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The energy dependence of the saturation scale in DIS at low x [☆]

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Abstract

Consistency of the previously suggested color-dipole representation of deep-inelastic scattering (DIS) and vector-meson production at low x with DGLAP evolution allows one to predict the exponent of the W^2 dependence of the saturation scale, $\Lambda^2_{\mathrm{sat}}(W^2) \sim (W^2)^{C_2}$. One finds $C_2^{\mathrm{theory}} = 0.27$ in agreement with the model-independent analysis of the experimental data from HERA on deep-inelastic electron scattering.
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The present Letter is concerned with deep-inelastic electron scattering (DIS) at low $x \cong Q^2/W^2 \ll 1$. In short, we analyse the consistency between DGLAP evolution of the nucleon structure function $F_2(x, Q^2)$ and the color-dipole picture. We find that the exponent C_2^{theory} that in our formulation of the color-dipole approach determines the energy dependence of the total photoabsorption cross section at large Q^2, $\sigma_{\gamma^* p}(W^2, Q^2) \sim (W^2)^{C_2}/Q^2$, or, equivalently, the energy dependence of the "saturation scale" $\Lambda^2_{\mathrm{sat}}(W^2) \sim (W^2)^{C_2}$, coincides with the result of previous fits to the experimental data, $C_2^{\mathrm{theory}} \cong C_2^{\mathrm{experiment}}$.

For $x \cong Q^2/W^2 \ll 1$, the photon–proton interaction is dominated by the interaction of the photon with the quark–antiquark sea in the proton. The proton structure function for $x \ll 1$ only contains the flavor-singlet quark distributions, and their evolution in Q^2 for $Q^2 \geqslant Q_0^2$ [1] is in good approximation determined by the gluon structure function alone [2],

$$\frac{\partial F_2(\frac{x}{2}, Q^2)}{\partial \ln Q^2} = \frac{R_{e^+e^-}}{9\pi} \alpha_s(Q^2) x g(x, Q^2). \quad (1)$$

The notation in (1) is the standard one, we only note $R_{e^+e^-} = 3\sum_f Q_f^2 = 10/3$, where Q_f denotes the quark charge and f runs over the contributing $(n_F = 4)$ flavors. In the physical picture underlying

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(1) the perturbatively calculable photon–gluon scattering amplitude is supplemented by the gluon structure function that parameterizes the unknown proton properties to be determined experimentally. The structure function $F_2(x, Q^2)$ is related to the flavor-blind, flavor-singlet quark distribution

$$x\Sigma(x, Q^2) = n_F\big(xq(x, Q^2) + x\bar{q}(x, Q^2)\big) \tag{2}$$

via

$$F_2(x, Q^2) = \frac{R_{e^+e^-}}{12} x\Sigma(x, Q^2). \tag{3}$$

In the color-dipole picture [3], valid at low $x \ll 1$ and any $Q^2 \geqslant 0$, in terms of the imaginary part of the virtual photon–proton forward-scattering amplitude, the process of $\gamma^* p$ scattering proceeds via the fluctuation of the photon into a $q\bar{q}$ pair that subsequently scatters on the proton via (the generic structure of) two-gluon exchange [4]. The properties of the proton are contained in the color-dipole cross section

$$\sigma_{(q\bar{q})p}(\vec{r}_\perp, W^2) = \int d^2 l_\perp \, \tilde{\sigma}_{(q\bar{q})p}(\vec{l}_\perp^{\,2}, W^2)\big(1 - e^{-i\vec{l}_\perp \vec{r}_\perp}\big) \tag{4}$$

that depends on the two-dimensional transverse quark–antiquark separation, $\vec{r}_\perp$ [3,5]. The function $\tilde{\sigma}(\vec{l}_\perp^{\,2}, W^2)$ is associated with the gluon transverse-momentum distribution in the proton, and the factor $(1 - \exp(-i\vec{l}_\perp \vec{r}_\perp))$ in (4) is characteristic of the (QCD) gauge-theory structure. This factor originates from the couplings of the two gluons to either the same quark (antiquark) or to a quark and an antiquark. Motivated by the mass dispersion relation of generalized vector dominance [6,7] or, equivalently, life-time arguments [8] on the hadronic (quark–antiquark) fluctuation of the virtual photon, the energy W appears as the second variable besides $\vec{r}_\perp$ or $\vec{l}_\perp$ in (4). We refer to the literature [3,5] for the explicit representation of the total virtual photoabsorption cross section in terms of the virtual-photon wave function describing the $q\bar{q}$ fluctuations of the photon and the dipole cross section in (4).

