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Forecasting Company Financial Distress Using the Gradient Measurement of Development and S-Curve

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Abstract

This paper shows that regardless of how good the financial situation is, sooner or later certain difficulties will appear. Hence, the idea has emerged to give the analytical form to the logistic law based gradient measurement (synthetic measure) of selected financial data, which enables forecasting financial distress. The presented proposal of determining the company development and the model of forecasting using the S-curve was tested with use of financial data regarding the two Polish company (with financial trouble and with good situation).

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1. Introduction

The aim of a warning forecast is to signal “early enough” unfavorable changes in selected business activity areas, described by time series. A warning forecast is, by nature, a long-term forecast; its characteristic feature is the fact that it does not give values of variables but only a warning against the possibility of unfavorable changes occurring.

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Warning forecasting consists in forecasting a decrease in business activity. On one hand, enterprises should be recipients of such forecasts, especially when they are preparing strategic targets of their activity. At the same time they should thoroughly assess the financial rates.

The warning forecast is constructed for any time series, whose correct trend is increasing. In practice, it rarely happens that financial quantities, describing a financial activity constantly increase, the growth period is followed by a stable or a decrease period. The warning forecasting’s task is to predict occurrence of a phase of decreasing values in the series. This is why the warning forecast is defined as follows Siedlecki (2006):

“The warning forecast is a formulated assumption based on the information given by time series, that in the next moment \( T_0 \) the state of the analyzed financial phenomenon will be lower than in \( T_0 - 1 \) moment.

The warning forecast formulated in \( T=n \) moment is true, when time series terms meet the condition:

\[
y^*_n - y^*_{T_0-1} < 0 \quad T_0 > n
\]

where \( y^*_n \) – future real series value”.

The truth of the warning forecast should be formed not on the basis of raw time series burdened with random errors, but on the basis of a smoothing function. The choice of the correct smoothing function \( f(T) \) has significant meaning in the warning forecast. In such case the condition of the truth of the warning forecast is as follows:

\[
f(T_0) - f(T_0 - 1) < 0
\]

Inequalities are the basic determination of the truth of the warning forecast. The warning forecast is de facto a quality forecast.

\( T_0 \) moment is called a warning forecast horizon. Too short a warning forecast horizon leads to low usability because there is not enough time for repairing the process performance.

In practice, the analysis and the warning forecast are not usually based on one time series, but on the whole series bundle, describing a selected fragment of the examined phenomenon.

In a company, the horizon of the warning forecast, which is the beginning of an unfavorable situation is the change in the sign of second differences of a trend function of a selected series – signaling devices.

A sign sequent of its first differences can be created for any function in \([1,n]\) period. In a similar way a sign sequent of \( f(t) \) function second differences can be created. Let ‘+’ mean positive (non-negative) and ‘−’ means negative first (or second) difference of such \( f(t) \) function. A warning occurs when in a smoothing function’s growth, its second differences show the change of the sign from ‘+’ into ‘−’, the warning disappears when the differences change the sign from ‘−’ into ‘+’.

The warning is permanent in time interval, starting from the point of inflection, via a maximum to a minimal point of smoothing function. A very rough analysis of the graph allows us to think that in the near future a decreasing growth of the series will be maintained.

2. Literature Review

To analyse and to forecast the warning construction it is best to apply an aggregated indicator that not only describes liquidity and debt, but also operational efficiency and profitability. In the long term we apply a synthetic measurement that would describe adequately the financial condition of the company. The choice of such an indicator-unit was first used in the multivariate statistical analysis method (multiple discriminant analysis) initiated by Beaver(1966), Altman(1968, 1983), Altman E., Haldeman R, Narayanan P. (1977). The effectiveness of the model and its popularity have made many researchers study the creation of such models indifferent countries. Altman’s most popular model modifications were Edminster’s model for small companies model, Ohlson (1980)

1 To make it easier we assume that the economic values which should determine fall (destimulant) can be transferred into stimulant with the use of certain methods. So, growing time series is an abstraction expressing desired course of an economic phenomenon.
was the first to apply logit models (logistic regression) and Źmijewski (1984) used probit models. From 1990 till today, the researchers created several models to forecast financial difficulties. The most important are Artificial Neural Networks ANN (Odom, Sharda 1990), k-nearest Neighbor KNN (Kerling, Podding 1994), Expected Default Frequency EDF (Crosbie, Vasicek 1997) and Support Vector Machines SVM (Vapnik 1995). Recent studies in financial distress forecasting have been very interesting: ANN (Ravishankar, Raval 2010, Sun, Shenoy 2007), EDF (Sommar, Shahnazarian 2009), SVM (Chen, Chen, Hsieh 2011).

