_{pi} = \max(0, f_{out}(\delta \eta_i)) \left( \frac{\partial f_{out}(\delta \eta_i)}{\partial \delta \eta_i} \right)^T$$

(11)

$$\tilde{e}_{si} = \max(0, f_{ini}(\delta \eta_i)) \left( \frac{\partial f_{ini}(\delta \eta_i)}{\partial \delta \eta_i} \right)^T$$

(12)

Next, an edge-based segmentation approach is utilized to avoid the collision among the members of the group and to ensure each vehicle is placed at a desired target in the formation [7]. The potential energy function for a segmented boundary can be defined as follows

$$P_{edge}(\delta \eta_i) = \frac{k_e}{2} \max \left(0, f_{edge}(\delta \eta_i)\right)^2 \pm \begin{cases} 0, & f_{edge}(\delta \eta_i) \geq 0 \\ \frac{k_e}{2} f_{edge}(\delta \eta_i), & f_{edge}(\delta \eta_i) < 0 \end{cases}$$

(13)

where $k_e$ is a positive constant. Differentiating (13) with respect to $\delta \eta_i$ gives

$$\left( \frac{\partial P_{edge}(\delta \eta_i)}{\partial \delta \eta_i} \right)^T = k_e \max \left(0, f_{edge}(\delta \eta_i)\right) \left( \frac{\partial f_{edge}(\delta \eta_i)}{\partial \delta \eta_i} \right)^T$$

(14)

which leads to

$$\tilde{e}_{ei} = \max(0, f_{edge}(\delta \eta_i)) \left( \frac{\partial f_{edge}(\delta \eta_i)}{\partial \delta \eta_i} \right)^T$$

(15)

To implement the multiplicative potential energy, let $P_l$ be the potential energy function associated with region boundary $RB_l, l = 1, 2, 3, \ldots, L$

$$RB_1: P_1(\delta \eta_i) = P_{p1}(\delta \eta_i) + P_{s1}(\delta \eta_i) + P_{edge1}(\delta \eta_i)$$

$$RB_2: P_2(\delta \eta_i) = P_{p2}(\delta \eta_i) + P_{s2}(\delta \eta_i) + P_{edge2}(\delta \eta_i)$$

$$\vdots$$

$$RB_L: P_L(\delta \eta_i) = P_{pL}(\delta \eta_i) + P_{sL}(\delta \eta_i) + P_{edgeL}(\delta \eta_i)$$

(16)

where $L$ is the number of desired boundaries. A multiplication method [4] is adopted in this paper, thus the total potential energy $P_T$ associated with the desired boundary in (16) is defined by

$$P_T(\delta \eta_i) = \prod_{l=1}^{L} P_l(\delta \eta_i) = P_1(\delta \eta_i) \times P_2(\delta \eta_i) \times \ldots \times P_L(\delta \eta_i)$$

(17)

where $P_l$ is defined in (16). The desired boundary produced from this multiplicative of the potential energy is the union of all the boundaries $RB_l$ that is $RB = RB_1 \cup RB_2 \cup \ldots \cup RB_L$. Note that $P_T$ has a minimum value of zero when $\eta_i$ is within any of the desired boundaries. Equation (17) expresses that the potential energy is at the minimum value (zero) at the desired target. This potential function will ensure that the AUV move toward the overall region produced by union of all the boundaries $RB_1, RB_2, \ldots, RB_L$. This function is useful when the AUV need to adapt the moving boundary, depending on the situation and environment such as avoiding obstacle on its path.

Partial differentiating the total potential energy function described by (17) with respect to $\delta \eta_i$ leads to

$$\left( \frac{\partial P_T(\delta \eta_i)}{\partial \delta \eta_i} \right)^T = \left( k_{p1} \tilde{e}_{p1} + k_{s1} \tilde{e}_{s1} + k_{e1} \tilde{e}_{e1} \right) \prod_{l=1}^{L} P_l(\delta \eta_i) + \left( k_{p2} \tilde{e}_{p1} + k_{s2} \tilde{e}_{s1} + k_{e2} \tilde{e}_{e1} \right) \prod_{l=2}^{L} P_l(\delta \eta_i) + \cdots + \left( k_{pl} \tilde{e}_{p1} + k_{sl} \tilde{e}_{s1} + k_{el} \tilde{e}_{e1} \right) \prod_{l=L}^{L} P_l(\delta \eta_i) \pm \Delta \xi_i$$

(18)

where the product rule is used to obtain the derivatives of products of two or more functions. When the MAUVs are outside the desired boundary, the control force $\Delta \xi_i$ described by (18) is activated to attract the MAUVs toward the desired boundary. When the AUV is inside the desired boundary, then the control force is zero or $\Delta \xi_i = 0$.