At sufficiently large Q^2, the dipole cross section in the limit of small interquark transverse separation, $\vec{r}_\perp^{\,2} \to 0$, becomes relevant. From (4), for $\vec{r}_\perp^{\,2} \to 0$,[2]

$$\sigma_{(q\bar{q})p}(\vec{r}_\perp, W^2) \cong \frac{1}{4}\vec{r}_\perp^{\,2}\pi \int d\vec{l}_\perp^{\,2}\, \vec{l}_\perp^{\,2} \tilde{\sigma}_{(q\bar{q})p}(\vec{l}_\perp^{\,2}, W^2). \tag{5}$$

By reformulating the γ^*-gluon-scattering approach underlying (1) in terms of the transverse position-space variable $\vec{r}_\perp$, one finds [9,10] that the gluon structure function $xg(x, Q^2)$ in (1) is proportional to the right-hand side of (5),

$$\alpha_s(Q^2)xg(x, Q^2) = \frac{3}{4\pi} \int d\vec{l}_\perp^{\,2}\, \vec{l}_\perp^{\,2} \tilde{\sigma}_{(q\bar{q})p}(\vec{l}_\perp^{\,2}, W^2). \tag{6}$$

The gluon structure function, according to (6), is proportional to the first moment of the gluon transverse-momentum distribution.[3] The validity of (6) is restricted to sufficiently large Q^2, where both the notion of the gluon structure function appearing in (1) as well as the $\vec{r}_\perp^{\,2} \to 0$ expansion of the dipole cross section in (5) are applicable.

Actually only transverse and longitudinal $(q\bar{q})^{J=1}_{T,L}$ (vector) states contribute to the imaginary part of the virtual forward-scattering Compton amplitude. The structure function $F_2(x, Q^2)$ may be represented [11] in terms of the $J = 1$ projections of the color-dipole cross section (4). The leading contribution to

$$\begin{aligned} F_2(x, Q^2) &= \frac{Q^2}{4\pi^2\alpha}\sigma_{\gamma^* p}(W^2, Q^2) \\ &= \frac{Q^2}{4\pi^2\alpha}\big(\sigma_{\gamma^*_T p}(W^2, Q^2) + \sigma_{\gamma^*_L p}(W^2, Q^2)\big) \end{aligned} \tag{7}$$

at sufficiently large Q^2 becomes[4]

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[2] Note that the scale for $\vec{r}_\perp^{\,2}$ that determines the validity of the expansion in (5) depends on the behavior of $\tilde{\sigma}(\vec{l}_\perp^{\,2}, W^2)$. Essentially, it is given by the effective or average value of $\vec{l}_\perp^{\,2}$ determined by $\tilde{\sigma}(\vec{l}_\perp^{\,2}, W^2)$.

[3] Note that the Q^2 dependence of the gluon structure function at fixed x is contained in $W^2 = Q^2/x$. This is at variance with the conventional assumption, where x occurs on the right-hand side in (6) and the Q^2 dependence is introduced via the upper limit, Q^2, of the integral in (6).

[4] The expression (8) for $F_2(x, Q^2)$ is obtained from (3.13) and (3.14) in Ref. [11] by expansion in powers of $\vec{l}_\perp^{\prime 2}/(4Q^2 + \vec{l}_\perp^{\prime 2})$.

$$\begin{aligned}
F_2(x, Q^2) &= \frac{Q^2}{36\pi^2} R_{e^+e^-}\left(\int d\vec{l}_\perp'^2 \frac{4\vec{l}_\perp'^2}{4Q^2+\vec{l}_\perp'^2}\bar{\sigma}_{(q\bar{q})_T^{J=1}}(\vec{l}_\perp'^2, W^2) + \frac{1}{2}\int d\vec{l}_\perp'^2 \frac{4\vec{l}_\perp'^2}{4Q^2+\vec{l}_\perp'^2}\bar{\sigma}_{(q\bar{q})_L^{J=1}}(\vec{l}_\perp'^2, W^2)\right) \\
&\cong \frac{R_{e^+e^-}}{36\pi^2}\left(\int d\vec{l}_\perp'^2 \vec{l}_\perp'^2 \bar{\sigma}_{(q\bar{q})_T^{J=1}}(\vec{l}_\perp'^2, W^2) + \frac{1}{2}\int d\vec{l}'^2 \vec{l}'^2 \bar{\sigma}_{(q\bar{q})_L^{J=1}}(\vec{l}'^2, W^2)\right), \qquad (8)
\end{aligned}$$

