9th Conference of the International Sports Engineering Association (ISEA)

Computational Homogenization Analysis Applied to Hyperelasticity for Porous Polymers

Akihiro Matsuda\(^a\), Nobuo Kawasaki\(^b\)

\(^a\)Faculty of Engineering, Information and Systems, University of Tsukuba, Tennodai 1-1-1, Tsukuba, Ibaraki, 3058573, Japan
\(^b\)Graduate School of Systems and Information Engineering, University of Tsukuba, Tennodai 1-1-1, Tsukuba, Ibaraki, 3058573, Japan

Accepted 02 March 2012

Abstract

The porous polymers are applied to sports equipment like a shoe sole, shock protector and grip tape of racquets. To analyze the mechanical properties of porous polymers under large deformation, the formulation of the homogenization analysis of hyperelasticity and mechanical loading test results of porous polymers for sports equipment were described in this paper. The computational simulation program of the hyperelasticity based on the homogenization method was newly developed. In order to evaluate the pore ratio on the mechanical characteristics of porous polymers, unit cell computation of microscopic structure were performed. In the mechanical loading tests, fundamental properties of porous polymers for sports equipment were shown to investigate the effect of microscopic pores diameter and material stiffness on the macroscopic mechanical characteristics. By the comparing the numerical simulation with experimental results, applicability of the proposed method to the porous polymer was shown.

© 2012 Published by Elsevier Ltd. Open access under CC BY-NC-ND license.

Keywords: Homogenization analysis; porous polymers; computer simulation; unit-cell calculation

1. Introduction

Porous polymers have developed in 1950s. Now, it has been applied to the shoe-sole, shock-protector, grips for sports equipment etc. The stiffness of the porous polymer depends on the original stiffness of polymer resin and microstructure (in Fig 1(a)). The homogeneous methods for the material which has periodic micro-structures like micro-pores in the porous polymers have been investigated numerically\([1][2][3][4]\).

* Akihiro Matsuda. Tel.: +81-29-853-5031 ; fax: +81-29-853-6103 .
E-mail address: a_matsuda@kz.tsukuba.ac.jp .
In this paper, a large-deformed homogenization method for polymer materials including micro-pores is investigated to propose a design method of mechanical characteristics of porous materials for sports by numerical analysis. In the formulation, uniform macroscopic deformation and periodic microscopic deformation were separated to consider microscopic structures on the macroscopic stress-strain relationships. To represent the large deformation of microscopic skeletal walls, the hyperelasticity was applied to the material modeling.

Then, uniaxial loading tensile tests of porous polymer for sports-equipment were conducted to get the stress-strain relationships, pore ratio of which were from 26% to 76%. The experimental and numerical predictions of stiffness reduction due to the pore ratio were compared to verify the applicability of our proposing method.

2. Formulation of Homogenization Method

The elastic media which have the periodic microstructures were supposed for the numerical simulation. In this section, the formulation of homogenization analysis for large deformation problem of a porous polymer was described with two different coordinates $X$ and $Y$. Here, $X$ is the macroscopic coordinate and $Y$ is the microscopic coordinate. The macroscopic behavior was defined in the macroscopic coordinate $X$, and the microscopic behavior was defined in the microscopic coordinate $Y$. The two coordinates are related by the scale rate $\varepsilon$ as follows:

$$ Y = \frac{X}{\varepsilon} \quad (1) $$

If the scale of microscopic structure is very small compared with the macroscopic structure, the scale rate $\varepsilon$ is very small. Therefore, macroscopic characteristics such as stiffness, stretch, stress and strain are given by the average on the microscopic structures. Also, the all microscopic structures are supposed to deform simultaneously. The total displacement of microstructure $y$ was separated to the uniform macroscopic displacement $Y$ and periodic microscopic displacement $w$ (in Fig 1(b)) as follows:

$$ y = \tilde{Y} Y + w \quad (2) $$
Here, $\bar{\mathbf{F}}$ is the deformation gradient tensor of macroscopic body and it is given as the boundary condition of numerical simulation. The microscopic deformation gradient is calculated from Eq. (2) as follows:

$$\mathbf{F} = \nabla \mathbf{y} = \bar{\mathbf{F}} + \frac{\partial \mathbf{w}}{\partial \mathbf{Y}}$$

(3)

The rate of displacement is also described by

$$\mathbf{v} = \dot{\mathbf{x}} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} \mathbf{Y} + \mathbf{w}$$

(4)

The velocity gradient tensor is calculated as follows:

$$\mathbf{L} = \mathbf{\tilde{L}} + \mathbf{L}_y$$

(5)

Here, $\mathbf{L}$ is the macroscopic velocity gradient tensor, $\mathbf{\tilde{L}}$ is the velocity gradient tensor given by macroscopic deformation and $\mathbf{L}_y$ is the velocity gradient rate tensor of periodic deformation. Therefore, deformation rate tensor $\mathbf{D}$ is calculated from above relation,

$$\mathbf{D} = \frac{1}{2} \left( \mathbf{L} + \mathbf{L}^T \right) = \mathbf{\tilde{D}} + \mathbf{D}_y$$

(6)

To describe formulation of the homogenization analysis applied to the large deformation problem of porous polymers, the following principle of virtual work in rate form is introduced. The periodic microscopic displacement rate $\mathbf{w}$ was solved in the FEM code and the total rate of displacement was given by Eq. (4).

$$\int \left[ \mathbf{\tilde{T}}_r + \mathbf{T} : (\nabla \mathbf{v} \cdot \delta \mathbf{v}) \right] d\mathbf{V} = \int \mathbf{g} \cdot \delta \mathbf{w} d\mathbf{V} + \int \mathbf{i} \cdot \delta \mathbf{v} d\mathbf{S}$$