We present a proposal of the model of forecasting a financial distress constructed with the taxonomic method of development based on the gradient distance (Siedlecka, Siedlecki (1990), Siedlecki, Papla (2013)).

The author proposes a method of determining a model of forecasting financial distress and financial downturns using the gradient measurement (synthetic measure) and the logistic function (logistic law) which was tested on the Polish two famous Polish companies. In this paper two methods of warning signal (possibility of crisis) estimation are used critical values of gradient measures and strategic bands.

3. The use of gradient methods for the analysis of financial ratios

A gradient method based on the determination of the taxonomic distances of examined objects is interesting for the purposes of warning forecasting and constructing a synthetic indicator. The method may also apply respective financial ratios of the object model.

Let us assume that the matrix X comprises financial ratios (observations of the studied phenomenon), which are stimulants $x_{it}$, where $i$ (number of ratio) = 1, 2, ..., $m$, $t$ (time) = 1, 2, ..., $n$, and $x_{it} \in \mathbb{R}$:

$$X = \begin{bmatrix} x_{i1} & \cdots & x_{in} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \quad (2)$$

And assume that two data points (poles) are determined:

Top – a pattern of development

$$P = \{p_{10}, \ldots, p_{m0}\} \quad (3)$$

Bottom

$$Q = \{q_{10}, \ldots, q_{m0}\} \quad (4)$$

Where

$$p_{01} = \max_t x_{it} \text{ and } q_{01} = \min_t x_{it} \quad (5)$$

Then QP segment defines the axis of a set of synthetic indicators. PQ vector gradient can be treated as a linear programming criterion function $\Phi(t)$:

$$\Phi(t) = \begin{bmatrix} p_{10} - q_{10} \\ \vdots \\ p_{m0} - q_{m0} \end{bmatrix}^T \begin{bmatrix} x_{i1} & \cdots & x_{in} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \quad (6)$$

The function $\Phi(t)$ shows the orthogonal projection of points. The values of this function are ordered synthetic indicators, where at any given time (point) $t$ the following value is obtained:

$$\varphi(t) = (p_{t0} - q_{t0}) \cdot x_{it} \quad (7)$$

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2 see Siedlecki, Papla (2013).
3 Destimulants and nominants have to be converted into stimulants.
In the analysis of financial indicators, in order to eliminate the effect of their size on the arrangement, conversions should be made from matrix X to Z by the unitarization

\[
Z_{it} = \frac{x_{it} - \min(x_{it})}{\max(x_{it}) - \min(x_{it})}
\]

(8)

If the zero unitarization method is employed (formula 8), points P and Q will have the following form: \( P = [1, ... 1], Q = [0, ... 0] \). In this case (see fig 1):

\[
\varphi(t) = \sum_{i=1}^{m} z_{it}
\]

(9)

And the measure of development (\( \mu_t \)) will be:

\[
\mu_t = \frac{\varphi(t)}{m}
\]

(10)

![Fig. 1. Orthogonal projection on the gradient for unitarized data](image)

If \( \mu_n \) (last measure) is close to 1, it denotes a good financial position. If it is closer to 0, the likelihood of financial distress is greater.

Assuming that the number of periods in is greater than 6 and the normalization method will be used for the predicted values base on pattern of observations of the studied phenomenon, for 2–4 years forecast horizon then the interpretation of \( \mu_t \) is as follows:

- \( \mu_t < 0.5 \) distress area, high probability of economic crisis,
- \( 0.5 < \mu_t < 0.7 \) grey area (warning signals – economic downturn),
- \( \mu_t > 0.7 \) safe area

The warning in the company forecasts model is the state of the phenomena on at time t determined on the basis of the relevant economic ratios describing its development. So a measure of the development of a proper conduct should show the following relationship: \( \varphi(1) < \varphi(2) < ... < \varphi(n) \), (or \( \mu_1 < \mu_2 < ... < \mu_n \)) which resembles a log-logistic (or logistic) curve, thus defining the development cycle of the economy (see Fig. 2).
Warning signals defined on the basis of the continuing decline in the value of measure $\mu_t$ or a decline hardly as shown in Figure 3. When analysing the trajectory of the company we should take into account the development of interference as a result of such changes in the economy (the economic downturn) and sector restructuring as well as random events.