Next, a vector $v_{ri}$ that is useful is defined

$$v_{ri} = f_i^{-1}(\tilde{\eta}_d - \delta \eta_i) - \alpha f_i^{-1} \Delta \xi_i$$

(19)

where $f_i^{-1}$ is the inverse of the Jacobian matrix, and $\alpha$ is a positive constant. The error term $\Delta \xi_i$ is given in (18). Based on the structure of (18) and (19) and the subsequent stability analysis, a filtered tracking error vector for multiple underwater vehicles is defined as

$$r_i(t) = v_i - f_i^{-1} \tilde{\eta}_d + \alpha f_i^{-1} \Delta \xi_i$$

(20)

In general, the development of the open-loop error system for $r_i(t)$ can be obtained by pre-multiplying the inertia matrix with the time derivative of $r_i(t)$ to yield

$$M_i \dot{r}_i + C_i(v_i) r_i + D_i(v_i) r_i + Y_{di}(\cdot) \theta_{di} = \tau_i$$

(21)
where
\[ Y_{di}(\cdot)\theta_{di} = M_i\dot{r}_i + C_i(v_i)\dot{r}_i + D_i(v_i)r_i + g_i(\eta_i) \] (22)
and the derivative of \( r_i \) in (22) is given as
\[ \dot{r}_i = f_i^{-1}\dot{\eta}_i + f_i^{-1}\dot{\eta}_d - \alpha_i f_i^{-1}\Delta \xi_i - \alpha_i f_i^{-1}\Delta \dot{\xi}_i \] (23)

Based on the error system development and the subsequent stability analysis, the proposed control law for MAUV is
\[ \tau_i = -f_i^T(\eta_i)\Delta \xi_i - K_{ui}r_i - f_i^T(\eta_i)\Delta \xi_i + Y_{di}(\cdot)\dot{\theta}_{di} \] (24)
where \( K_{ui} \) is constant positive matrix. The estimated parameters \( \dot{\theta}_{di} \) are updated using the following update law
\[ \dot{\theta}_{di} = -L_{di} Y_{di}^T(\cdot)\tau_i \] (25)
where \( L_{di} \) is symmetric positive definite. Substituting (24) into (21) produces a closed-loop dynamic for \( r_i(t) \) as follows
\[ M_i\dot{r}_i = -C_i(v_i)r_i - D_i(v_i)\dot{r}_i - f_i^T(\eta_i)\Delta \xi_i + Y_{di}(\cdot)\theta_{di} \] (26)
where \( \dot{\theta}_{di} = \dot{\theta}_{di} - \theta_{di} \) denotes the parameter estimation error. Next, the following non-negative function is introduced to analyze the stability of the proposed control law
\[ V = \sum_{i=1}^{N} \frac{1}{2} r_i^T M_i \dot{r}_i + \sum_{i=1}^{N} \frac{1}{2} \theta_{di}^T L_{di}^{-1} \dot{\theta}_{di} + P_T(\delta \eta_i) \] (27)

Differentiating \( V \) with respect to time and using the update law (25) yields
\[ \dot{V} = \sum_{i=1}^{N} r_i^T M_i \dot{r}_i - \sum_{i=1}^{N} \dot{\theta}_{di}^T L_{di}^{-1} Y_{di}^T(\cdot)\tau_i \]
\[ + \sum_{i=1}^{N} (k_{pi1} \dot{e}_{pi1} + k_{sli} \dot{e}_{sli} + k_{e11} \dot{e}_{e11})(\delta \eta_i)^T \prod_{l \neq i} P_l(\delta \eta_l) \]
\[ + \sum_{i=1}^{N} (k_{pi1} \dot{e}_{pi2} + k_{sli} \dot{e}_{sli} + k_{e12} \dot{e}_{e12})(\delta \eta_i)^T \prod_{l \neq i} P_l(\delta \eta_l) \]
\[ + \sum_{i=1}^{N} (k_{pi1} \dot{e}_{pi2} + k_{sli} \dot{e}_{sli} + k_{e12} \dot{e}_{e12})(\delta \eta_i)^T \prod_{l \neq i} P_l(\delta \eta_l) \] (28)

Utilizing equation (19), (20), a closed-loop dynamic (26) and cancelling the common terms leads to
\[ \dot{V} = - \sum_{i=1}^{N} r_i^T D_i(v_i)\dot{r}_i - \sum_{i=1}^{N} r_i^T K_{ui}r_i - \sum_{i=1}^{N} \alpha_i^2 \Delta \xi_i^T \Delta \xi_i \leq 0 \] (29)

where Property 3 is used. Now, a new theorem can be stated as follows:

**Theorem:** Given a closed-loop of MAUVs in (26), the proposed adaptive control law (24) and the update parameter laws (25) guarantees the convergence of \( \eta_i \) for all \( i = 1, 2, ..., N \) into a dynamic region boundary in the sense that \( \Delta \xi_i \to 0 \) and \( r_i \to 0 \), as \( t \to \infty \).

