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# Micromechanical Analysis on the Failure Criterion of Ductile Material

M.M. Liu<sup>a</sup>, J.J. Chen<sup>a,\*</sup><sup>a</sup>East China University of Science and Technology, School of Mechanical and Power Engineering, 130 Meilong Rd, Shanghai 200237, China

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## Abstract

Based on finite element simulation, the void growth behavior in a 3D cell model subjected to low stress triaxiality was studied. The distribution rule of stress-strain near the void was investigated, as well as the developing trend of volume of void. The analysis results indicated that it is not appropriate to regard void volume fraction as a damage parameter under the low stress triaxiality condition. Further research was carried out to verify the reliability of the fracture strain as element failure criterion by comparing the uniaxial tension simulation and experiment results. More studies were conducted to analyze the void growth under low stress triaxiality by introducing the critical equivalent plastic strain as the material failure parameter.

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*Keywords:* Critical equivalent plastic strain; micromechanical analysis; Finite element simulation

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## 1. Introduction

It is well known that material's microstructure, such as micro crack, void and inclusion, has a significant effect on ductile fracture which is generally regarded as the result of voids generation, growth and coalescence. As a result, the void behaviour inside ductile material has received a great number of attentions recently by different experimental, theoretical and numerical researches. The work of McClintock [1], where he analysed the growth discipline of cylindrical voids distributed regularly inside infinite rigid-plastic matrix, showed that stress triaxiality has a considerable influence on void growth. Rice and Tracy [2-3] further investigated the influence of stress state on voids growth by introducing spherical void model and pointed out that voids grow rapidly with the increasing of

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\* Corresponding author. Tel.: +86-021-64253622; fax: +86-021-64253622.

E-mail address: [chinalmm@mail.ecust.edu.cn](mailto:chinalmm@mail.ecust.edu.cn)

stress triaxiality. By defining void volume fraction as damage parameter, Gurson [4-5] proposed a new model of spherical or cylindrical voids inside finite rigid-plastic matrix, and then provided a more practical and systematic constitutive theory. Considering the interaction of voids, Tvergaard and Needleman [6] modified the Gurson model so that the model could be more accurate and practical. From then on, researchers mainly paid their attention to conditions of high stress triaxiality and generally considered void volume fraction as damage parameter to evaluate void growth. However, the traditional GTN model is only valid in the high stress triaxiality condition and will cause obvious deviant in low stress triaxiality condition such as shear failure. Nahshon and Hutchinson [7] proposed a modified Gurson model to account for shear failure. Experiment and simulations performed by Barsoum and Faleskog [8-10] showed that ductile failure results from the interaction of stress triaxiality and Lode parameter and also showed that a systematic research on void growth of matrix subjected to low stress triaxiality is needed.

In the current study, the classical plastic flow theory was introduced to investigate the void growth behavior. Based on finite element simulation, a 3D cell model subjected to low stress triaxiality, with a spherical void located in center was adopted. The distribution rule of stress and strain was investigated, as well as the developing trend of volume of void, which showed that under the low triaxiality condition there are some limitations of the GTN model which regarded void volume fraction as damage parameter. A new parameter to characterize the damage behavior of ductile material subjected to low stress triaxiality is needed. Further research was carried out to investigate the void growth and material failure under different stress triaxiality, especially low stress triaxiality by introducing fracture strain as failure criterion. A new criterion based on critical equivalence plastic strain to evaluate material damage behavior was then developed.

## 2. Micromechanical model

### 2.1. Cell model

A micromechanical model developed in Barsoum and Faleskog [10] was employed. Due to the voids regular array distribution, attention could mainly focus on a 3D unit cell as indicated in Fig. 1. The length of the cell in Y direction must be larger enough to avoid interaction with other rows of voids, i.e.  $L_2 > L_1, L_3$ . According to the research of Pardoen and Hutchinson [11], the initial size of the cell was given by  $L_2 = 2L_1 = 2L_3 = 2L_0$ , so that the interaction of the Y direction could be neglected. The cell contained one void located in its centre, initially of spherical shape with radius  $R_0$ . The initial ratio of void size to void spacing was defined as  $\chi = \frac{2R_0}{L_0}$ . The initial void volume was  $v_{h0} = \frac{4}{3} \pi R_0^3$ , and the void volume relative incremental was  $P = \frac{v_h - v_{h0}}{v_{h0}} \times 100\%$ .