(7)

Here, $\mathbf{\tilde{T}}_r$ is the Truesdell stress rate, $\mathbf{g}$ is the rate of body force and $\mathbf{i}$ is rate of traction on the boundary $\mathbf{S}$. The following hyperelasticity was applied to the constitutive equation to the mechanical characteristic of polymer skeletal wall.

$$\mathbf{S} = 2 \frac{\partial W(C)}{\partial C}$$

(8)

$\mathbf{S}$ is the 2nd-Piola Kirchhoff stress and $W(C)$ is the stored energy function of hyperelasticity. $\mathbf{C}$ is the right Cauchy-Green deformation tensor. The Truesdell stress rate and 2nd-Piola Kirchhoff stress have following relationships.

$$\mathbf{\tilde{T}}_r = \frac{1}{J} \mathbf{F} \cdot \mathbf{\dot{S}} \cdot \mathbf{F}^T$$

(9)

In this research, we applied the following equation to the stored energy function of hyperelasticity[5].

$$W(C) = C_1 (I_1 - 3) + C_2 (I_2 - 3) + p(J - 1)$$

(10)

$C_1$ and $C_2$ are material moduli. $I_1$ and $I_2$ are the 1st and 2nd principal invariants of the right Cauchy-Green deformation tensor $\mathbf{C}$, respectively. $p$ is the hydrostatic pressure and $J$ is the determinant of the deformation gradient tensor $\mathbf{F}$. For the numerical simulation, the mixed variational principle with perturbed Lagrange-multiplier was applied to the simulation code.
3. Tensile Loading Test and Microscopic Observation of Porous Polymer

3.1. Tensile loading test

In this section, tensile loading tests and microscopic observation of porous polymer were conducted. For the tensile loading tests, an-uniaxial loading test machine (in Fig 2(a)) was applied. Dumbbell shaped test specimens of polyurethane form were applied to the tensile loading test to evaluate the relationships between tensile stiffness and pore ratio. The pore ratios of test specimens were 26%, 61%, 66% and 76%, respectively (in Fig 2(b)). The original density of the resin is 1.0g/cm$^3$ at 0% of pore ratio.

Relationships between nominal stress and strain of tensile loading test were shown in Fig 3(a). The effect of the pore ratio on the tensile stiffness and linear tendency of the relation were plotted in Fig 3(b). From the Fig 3(b), the tensile stiffness of original polymer resin without microstructural pores was predicted as 4.6MPa in small strain region.

![Tensile loading test machine](image1)

![Dumbbell specimens](image2)

![Fig. 2. (a) Tensile loading test machine; (b) Dumbbell specimens developed by the INOAC COPR. Japan](image3)

![Fig. 3. (a) Tensile loading test results; (b) Relationships between the pore ratio and the tensile stiffness](image4)
3.2. Microscopic observation

Microscopic observations for porous polymer, the pore ratio of which is 80% were conducted to investigate about deformation of microscopic structures with uniformly macroscopic deformation. Compressed deformation was applied to the specimens and microscopic strain was measured by using the microscope system. Microscopic structure and relationships between macroscopic and microscopic strain are shown in Fig 4. Microscopic strain in compressed direction was corresponding to the macroscopic strain and microscopic strain in lateral direction was very small compared to the macroscopic strain. The result means that the Poisson ratio of this specimen was a very small value.

4. Numerical Simulation of Porous Polymer

Deformation of microstructure of porous polymer was simulated by the developed FEM code. The simulation program was coded originally using FORTRAN under the LINUX operating system. The unit cell for homogenization analysis is shown in Fig 5(a). Material moduli of hyperelasticity were approximated by the tensile loading test results. The unit cells for homogeneous analysis is shown in Fig 5(a). The Boundary condition of deformation was corresponded to the deformation of microscopic observation which is shown in Fig. 4(b). Moreover, Tensile deformation was applied to the unit cell in

Fig. 4. (a) Microscopic structure of porous polymer; (b) Relationships between macroscopic and microscopic strain

Fig. 5 (a) Unit cell for homogenization analysis(pore ratio=30%); (b) Mean stress distribution of 2x2 unit cells( Pore ratio is 30%, Tensile direction is B); (c) Mean stress distribution of 2x2 unit cells (Pore ratio is 50%, Tensile direction is B)
the two directions A and B in Fig 5(a). Mean stress distributions of numerical simulations were shown in Fig 5(b) and Fig 5(c). The relationships between tensile stress and strain calculated by the FEM code were shown in Fig. 6(a). Effect of the pore ratio on the mechanical characteristics and nonlinearity of stress-strain curves were reproduced well by the numerical simulation. The reductions of the initial stiffness due to the pore ratio given by the simulation and loading tensile tests are compared in Fig 6(b). Numerical calculation shows good agreement with the experimental results.

![Graph](image)

**Fig. 6.** (a) Relationships between nominal strain and tensile stiffness calculated by developed FEM code (Direction B); (b) Relationships between the pore ratio and the tensile stiffness given by numerical simulation and tensile loading tests

### 5. Conclusion

The formulation of the homogenization analysis of hyperelasticy and mechanical loading test results of porous polymers for sports equipment were shown in this paper. From the tensile loading tests, mechanical properties of porous polymers for sports equipment were shown to investigate the effect of microscopic pores on the macroscopic mechanical characteristics. The applicability of developed homogenization simulation programs to the sports materials was verified by comparing with experimental results.

### References