The analysis measures the development of the financial distress. One way is to analyse the stability of increments and the volatility of financial ratios. So it can be argued that the warning signals decrease or increase beyond the allowable deviation from the trend function (modified S-curve – log-logistic), designated by strategic bands.

4. Warning signals analysis based on two Polish Companies

Warning signals tracing, using a financial cycle of development should include: selection of suitable financial data, a suitable function for its forecast and variation and stability of the increment analysis.

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4 Source: own study based on Argenti (1976)
5 Source: own study based on Argenti (1976)
One of the most important tasks in this case is to select the financial data and match a suitable function. The selection mainly depends on a given company branch, the period for which warning signals are supposed to be generated, and on a subjective appraisal of a person analyzing those signals.

One of the ways of financial warning signals analysis, based on development of a company cycle, is to analyze the increment stability and variation of selected financial parameters.

The concept of warning forecasts uses the annual values of selected indicators for listed Polish companies which are in different phase of development (before financial crisis in 2008):

- KGHM SA – one of the largest and most well-known Polish listed companies.
- Krosno SA – the company in bankruptcy recently on the Polish market.

The analysis, based on the literature (e.g. Altman, E. (1983), Ravisankar, P. & Ravi, V. (2010)) and own studies, selected 7 (from 15) indicators using the method of distribution convexity:

- 0-unitarization method
- Arrangement of data in ascending order
- Determination of the frequency \( w_{ij} = \frac{1}{m} \)
- Determination of the ratio \( \vartheta_j = 1 - \sum_{x_{ij} \leq 0.5} \), and \( w_{ij} < 0.5 \)

Table 1 presents selected data for the Polish companies. Nominants and destimulants have been converted into stimulants respectively:

- nominants:
  \[ x_{it} = -|x_{it} - \text{median}(x_{i})|, \]

- destimulants:
  \[ x_{it} = \max_{t} x_{it} - x_{it}. \]

Table 1. Presents selected data for chosen companies.

<table>
<thead>
<tr>
<th>KGHM</th>
<th>ROI</th>
<th>Current ratio</th>
<th>Receivables turnover</th>
<th>EV/Debt</th>
<th>TIE</th>
<th>Revenue growth</th>
<th>Cfop/EV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.44</td>
<td>0.44</td>
<td>0.22</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.33</td>
</tr>
<tr>
<td>V</td>
<td>0.546904</td>
<td>0.600907</td>
<td>0.760914</td>
<td>0.916523</td>
<td>0.975367</td>
<td>0.850245</td>
<td>0.600845</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Krosno</th>
<th>ROI</th>
<th>Current ratio</th>
<th>Receivables turnover</th>
<th>EV/Debt</th>
<th>TIE</th>
<th>Revenue growth</th>
<th>Cfop/EV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.43</td>
<td>0.43</td>
<td>0.43</td>
<td>0.29</td>
<td>0.43</td>
<td>0.43</td>
<td>0.43</td>
</tr>
<tr>
<td>V</td>
<td>0.837841</td>
<td>0.673266</td>
<td>0.696971</td>
<td>1.173815</td>
<td>0.847471</td>
<td>0.844351</td>
<td>0.517368</td>
</tr>
</tbody>
</table>

On the basis of formulas 6, 8, 9 gradient measurement \( \mu_t \) is assigned respectively:

- KGHM SA determining \( \mu_t \) based on 2001–2008 and 2001–2009

The parameters of gradient measurement \( \mu_t \) presented in Table 2 and figures 4, 5.
Table 2. Gradient measurement $\mu_t$ for KGHM and Krosno.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>0.121734</td>
<td>0.121734</td>
<td>0.17228</td>
<td>0.271507</td>
</tr>
<tr>
<td>2002</td>
<td>0.253137</td>
<td>0.25294</td>
<td>0.362054</td>
<td>0.404798</td>
</tr>
<tr>
<td>2003</td>
<td>0.334939</td>
<td>0.334486</td>
<td>0.561442</td>
<td>0.609729</td>
</tr>
<tr>
<td>2004</td>
<td>0.394368</td>
<td>0.392628</td>
<td>0.789805</td>
<td>0.804203</td>
</tr>
<tr>
<td>2005</td>
<td>0.571945</td>
<td>0.568923</td>
<td>0.528624</td>
<td>0.632489</td>
</tr>
<tr>
<td>2006</td>
<td>0.708316</td>
<td>0.702417</td>
<td>0.361442</td>
<td>0.361047</td>
</tr>
<tr>
<td>2007</td>
<td>0.780435</td>
<td>0.774149</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>0.545068</td>
<td>0.538894</td>
<td></td>
<td>−0.5419 (base on period 2001–2007)</td>
</tr>
<tr>
<td>2009</td>
<td></td>
<td></td>
<td>0.615741</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td></td>
<td></td>
<td>0.648943 (base on period 2001–2009)</td>
<td></td>
</tr>
</tbody>
</table>