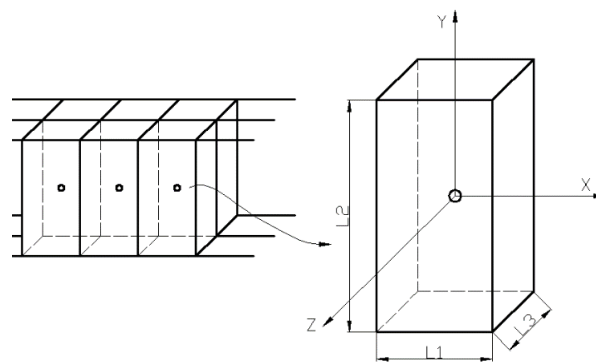


Fig. 1. Cell model.

The 3D cell was numerically analysed by the finite element software (ABAQUS). Symmetry allowed for simulating of the  $Z > 0$  half of the cell as showed in Fig. 2(a). A typical mesh with  $\chi = 0.4$  was showed in Fig. 2(a),

which consisted of 92780 8-node linear-reduced integration elements (C3D8R). To obtain accurate numerical results, the meshes were refined near the middle of the cell as showed in Fig. 2(b). Symmetry boundary condition was applied on the surface Z=0 and loads were applied as showed in Fig. 2(c) to obtain different stress triaxiality.

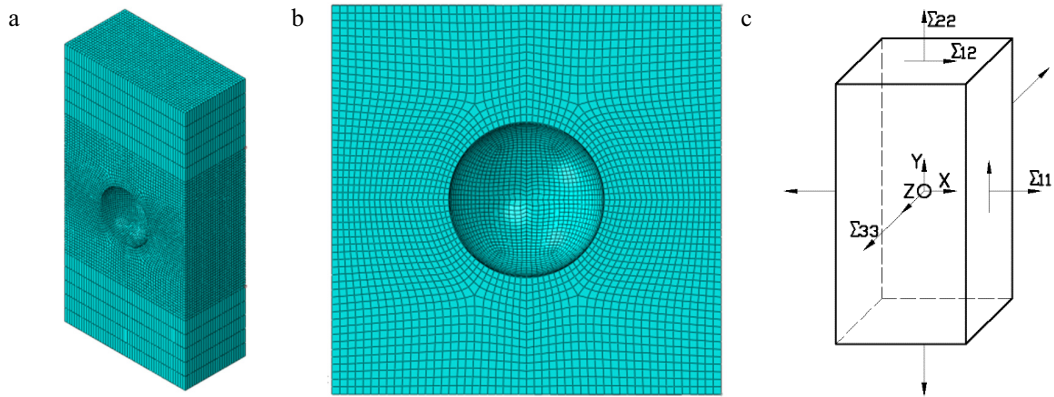


Fig. 2. (a) calculating model; (b) local scheme of mesh; (c) scheme of load combination.

2.2. Material

Cold rolled silicon steel was considered as the material and the material was assumed to be elastic-plastic with isotropic hardening and uniaxial behavior. Elastic-plastic parameters were obtained from tensile experiment and fitting curve as indicated in Fig.3. The true stress-strain behavior was given by Eq. (1):

$$\sigma = \begin{cases} E\varepsilon & (\varepsilon \leq \varepsilon_0) \\ K\varepsilon^N & (\varepsilon > \varepsilon_0) \end{cases} \tag{1}$$

Where E denotes Young’s modulus, E = 205000MPa. K denotes hardening parameter, K = 502MPa. N denotes strain hardening exponent, N = 0.096.

