

9th Conference of the International Sports Engineering Association (ISEA)

The validity of a rigid body model of a cricket ball-bat impact

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Accepted 02 March 2012

Abstract

Rigid body impact models have been used in a number of racket and bat sports to better understand how physical properties such as mass, moment of inertia and balance point can affect ball rebound speed. Cricket is sport whereby players can select their preferred bat with a wide range of different physical properties. No previous studies have attempted to validate the use of rigid body impact models in cricket, and player choices are typically made through intuition with little consideration of impact mechanics. This study measured the performance of three different cricket bats in freely suspended impact tests, and compared the results to predictions made by a rigid body model. Ball rebound speed was measured using high speed video on impacts locations across the blade. The physical properties of the different bats were measured and used as the input for the rigid body model predictions. It was found that for impact locations close to the bat's centre of mass, the rigid body model worked well, but some discrepancies were found as the impact location moved away from the centre of mass. These discrepancies were believed to be caused by the large vibrations evident during the impacts (a clear violation of the model's rigid body assumption) and the erroneous method that was employed to measure the bats coefficient of restitution. It was concluded that using a rigid body model to describe the impact of a cricket ball with a cricket bat is valid as a first approximation and that it has significant value in terms of exploring how changing a bat's physical properties may affect its performance.

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Keywords: Cricket; freely suspended; impact; rigid body modelling

1. Introduction

The game of cricket has evolved over the last 40 years with the introduction of new, shorter formats (e.g. one day, Twenty20). Consequently, the cricket bat has seen many innovations in an attempt to allow the batsmen to produce results in keeping with changing tactical demands. For example, the shorter game requires the batsman to score faster and strike more powerful shots more often. Anecdotally, many players choose lighter bats for long test matches and heavier bats for the more attacking roles required in one day matches and Twenty20 games.

Other than experience and personal preference, currently there is no method for a player to select an appropriate bat that will produce optimum results for a particular format of the game. Cricketers would be keen to know how their performance might change for bats of varying physical characteristics, such as mass, balance point and moment of inertia. As well as affecting the efficiency of the ball-bat impact, these physical properties also affect the amount of effort required by the batsman to swing the bat, and to control it during the swing [1].

Rigid body models have been used extensively to predict equipment behaviour in sporting disciplines such as tennis [2, 3] and baseball [4]. Researchers have produced rigid body models of the bat/racket to predict how performance will change with different physical characteristics thereby providing insights for design optimization and innovation. Rigid body models assume that the material from which the implement is made is uniform throughout, and is infinitely stiff. They are unable to account for energy losses that occur from deformation and vibration. Nonetheless, rigid body models have been proven to be effective in sport applications, and provide a solid platform from which more complex models can be developed.

Previous studies have investigated the cricket bat's inertial and vibration characteristics [5] and methods such as finite element analysis and modal models have been also been used to investigate design performance [6]. However, no previous authors have attempted to model the cricket ball-bat impact through rigid body techniques, despite its widespread use in other sports. It appears that authors have adopted complex modeling techniques before first considering more simple approaches.

Therefore, the purpose of this study is to apply a rigid body model to a range of different cricket bats, and to then attempt to validate the model's accuracy through experiment.

2. The model

The model using in this research is adapted from the work of Brody on tennis ball-racket impacts [2]. It uses the principles of conservation of linear and angular momentum to determine the velocity of the cricket ball after impact with a freely suspended cricket bat. The rebound velocity of a ball after impact is commonly used as measure of performance for freely suspended impacts and is therefore the parameter of primary concern [7, 8]. The model is one dimensional assuming that all impacts will occur on the central axis and no twisting will occur in the polar plane. By conservation of linear momentum,

$$m_b v_b + MV = m_b v'_b + MV' \quad (1)$$

where m_b is the mass of the ball, M is the mass of the bat, v_b and v'_b are the initial and final ball velocities, respectively and V and V' are the initial and final bat speeds, respectively, measured at the centre of mass (COM).

Conservation of angular momentum is described by the equation;

$$I\omega + m_b v_b z = I\omega' + m_b v'_b z \quad (2)$$

where I is the moment of inertia about the COM, ω and ω' are the angular velocities of the bat before and after impact, and z is the distance from the COM to the impact location.