where $\vec{l}'^2$ is related to the gluon transverse momentum and the light-cone variable z via

$$\vec{l}'^2 = \frac{\vec{l}_\perp^2}{z(1-z)}. \qquad (9)$$

Moreover, also the gluon structure function (6) may be represented in terms of the longitudinal part of the $J=1$ projection of the color-dipole cross section [11],

$$\alpha_s(Q^2)xg(x, Q^2) = \frac{1}{8\pi}\int d\vec{l}_\perp'^2 \vec{l}_\perp'^2 \bar{\sigma}_{(q\bar{q})_L^{J=1}}(\vec{l}_\perp'^2, W^2). \qquad (10)$$

In terms of the "saturation scale"

$$\Lambda_{\rm sat}^2(W^2) \equiv \frac{\pi}{\sigma^{(\infty)}}\int d\vec{l}_\perp'^2 \vec{l}_\perp'^2 \bar{\sigma}_{(q\bar{q})_L^{J=1}}(\vec{l}_\perp'^2, W^2), \qquad (11)$$

where the constant $\sigma^{(\infty)}$ will be explicitly defined below (compare (40)), (10) becomes

$$\alpha_s(Q^2)xg(x, Q^2) = \frac{1}{8\pi^2}\sigma^{(\infty)}\Lambda_{\rm sat}^2(W^2). \qquad (12)$$

So far our considerations have exclusively been based on the two-gluon exchange structure embodied in the form of the color-dipole cross section (4). To proceed, we assume the flavor-singlet distribution (2) and the gluon distribution (12) to have identical dependence on the kinematic variables x and Q^2. In our case, x and Q^2 appear in the combination $W^2 \cong Q^2/x$. Both $x\Sigma(x, Q^2)$ and $\alpha_s(Q^2)xg(x, Q^2)$ must then be proportional to $\Lambda_{\rm sat}^2(W^2)$. Since $F_2(x, Q^2)$ is proportional to $x\Sigma(x, Q^2)$, compare (3), also $F_2(x, Q^2)$ in (8) must be proportional to $\Lambda_{\rm sat}^2(W^2)$.

Since, moreover, the longitudinal term on the right-hand side in (8) is proportional to the gluon structure in (10) and (12), also the transverse contribution to $F_2(x, Q^2)$ in (8) must be proportional to $\Lambda_{\rm sat}^2(W^2)$. In terms of the integrals in (8), the above requirement on the flavor-singlet quark and the gluon distribution thus becomes

$$\int d\vec{l}_\perp'^2 \vec{l}_\perp'^2 \bar{\sigma}_{(q\bar{q})_T^{J=1}}(\vec{l}_\perp'^2, W^2) = r\int d\vec{l}_\perp'^2 \vec{l}_\perp'^2 \bar{\sigma}_{(q\bar{q})_L^{J=1}}(\vec{l}_\perp'^2, W^2). \qquad (13)$$

Note that the constant r is related to the longitudinal to transverse ratio

$$\frac{\sigma_{\gamma_L^* p}(W^2, Q^2)}{\sigma_{\gamma_T^* p}(W^2, Q^2)} = \frac{1}{2r}. \qquad (14)$$

Our previous analysis [12–15] of the experimental data on DIS was based on the equality of

$$\bar{\sigma}_{(q\bar{q})_T^{J=1}}(\vec{l}_\perp'^2, W^2) = \bar{\sigma}_{(q\bar{q})_L^{J=1}}(\vec{l}_\perp'^2, W^2), \qquad (15)$$

i.e., on

$$r = 1. \qquad (16)$$

We found consistency with the experimental data, including [14] the available information on the longitudinal virtual photoabsorption cross section. We will henceforth put $r=1$. Upon inserting (13) and introducing $\Lambda_{\rm sat}^2(W^2)$ from (11), $F_2(x, Q^2)$ from (8) becomes

$$F_2(x, Q^2) = \frac{R_{e^+e^-}}{36\pi^3}\sigma^{(\infty)}\Lambda_{\rm sat}^2(W^2)\left(1+\frac{1}{2}\right), \qquad (17)$$

where the sum on the right-hand side refers to the sum of the transverse and longitudinal parts.