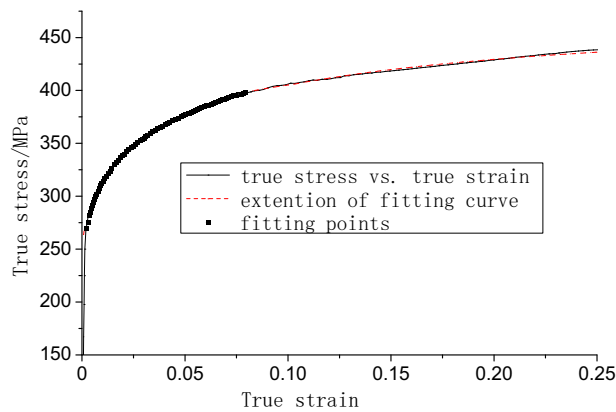


Fig. 3. Fitting curve of true stress vs. true strain.

### 3. Analysis and discussion

Different stress triaxiality were obtained by adjusting the stress combination according to Eq. (2):

$$T = \frac{\sum_h}{\sum_e} \tag{2}$$

Where  $\sum_h$  denotes the mean value of macroscopic stress,  $\sum_h = \frac{1}{3} \mathbf{S}_{ii}$ .  $\sum_e$  denotes the von Mises value of macroscopic stress,  $\sum_e = \sqrt{\frac{3}{2} \mathbf{s}_{ij} \mathbf{s}_{ij}}$ .

#### 3.1. Micromechanical analysis of matrix subjected to low stress triaxiality

Since the case of shear stress is a typical stress state of low stress triaxiality, the analysis of void behaviour was mainly focus on the case of shear stress in present section. In the case of shear stress, loads were applied as  $\sum_{11} = \sum_{22} = \sum_{33} = 0$ , and  $\sum_{12}$  was proportional applied to keep  $T=0$ . A model with  $\chi = 0.4$  was chosen to calculate.

##### 3.1.1. Analysis of distribution of stress and strain

Taken  $\sum_{12} = \sigma_s$  to analyse, the distribution of Mises stress and equivalent plastic strain were displayed in Fig. 4(a), (b) respectively. Mises stress was anti-symmetric distributed relative to XOZ surface and YOZ surface. Stress in remote area was more uniform than area near void, and the maximum stress value appears on the inner surface of void. Stress along the X and Y axis was higher than that along 45° direction, so the lower stress area distributed like a butterfly. Equivalent plastic strain (PEEQ) had a similar distribution as was showed in Fig. 4(b).

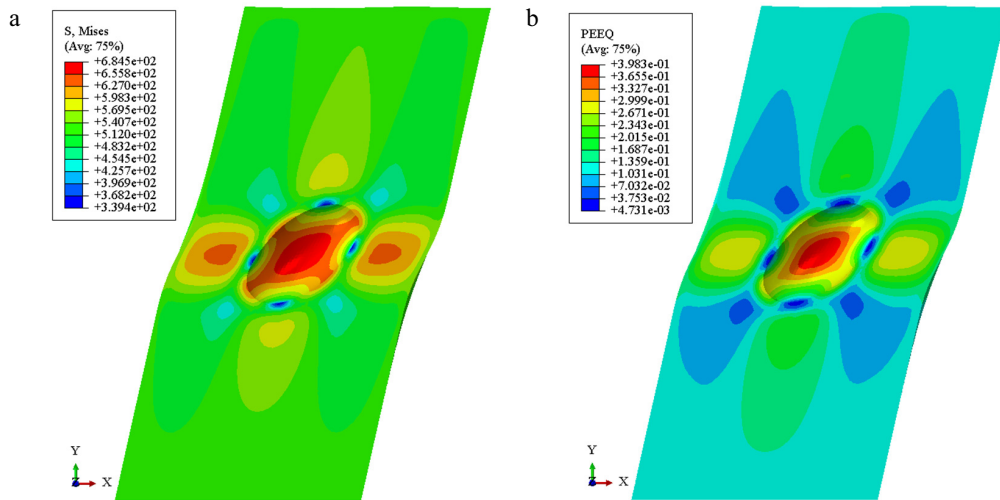


Fig. 4. (a) distribution of Mises stress; (b) distribution of PEEQ.

##### 3.1.2. Analysis of void volume fraction

Void volume fraction was generally considered as a parameter to evaluate material damage in classical numerical model. While when matrix subjected to shear stress, the void volume decreased, instead of increased, as proportionally increasing the shear stress after yield, as was showed in Fig. 5. That went against the theory proposed by Chaoche [12] based on thermodynamics that damage parameter increases monotonically. Therefore, void volume fraction is not suitable to characterize void growth when matrix subjected to low stress triaxiality.