The bat velocity at the impact point, VIP, is;

$$V_{IP} = V + \omega z \quad (3)$$

Using the definition of the coefficient of restitution;

$$e = \frac{V'_{IP} - v'_b}{v_b - V_{IP}} \quad (4)$$

where V'_{IP} is the velocity of the impact point of the bat after impact, it is possible to combine equations 1 to 3 to solve for the final ball velocity,

$$v'_b = \frac{v_b \left(\frac{m_b}{M} + \frac{m_b z^2}{I} - e \right) + V(1+e) + \omega z(1+e)}{\left(1 + \frac{m_b}{M} + \frac{m_b z^2}{I} \right)} \quad (5)$$

where e is the coefficient of restitution.

For a freely suspended bat, terms involving V and ω will become zero, so the equation 5 reduces to;

$$v'_b = \frac{v_b \left(\frac{m_b}{M} + \frac{m_b z^2}{I} - e \right)}{\left(1 + \frac{m_b}{M} + \frac{m_b z^2}{I} \right)}. \quad (6)$$

3. Determining model parameters

The rigid body model uses a set of fixed parameters that describe the physical properties of the bat. These parameters are bat mass, moment of inertia about the COM, coefficient of restitution, and the location of the COM. By knowing these parameters the model is able to predict the rebound velocity of the ball at any known impact location along the blade.

The moment of inertia was found by using methods similar to those described by Brody [9]. A rod was attached to the end of the handle of the bat. The rod was placed on parallel knife edges which allowed the free swing of the bat, and the time period of one oscillation was determined from measuring 10 oscillations and calculating the mean. The bat was released from angles no larger than 10° so that small angle approximation applied. The moment of inertia about the handle end can be found using,

$$I_{handle} = \frac{T^2 m_b g d}{4\pi} \quad (7)$$

where m_b is the mass of the bat, g is the acceleration due to gravity, T is the period of oscillation and d is the distance from the handle end to the centre of mass. However, this method provides the moment of inertia about the handle end, and not about the COM as required. To rectify this, the parallel axis theorem was used to translate the moment of inertia to the axis desired.

Bat mass, and the location of its centre of mass were measured using bespoke equipment developed by the Centre for Sports Engineering Research at Sheffield Hallam University. The apparatus is able to measure mass to within 0.0001 kg and the location of COM to within 0.0001 m.

The coefficient of restitution was determined by firing a bola practice cricket ball at the freely suspended bat. The ball was launched using a bowling machine and was targeted on the bat's centre of mass. The ball's inbound velocity, rebound velocity and the bat's recoil velocity were all measured using high speed video techniques and the coefficient of restitution was calculated using the equation below;

$$e = \frac{v_b' - V'}{v_b} \quad (8)$$

where, V' is the recoil velocity of the bat's centre of mass.

4. Validation experiments

The rigid body model was evaluated by conducting impact experiments on three, brand new, high quality cricket bats with a range of physical properties. The physical properties of the bats were ascertained using the methods described in the previous section, and are summarized in Table 1.

Table 1. A summary of the physical properties of the three cricket bats tested. Centre of mass location is measured from the end of the handle

	Mass (kg)	Centre of mass location (m)	Moment of inertia (kgm^2)	Coefficient of restitution
Bat 1	1.175	0.491	0.0652	0.511
Bat 2	1.161	0.527	0.0558	0.519
Bat 3	1.090	0.518	0.0544	0.509

The bats were freely suspended on a rigid frame and supported by a pin at the end of the handle. Bola practice cricket balls of average mass 80 grams were launched at the cricket bats using a Bola bowling machine. A Phantom v4 high speed video system was positioned perpendicular to the impact plane to measure the inbound and rebound velocity as well as the impact location. A second high speed video system was positioned behind the bowling machine in the same orientation as the impact plane to determine whether the balls struck the bats in the centre of their blades. Impacts that did not strike the bat in the centre of the blade were discarded as they would require additional degrees of freedom to model. The experimental setup allowed for the height of the bat to be incrementally changed such that impacts could be measured at various locations along the blade. The resulting high speed video footage was calibrated and manually digitized using bespoke software. The speed of the bowling machine remained fixed throughout testing and the ball impact velocity averaged 27.7 ms^{-1} with a standard deviation of 0.3. Approximately 25 successful impacts were recorded on each bat.