We now insert $F_2(x, Q^2)$ from (17) and the gluon distribution (12) into the DGLAP-evolution equation (1), to find the interesting constraint

$$\frac{\partial}{\partial \ln W^2}\Lambda_{\rm sat}^2(2W^2) = \frac{1}{3}\Lambda_{\rm sat}^2(W^2), \qquad (18)$$

or, alternatively, in terms of the observable $F_2(x, Q^2)$,

$$\frac{\partial}{\partial \ln W^2}F_2(2W^2) = \frac{1}{3}F_2(W^2). \qquad (19)$$

We adopt a power-law ansatz for $\Lambda_{\rm sat}^2(W^2)$,

$$\Lambda_{\rm sat}^2(W^2) = B\left(\frac{W^2}{W_0^2}+1\right)^{C_2} \cong B\left(\frac{W^2}{W_0^2}\right)^{C_2}, \qquad (20)$$

identical in form to the one previously employed [12–15] in (successful) fits to the experimental data on the virtual photoabsorption cross section,

$$\sigma_{\gamma^* p}\big(W^2, Q^2\big) = \sigma_{\gamma^* p}\big(\eta\big(W^2, Q^2\big)\big), \tag{21}$$

and on deeply virtual Compton scattering (DVCS) in terms of the scaling variable [12,13][5]

$$\eta\big(W^2, Q^2\big) \equiv \frac{Q^2 + m_0^2}{\Lambda_{sat}^2(W^2)}. \tag{22}$$

Inserting the power-law ansatz (20) into (18), we deduce

$$C_2 = \frac{1}{3}\left(\frac{1}{2}\right)^{C_2}, \tag{23}$$

and accordingly[6]

$$C_2^{theory} = 0.276. \tag{24}$$

The theoretically deduced magnitude of the exponent $C_2^{theory} = 0.276$ in (24) is in agreement with the model-independent fit [12,13] to the experimental data,

$$C_2^{exp} = 0.275 \pm 0.06. \tag{25}$$

The model-independent fit assumes that $\sigma_{\gamma^* p}(W^2, Q^2)$ may be represented by a smooth function of the scaling variable $\eta(W^2, Q^2)$ in (22) with $\Lambda_{sat}^2(W^2)$ from (20). As a consequence of the generality of the ansatz, any theoretical prejudice with respect to the empirical validity of scaling in $\eta(W^2, Q^2)$ is excluded.

The fit based on an explicit ansatz for the color-dipole cross section (compare (32)) gave the more precise result

$$C_2^{exp} = 0.27 \pm 0.01 \tag{26}$$

in agreement with our theoretical result (24).

To summarize: the choice of W as the relevant variable in the color-dipole approach together with the requirement that the singlet quark distribution and the gluon distribution have identical dependence on the kinematic variables, $W^2 = Q^2/x$ in our case, converts the DGLAP-evolution equation (1) into a constraint that allows one to predict the exponent C_2. The agreement with experiment supports the validity, at least as a relevant approximation, of the underlying assumptions.

The choice of $W^2 \cong Q^2/x$ as the relevant variable in our approach was motivated by the color-dipole approach and its generalized vector-dominance interpretation. We note that the dependence (17)

$$F_2 \sim \big(W^2\big)^{C_2} \tag{27}$$

is closely related to the so-called singular solution of the gluon evolution equation, where[7]

$$F_2 \sim \ln Q^2 x^{-\lambda} \simeq \frac{\ln Q^2}{(Q^2)^\lambda}\big(W^2\big)^\lambda \cong \big(W^2\big)^\lambda \tag{28}$$

with $\lambda \geqslant 0.25$ being fixed and equal to the input value at all Q^2. In a restricted but relevant range of Q^2, (27) is similar to (28).

The conventional application of the color-dipole approach [3] to DIS does not explicitly introduce the $J = 1$ projection of the color-dipole forward-scattering amplitude. One starts by an assumption on $\tilde\sigma(\vec l_\perp^{\,2}, W^2)$ in (4), rather than its $J = 1$ projection, $\bar\sigma_{(q\bar q)^{J=1}_{T,L}}(\vec l_\perp^{\,\prime 2}, W^2)$. We briefly elaborate on how our approach can be formulated in terms of $\tilde\sigma(\vec l_\perp^{\,2}, W^2)$.