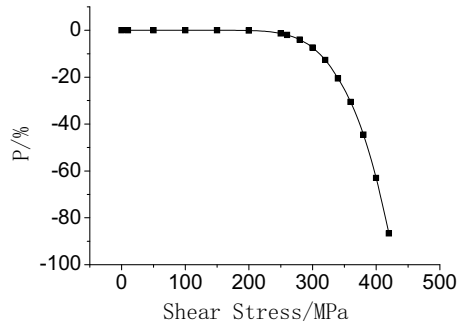


Fig. 5. Curve of shear stress vs.  $P$ .

### 3.2. Analysis of void growth and material failure

Zheng Changqing [13] presented a function of fracture strain and stress triaxiality in his work. Studies of Yingbin Bao, Tomasz Wierzbicki [14] also showed that the failure strains of material subjected to different stresses are directly related to stress triaxiality. Based on these researches, fracture strain was introduced in numerical computing by applying VUSDFLD subroutine in ABAQUS. The fracture strain  $\epsilon_f$  was defined as Eq. (3):

$$\epsilon_f = \alpha \cdot e^{-\frac{3}{2}T} \tag{3}$$

Where  $\alpha$  denotes fracture coefficient.  $T$  denotes stress triaxiality.

As could be seen in Fig. 6, the strain-stress behavior was strongly influenced by  $\alpha$ . By comparing the curve of experiment and simulation, we could reach a conclusion that the result of simulation is in best accordance with the result of experiment when  $\alpha = 0.8$ . Therefore, the fracture coefficient was chosen as  $\alpha = 0.8$ .

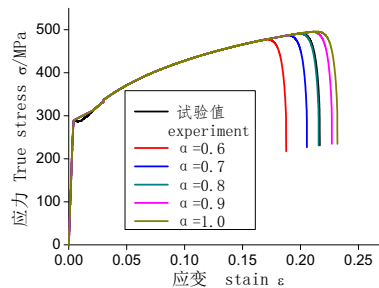


Fig. 6. Curve of true strain vs. true stress of experiment and simulation.

#### 3.2.1. Analysis of void behavior of model under uniaxial tension

The loads were applied as  $\Sigma_{11}=\Sigma_{33} = \Sigma_{12}=0$  and  $\Sigma_{22}$  proportional increased to keep stress state of uniaxial tension. A model with  $\chi = 0.4$  was selected to analysis. As could be seen from Fig. 7, the PEEQ in localization band increased as Mises stress increasing after yield. As deformation progressed, a maximum Mises stress was reached when  $\Sigma_{22} = 1.6\sigma_s$ , while localization occurred and material failed.

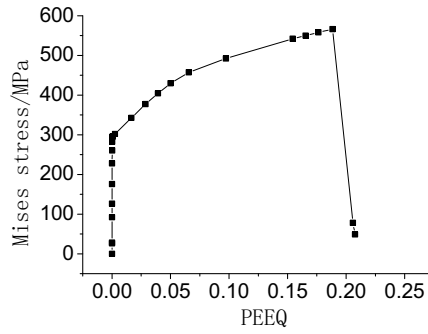


Fig. 7. Curve of PEEQ vs. Mises stress of model under uniaxial tension.

Figure 8. and Fig. 9. showed the deformation of void corresponding to in-situ tensile experiment and simulation, respectively. The experimental appearances of void in Fig. 8. (a)-(c) and the simulating appearances of void in Fig.9. (a)-(c) corresponded to stress state of  $\epsilon_T = 0.01, 0.1, 0.21$ . As we could see from the figure, the void stayed spherical when  $\sum_{22} < \sigma_s$ . Mises stress increased and void extended as tensile load continually increasing. The high stress gradually localized in XOZ plane and void grown into ellipsoid as was showed in Fig.8. (b) and Fig.9. (b). Carrying capacity of matrix decreased and material failure occurred when the deformation became highly non-uniform and localized into a thin band in centre at  $\epsilon_T = 0.21$ . It is clear that void extended along X axis both in simulation and experiment and the expanded trends kept accordant, which verified that it is reliable to apply fracture strain to evaluate element failure.