As shown in Table 2 and Figure 4 Krosno in 2005, the company generated a weak signal warning in 2006 (and 2007 was transformed into a strong signal. The trajectory of development resembles a bankrupt company (see fig 3). In 2009 Krosno went bankrupt.

Fig.4. Warning signals for Krosno SA respectively for 2005 and 2007

The trajectory of a healthy company is presented by KGHM. This company in 2008 had a weak warning signal, which could be a result of the economic situation in Poland and in the world.
5. Warning signals analysis of KGHM’s trajectory

In this point we are examining the trajectory of KGHM using a logistic function and strategic bands from 2001-
2010 defined by the trajectory and the raised lower limit (strategic bands) determined on the basis of RSME (Root
Mean Square Error). The logistic function (Hellwig, Siedlecki 1989, Siedlecki 2011) expressed by the following
formula:

\[ f(t) = \frac{a}{1 + e^{b-c}^t + d}, \]

To determine the phases of a logistic growth we will use measurement \( A(\tau) \) which is expressed by following
formula:

\[ A(\tau) = \begin{cases} \frac{y_t}{a}, & y_t < 1 \\ \frac{a}{1}, & otherwise \end{cases} \]

Where: \( y = \max_{t<\tau} x_t \)

When examining the logistic function, we can show its basic properties

\[ A(\tau) = \frac{1}{1 + e^{b-c}^\tau} \]

The interpretation of \( A(\tau) \) is as follows
- Start up: \( A(\tau) < 0.15 \),
- Intensive growth: \( 0.15 < A(\tau) < 0.85 \),
- Stable growth: \( A(\tau) > 0.85 \)

The measure \( \mu_t \) of KGHM decreased in 2008 and then went back in 2009-2010 to the logistic function. The
application of logistic to define and forecast KGHM development based on measurement \( \mu_t \) has been shown in
Fig. 5 and 6. The figures show 2001–2007 (when the trajectory was valid) smoothing, forecasting phases of growth
using the discussed logistic up to the pattern (The residuals are stationary. Test statistic (KPSS test) = 0.373369):

\[ f(t) = \frac{0.730887}{1 + e^{0.2713658-0.7597}^t} + 0.104701, \]
Where RSME=0.07232 and

\[ A(\tau) = \frac{1}{1 + e^{3.2713658 - 0.7593t}} \]

KGHM, which in the tested period, signals the transition to another growth (mature) phase, the sign of transition is the \( A(\tau) \) measurement higher then 0.85 in 2007 (weak warning signal) which means that the growth rate significantly slowed. The transition to intensive was signaled in 2002 phase (after the economic slowdown of 2000–2001). The analysis of strategic bands shows, in a forecasting period that weak warning signals were generated in 2008 (financial crisis in global market). However, it has not been confirmed in the coming years, because the measurement \( \mu_t \) went back in 2009–2010 to the healthy trajectory.

![Graph 6: Strategic bands forecast for 2008–2010 for KGHM based on \( \mu_t \) in 2001-2007](image)

![Graph 7: Phase of growth estimation for KGHM based on 2001–2007](image)
6. Conclusion

According to the analysis presented in the paper, it was found that forecast warnings based on the proposed model formed well in advance of the coming financial risks for Krosno SA and normal development for KGHM. As we can see from the analysis, the selected financial parameters are very useful for assessing the financial companies. Company managements can use them for the purposes of strategic development planning. The use of gradient measurement $\mu_t$ seems an easy and efficient tool to predict the warning signals. This measure also allows you to identify the most favourable moment for the company and also allows you to identify the phases of the business with the use of the logistic function (Siedlecki 2012). A measure of development is also a very good tool for grouping companies and dividing them into bankrupt and healthy companies.

The presented analysis of the development and the setting of the phases of business development based on the logistic function can be extended to the analysis of its first and second derivatives and testing for more companies.

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