Consider the ansatz

$$\tilde\sigma\big(\vec l_\perp^{\,2}, W^2\big) = \frac{\sigma^{(\infty)}}{\pi}\delta\left(\vec l_\perp^{\,2} - \frac{1}{6}\Lambda_{sat}^2\big(W^2\big)\right). \tag{29}$$

Inserting (29) into (6), we immediately recover the gluon structure function (12). Evaluation of the $J = 1$ parts of (29) yields

$$\bar\sigma_{(q\bar q)^{J=1}_{L,T}}\big(\vec l_\perp^{\,\prime 2}, W^2\big) = f_{L,T}\big(\vec l_\perp^{\,\prime 2}, \Lambda_{sat}^2\big(W^2\big)\big)\theta\left(\vec l_\perp^{\,\prime 2} - \frac{2}{3}\Lambda_{sat}^2\big(W^2\big)\right), \tag{30}$$

the explicit form of the function $f_{L,T}(\vec l_\perp^{\,\prime 2}, \Lambda_{sat}^2(W^2))$ being irrelevant in the present context. We only note the normalization of (30)

$$\int d\vec l_\perp^{\,\prime 2}\, \bar\sigma_{(q\bar q)^{J=1}_T}\big(\vec l_\perp^{\,\prime 2}, W^2\big) = \int d\vec l_\perp^{\,\prime 2}\, \bar\sigma_{(q\bar q)^{J=1}_L}\big(\vec l_\perp^{\,\prime 2}, W^2\big) = \frac{\sigma^{(\infty)}}{\pi}. \tag{31}$$

[5] For the sake of clarity, we introduced the notation $\Lambda_{sat}^2(W^2) \equiv \Lambda^2(W^2)$ for the quantity previously denoted by $\Lambda^2(W^2)$.

[6] The arguments leading to (24) were implicitly used in the third paper of Ref. [13] without, however, fully realizing their significance.

[7] Compare the discussion in Section 7 of Ref. [16] and the literature quoted there, in particular Ref. [17].

Evaluating the longitudinal part of $F_2(x, Q^2)$ in (8) by substituting (30), we recover our previous result (17) for the longitudinal contribution. The normalization (31) suggests to approximate (30) by

$$\bar{\sigma}_{(q\bar{q})_T^{J=1}}(\vec{l}_\perp^{\prime 2}, W^2) = \bar{\sigma}_{(q\bar{q})_L^{J=1}}(\vec{l}_\perp^{\prime 2}, W^2) = \frac{\sigma^{(\infty)}}{\pi}\delta(\vec{l}_\perp^{\prime 2} - \Lambda_{\text{sat}}^2(W^2)). \tag{32}$$

With (32) inserted into (8), we now obtain not only the longitudinal, but also the transverse part of F_2 given in (17). The direct evaluation of F_2, inserting (30), however, yields

$$F_2(x, Q^2) = \frac{R_{e^+e^-}}{36\pi^3}\sigma^{(\infty)}\Lambda_{\text{sat}}^2(W^2) \times \left(\frac{1}{2}\ln\frac{6Q^2}{\Lambda_{\text{sat}}^2(W^2)} + \frac{1}{2}\right) \tag{33}$$

in distinction from (17). Our requirement of identical singlet quark and gluon distributions that is contained in (13), (15) and (32) is not fulfilled by the ansatz (29). It is not known whether an ansatz for $\tilde{\sigma}(\vec{l}_\perp^2, W^2)$ can be given such that (13) with $r = 1$ be valid. For the time being, we have to accept the equality (32) as a valid approximation for the $J = 1$ projections that describes the experimental data on DIS in the low x diffraction region with the predicted value of the exponent C_2.

Encouraged by the result (24), we now examine the coupled system of equations for singlet quark and gluon evolution.