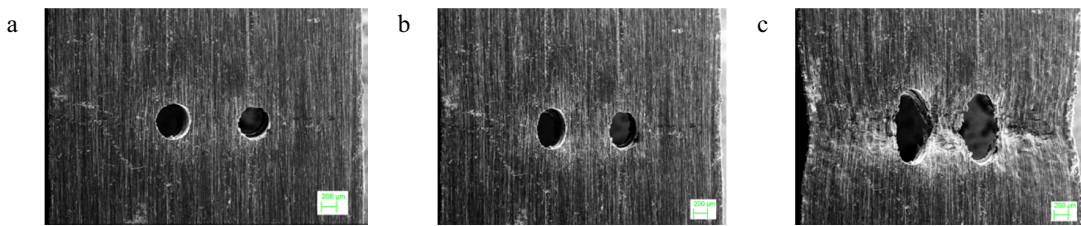


Fig. 8. (a)  $\epsilon_T = 0.01$ ; (b)  $\epsilon_T = 0.10$ ; (c)  $\epsilon_T = 0.21$ .

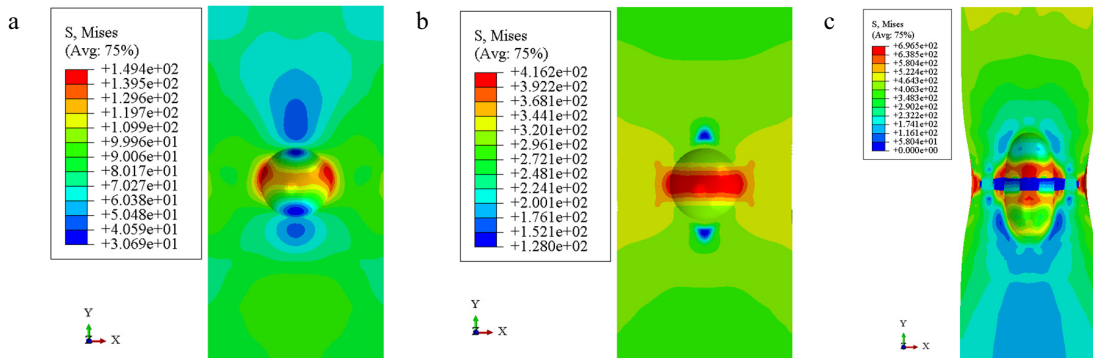


Fig. 9. (a)  $\epsilon_T = 0.01$ ; (b)  $\epsilon_T = 0.10$ ; (c)  $\epsilon_T = 0.21$ .

3.2.2. Analysis of void behavior of model under low stress triaxiality

Applied  $\Sigma_{11}=\Sigma_{22}=\Sigma_{33}$  and  $\frac{\Sigma_{11}}{\Sigma_{12}} = 0, \frac{\sqrt{3}}{10}, \frac{\sqrt{3}}{5}, \frac{3\sqrt{3}}{10}, \frac{2\sqrt{3}}{5}, \frac{\sqrt{3}}{2}$  to keep the Lode parameter identical and obtain  $T=0, 0.1, 0.2, 0.3, 0.4, 0.5$ . A model with  $\chi = 0.4$  was selected to analysis.

Defined the value of equivalent plastic strain at localization as critical equivalent plastic strain  $\epsilon_{ep}^c$ . As could be seen from Fig.10,  $\epsilon_{ep}^c$  was strongly influenced by  $T$ . generally, a smaller value of  $\epsilon_{ep}^c$  corresponds to a larger  $T$ . The relationship of  $\epsilon_{ep}^c$  and  $T$  can be given by Eq. (4):

$$\epsilon_{ep}^c = 1.125T^3 - 0.865T^2 - 0.082T + 0.175 \tag{4}$$

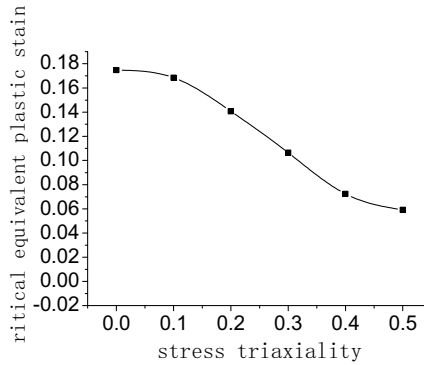


Fig. 10. Curve of stress triaxiality vs. critical equivalent plastic strain.