**Proof:** See [8] for proof.

**Remark:** The proposed dynamic control concept can be extended to the case of a scaling region boundary. In this case, continuous first partial derivatives in (4) needs to be exploited such that \( \delta \eta_{si} = S(\eta_i - \eta_d) \in \mathbb{R}^6 \); \( S(t) \) is a time-varying and nonsingular scaling factor that is defined as [9]
\[ S = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix} \] (30)
where \( S_1 \) is the scaling matrix of \( \eta_i \) and \( S_2 \) is the scaling matrix of \( \eta_2 \). The scaling of the orientation of MAUVs is not required in general, so \( S_2 \) can be set as an identity matrix. Thus, the scaling matrix \( S_1 \) is given by
\[ S_1 = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix} \] (31)
where \( s_x(t) \), \( s_y(t) \) and \( s_z(t) \) are scaling factors.

**4. Simulation study**

In this section, a simulation study is carried out to assess the efficacy of the proposed dynamic region boundary-based control law for multiple underwater vehicles. The vehicles are required to achieve a vertical line formation in this simulation. An ODIN with full 6-DOF [10] is chosen as autonomous underwater vehicle model for numerical simulation. In simulation, the following inequality functions are defined for a boundary of a spherical region
\[ f_{OUT_R} = (x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2 \leq \kappa_i^2 \] (32)
\[ f_{IN_1} = (x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2 \geq \kappa_i^2 \] (33)

while the subsequent inequality functions are defined for a boundary of a ellipsoid region

\[ f_{OUT_2} = (x_i - x_0)^2 + \left( \frac{y_i - y_0}{4} \right)^2 + (z_i - z_0)^2 \geq \left( \frac{\kappa_i^2}{4} \right) \] (34)

\[ f_{IN_2} = (x_i - x_0)^2 + \left( \frac{y_i - y_0}{4} \right)^2 + (z_i - z_0)^2 \geq \left( \frac{\kappa_i^2}{4} \right) \] (35)

Equations (32) and (34) represent the outer sub-regions and (33) and (35) denote the inner sub-regions. \( \kappa_i \equiv \kappa_i^- \) is a time-varying tolerance vector. A group of underwater vehicles are required to track a straight-line trajectory with green and magenta (cross-section lines) trajectory is the horizontal basis position initialized at \([1.45 - 0.6\ 0]\) m. The green and magenta designate spherical and ellipsoid regions, respectively. The solid blue lines represent the position of AUVs at various time instances.

In this simulation, the first vehicle is initialized to the position \( \eta_{11}(0) = [1.35 - 1 - 1]\) m while the second, third and forth vehicles are placed to the position \( \eta_{12}(0) = [1.4 - 1.3\ 1]\) m, \( \eta_{13}(0) = [1.4\ 0\ 1.5]\) m and \( \eta_{14}(0) = [1.35\ 0 - 0.5]\) m, respectively. The orientations of all AUVs are kept constant during simulation with the allowable errors, denoted by \( \kappa_x, \kappa_y \) and \( \kappa_z \), are set to 0.1 rad, and the initial values are \( \eta_{1i}(0) = [0\ 0\ 0]\) rad; \( i = 1, 2, 3 \) and 4. To ensure each vehicle move towards its desired position in triangular formation, the following inequality function is defined as

\[ f_{edge}(\delta\eta_{1i}) = \begin{cases} (x_i - x_0) + R_i x - \kappa_x \geq 0, \\ (y_i - y_0) + R_i y - \kappa_y \geq 0, \\ (z_i - z_0) + R_i z - \kappa_z \geq 0, \end{cases} \] (36)

where the vector \( [R_{i x} \ R_{i y} \ R_{i z}]^T \) for \( i = 1, 2, 3 \) and 4 is the circumradius for an \( i^{th} \) AUV. \([\kappa_x \ k_y \ k_z]^T \) is a tolerance vector. The control gains are set to the following:

\[ k_{pi} = 218; k_{ei} = k_{el} = 156.4; \alpha_l = 1; \]

\[ K_{al} = diag([2.08\ 2.08\ 2.08\ 1.1\ 1.1\ 1.1]) \]

As can be seen from Fig. 1 and Fig. 2, each AUV initially converges into the desired position on a boundary line to form a vertical line formation at various time instances.

5. Conclusion

In this paper, a new dynamic region boundary-based method has been proposed for a group of autonomous underwater vehicles. This control technique enables the multiple AUVs to perform a specific formulation while undergoing an underwater tracking task. It has been shown that MAUVs are able to track a desired moving boundary produced by the union of two or more boundaries. Moreover, each AUV navigates into a specific location on the boundary lines or surfaces while the target itself is moving. It is achieved using an edge-based segmentation approach. The Lyapunov-like function is used to analyze the stability of the controller. Simulation results have been presented to demonstrate the performance of the proposed tracking controller.
Fig. 2. Planar trajectories of MAUVs that are illustrated in (a) XY-plane, (b) XZ-plane and (c) YZ-plane.

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References