Substituting the gluon structure function (12) and the singlet quark distribution $x\Sigma(x, Q^2)$ from (3) with (17), into the evolution equations, with the power-law (20) for $\Lambda_{\text{sat}}^2(W^2)$ and the notation $t \equiv \ln Q^2$, one finds

$$\frac{\partial \Lambda_{\text{sat}}^2(W^2)}{\partial \ln W^2} = \Lambda_{\text{sat}}^2(W^2)\int_x^1 dy\left(\frac{\alpha_s(t)}{2\pi}P_{qq}(y) + P_{qg}(y)\right)y^{C_2}, \tag{34}$$

and

$$\frac{\partial \Lambda_{\text{sat}}^2(W^2)}{\partial \ln W^2} = \Lambda_{\text{sat}}^2(W^2)\frac{1}{\alpha_s(t)}\frac{d\alpha_s(t)}{dt} + \Lambda_{\text{sat}}^2(W^2)\frac{\alpha_s(t)}{2\pi} \times \int_x^1 dy\left(\frac{\alpha_s(t)n_F}{\pi}P_{gq}(y) + P_{gg}(y)\right)y^{C_2}. \tag{35}$$

The first equation, (34), without relying on the approximation contained in the right-hand side of (1), describes the evolution of the flavor singlet quark distribution, while the second equation, (35), describes the evolution of the gluon distribution. By noting that $\partial\Lambda_{\text{sat}}^2(W^2)/\partial\ln W^2 = C_2\Lambda_{\text{sat}}^2(W^2)$, and upon evaluating the integrals on the right-hand side, in (34), we obtain

$$C_2 = 0.044\alpha_s(t) + \frac{C_2^2 + 3C_2 + 4}{2(C_2+1)(C_2+2)(C_2+3)}. \tag{36}$$

The numerical value of 0.044 in the (small) C_2-dependent correction proportional to $\alpha_s(t)$ in (36) was obtained by inserting $C_2 = 0.276$. Solving (36) for C_2, we find

$$C_2 = 0.044\alpha_s(t) + 0.260 \cong 0.265, \tag{37}$$

upon disregarding the (weak) Q^2 dependence of α_s at large Q^2 and inserting $\alpha_s = 0.11$.

A similar approach, when applied to the gluon-evolution equation (35), leads to

$$C_2 = \frac{1}{\alpha_s(t)}\frac{d\alpha_s(t)}{dt} + C_3(C_2)\alpha_s^2(t) + C_4(C_2)\alpha_s(t) \cong 0.275. \tag{38}$$

The dependence of the coefficients $C_3(C_2)$ and $C_4(C_2)$ on C_2 is directly calculated from (35). The value of $C_2 = 0.275$ was obtained by consistently solving (38), using $\alpha_s(t) = 0.11$.

As a result of our analysis of the complete evolution equations, by comparing (37) and (38) with (24), we conclude that the power-law ansatz (20) with a constant (Q^2-independent) value of C_2^{theory} of magnitude $C_2^{\text{theory}} \cong 0.276$ according to (24) is consistent with evolution. The additional α_s-dependent contributions in (34) which have been ignored in (1) hardly affect the value of the exponent C_2.

So far in this Letter we were concerned with DIS at low $x \ll 1$ and sufficiently large $Q^2 \gg \Lambda_{\text{sat}}^2(W^2)$ where the QCD-improved parton picture and the color-dipole picture are dual descriptions of the underlying physics.

For details on the extension to $Q^2 \ll \Lambda_{\text{sat}}^2(W^2)$ and the (successful) description of the experimental data based on the ansatz (32), we refer to Refs. [13–15]. We only note the $Q^2 \to 0$ limit in addition to the large-Q^2 limit

$$\begin{aligned}
&\sigma_{\gamma^* p}(W^2, Q^2) \\
&\quad = \sigma_{\gamma^* p}(\eta(W^2, Q^2)) \\
&\quad = \frac{\alpha}{3\pi} R_{e^+e^-} \sigma^{(\infty)} \\
&\qquad \times \begin{cases} \ln \frac{\Lambda^2_{\text{sat}}(W^2)}{Q^2+m_0^2} & (Q^2 \ll \Lambda^2_{\text{sat}}(W^2)), \\ \frac{\Lambda^2_{\text{sat}}(W^2)}{2Q^2} & (Q^2 \gg \Lambda^2_{\text{sat}}(W^2)). \end{cases}
\end{aligned} \tag{39}$$

With $\Lambda^2_{\text{sat}}(W^2)$ from (11), the asymptotic limit of $Q^2 \gg \Lambda^2_{\text{sat}}(W^2)$, in (39) coincides with (17). We also note

$$\sigma^{(\infty)} = \pi \int d\vec{l}_\perp^{\prime 2} \, \bar{\sigma}_{(q\bar{q})^{J=1}_L p}(\vec{l}_\perp^{\prime 2}, W^2). \tag{40}$$

Hadronic unitarity requires $\sigma^{(\infty)}$ to at most show a weak W dependence. The fit to the experimental data led to $\sigma^{(\infty)} = \text{constant}$.