To exemplify the influence of stress triaxiality on void growth under low stress triaxiality, a model subjected to shear stress was show in Fig. 11. Fig. 11 (a)-(c) represented the contours of the Mises stress at stages of  $\Sigma_{12} = 0.2\sigma_s$ ,  $\Sigma_{12} = 0.8\sigma_s$ ,  $\Sigma_{12} = \sigma_s$ . Mises stress was proportional to macro shear stress. Void hardly extended before  $\Sigma_{12} = 0.5\sigma_s$ . With macro shear stress increasing, the matrix inclined gradually, as well as void deformed. Micro imperfections accumulated on the localization band which resulted in matrix fracture along the 45° direction.

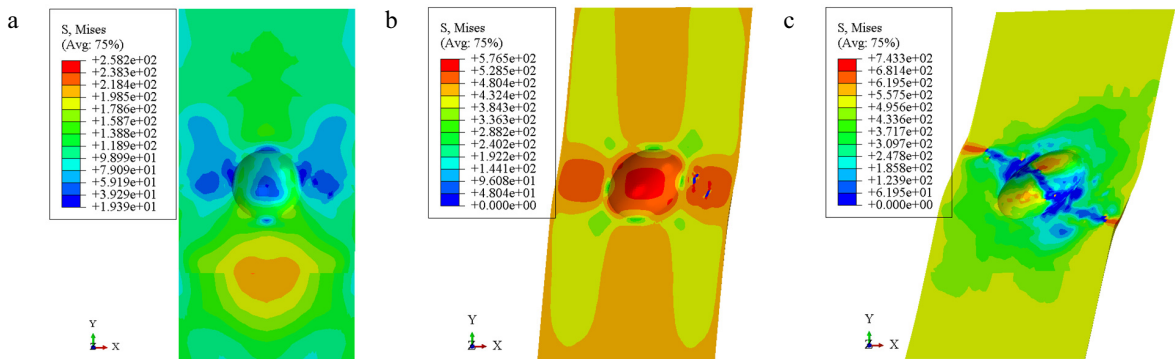


Fig. 11. (a)  $\Sigma_{12} = 0.2\sigma_s$ ; (b)  $\Sigma_{12} = 0.8\sigma_s$ ; (c)  $\Sigma_{12} = \sigma_s$ .

4. Conclusion

Based on the finite element simulation (ABAQUS), the distribution rule of stress and strain was investigated, as well as the developing trend of volume of void and matrix, which verified the limitation of the GTN model. A new parameter, fracture strain, to characterize the damage behaviour of ductile material subjected to low stress triaxiality



was introduced. A new criterion based on critical equivalence plastic strain to evaluate material damage behaviour was developed as well. The present study was concluded in the following:

- The void volume decreased with increasing macro loads in stress state of low stress triaxiality, which indicated that theory which considered void volume fraction as damage parameter is not suitable to stress state of low stress triaxiality.
- For uniaxial tension state, both results of experiment and simulation showed that void extended along X axis and the expanded trends kept accordant, which verified that it is reliable to apply fracture strain to evaluate element failure. Material failure occurred when the deformation became highly non-uniform and localized into a thin band in center when  $\sum_{22} > 1.6\sigma_s$ .
- $\varepsilon_{ep}^c$ , which is strongly effected by  $T$ , was introducing to characterize the damage of material. Generally, a smaller value of  $\varepsilon_{ep}^c$  corresponds to a larger  $T$ . The relationship of  $\varepsilon_{ep}^c$  and  $T$  can be given by  $:\varepsilon_{ep}^c = 1.125T^3 - 0.865T^2 - 0.082T + 0.175$ .

### Acknowledgment

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