5. Results

Figure 1 shows comparisons between the experimentally measured ball rebound velocities and the predicted rebound velocities from the rigid body model for the three different cricket bats. All three plots show a generally good agreement between the experimentally measured ball rebound velocities and the predicted rebound velocities. The experimental results show Bat 1 to have the highest maximum ball rebound velocity, and this is matched by the model. Equally, the experimental data shows Bat 2 to perform better during impacts towards its toe, and this is also matched by the model.

For each bat, the maximum rebound velocity occurs at the centre of mass. As the impact location moves away from the centre of mass, the ball rebound velocity diminishes rapidly due to a reduction in the bat's effective mass. Each bat has a subtly different modeled profile due to its different mass, centre of mass location, moment of inertia, and coefficient of restitution. As the coefficient of restitution was

similar for all three bats it is not surprising that the heaviest bat (Bat 1) was found to possess the maximum predicted rebound velocity.

In all three cases, there is a better agreement at the higher rebound velocities (impacts close to the centre of mass) as compared to the lower rebound velocities (impacts at a distance away from the centre of mass). The good agreement at impacts close to the centre of mass is to be expected as the coefficient of restitution was measured at this point. For impacts away from the centre of mass, the model tends to over predict the rebound velocity of the ball.

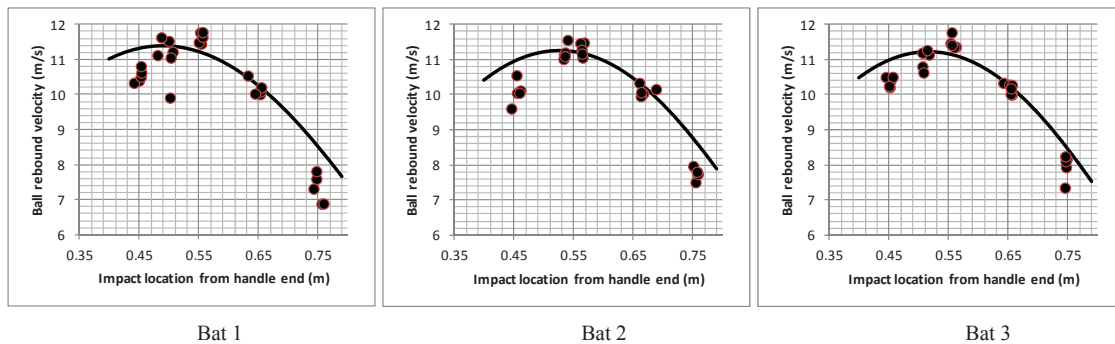


Fig. 1. A comparison of experimentally measured ball rebound velocities and the rigid body model predictions for all three cricket bats

6. Discussion

In these experiments, the coefficient of restitution was determined by firing a ball at the centre of mass of the freely suspended bat. As Cross describes in his work on rigid body models [10], the coefficient of restitution changes depending on the ball impact location on a freely suspended bat or racket. In this instance, the coefficient of restitution that was measured is likely to be lower than its true value since it will have been somewhat affected by the large anti-node vibrations that occur at the centre of mass of a cricket bat. With hindsight it would have been more appropriate to measure the coefficient of restitution by rigidly clamping the bat in a suitable bespoke frame.

Despite the erroneous method that was employed to measure the bats' coefficient of restitution, the rigid body model performs remarkably well. When analyzing the high speed video footage it is evident that the bat undergoes significant vibration during impact with large amplitude. It is therefore somewhat surprising that the rigid body model works as well as it does. Presumably the elastic properties of wood allows for vibrations and deformations of the system without causing significant losses of energy. Therefore, the rigid body approximation 'works' since the vibrations of the bat do not considerably reduce the ball rebound velocity.

This study validates the rigid body method as a first approximation for the cricket ball-bat impact and opens the door for the method to be used as a tool to explore how changing a bat's physical properties will affect its overall performance. However, the model does not account for how the player will inevitably find it harder to swing a heavier bat, or a bat with a larger moment of inertia. Understanding how different players are able to swing bats with different physical properties is an essential, and yet missing piece of the equation if one is to be able to find the optimum bat characteristics for a particular style of play. Understanding the relationship between a cricket bat's physical properties and swing speed is the focus of future research.

7. Conclusions

The experimental results reported in this paper demonstrate that using rigid body models to describe the impact of a cricket ball with a cricket bat is a valid first approximation. Generally good agreement was found between experimental results and model predictions. The method serves as a valid and potentially useful tool for exploring how changing a bat's physical properties will affect its overall performance.

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