It is worth noting that the virtual photoabsorption cross section, or $F_2(x, Q^2)$, for $x = Q^2/W^2 \ll 1$ and any Q^2 only depends on the integrated quantities in (11) and (40).

From (39), at any Q^2, for sufficiently large energy, such that $\Lambda^2_{\text{sat}}(W^2) \gg Q^2$ and $\eta(W^2, Q^2) \to 0$, the virtual photoabsorption cross section approaches the "saturation limit" of [12,13]

$$\lim_{\substack{W^2\to\infty \\ Q^2 \text{ fixed}}} \frac{\sigma_{\gamma^* p}(\eta(W^2, Q^2))}{\sigma_{\gamma p}(W^2)} = 1. \tag{41}$$

The quantity $\Lambda^2_{\text{sat}}(W^2)$ indeed sets the scale for the limiting behavior (41). The terminology "saturation scale" for the effective gluon transverse momentum squared, $(1/6)\Lambda^2_{\text{sat}}(W^2)$, originating from the underlying two-gluon exchange, is indeed appropriate.

In conclusion: the previously formulated color-dipole approach to DIS (the generalized vector dominance/color dipole picture, GVD-CDP) has been examined with respect to the underlying singlet quark and gluon distribution. Both of these distributions being proportional to the saturation scale, $\Lambda^2_{\text{sat}}(W^2)$, we find that the evolution equations lead to a remarkable constraint on the value of the exponent of the W^2 dependence that agrees with the experimental result.

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References

[1] G. Altarelli, G. Parisi, Nucl. Phys. B 126 (1977) 298;
V.N. Gribov, L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438;
L.N. Lipatov, Sov. J. Nucl. Phys. 20 (1975) 96;
Y.L. Dokshitzer, Sov. Phys. JETP 46 (1977) 641.
[2] K. Prytz, Phys. Lett. B 311 (1993) 286.
[3] N.N. Nikolaev, B.G. Zakharov, Z. Phys. C 49 (1991) 607.
[4] F.E. Low, Phys. Rev. D 12 (1975) 163;
S. Nussinov, Phys. Rev. Lett. 34 (1975) 1286;
S. Nussinov, Phys. Rev. D 14 (1976) 246;
J. Gunion, D. Soper, Phys. Rev. D 15 (1977) 2617.
[5] J.B. Kogut, D.E. Soper, Phys. Rev. D 1 (1970) 2901;
J.B. Bjorken, J.B. Kogut, Phys. Rev. D 3 (1971) 1382.
[6] J.J. Sakurai, D. Schildknecht, Phys. Lett. B 40 (1972) 121;
B. Gorczyca, D. Schildknecht, Phys. Lett. B 47 (1973) 71.
[7] V.N. Gribov, Sov. Phys. JETP 30 (1970) 709.
[8] B.L. Joffe, Phys. Lett. 30 (1968) 123.
[9] N.N. Nikolaev, B.G. Zakharov, Phys. Lett. B 332 (1994) 184.
[10] L. Frankfurt, A. Radyushkin, M. Strikman, Phys. Rev. D 55 (1997) 98.
[11] M. Kuroda, D. Schildknecht, Phys. Rev. D 67 (2003) 094008.
[12] D. Schildknecht, Diffraction 2000, Nucl. Phys. B (Proc. Suppl.) 99 (2001) 121;
D. Schildknecht, in: G. Bruni, et al. (Eds.), The 9th International Workshop on Deep Inelastic Scattering, DIS 2001, Bologna, Italy, World Scientific, Singapore, 2002, p. 798.
[13] D. Schildknecht, B. Surrow, M. Tentyukov, Phys. Lett. B 499 (2001) 116;
G. Cvetic, D. Schildknecht, B. Surrow, M. Tentyukov, Eur. Phys. J. C 20 (2001) 77;
D. Schildknecht, B. Surrow, M. Tentyukov, Mod. Phys. Lett. A 16 (2001) 1829.
[14] D. Schildknecht, M. Tentyukov, hep-ph/0203028.
[15] M. Kuroda, D. Schildknecht, Eur. Phys. J. C 37 (2004) 205, hep-ph/0309153.
[16] A.M. Cooper-Sarkar, R.C.E. Devenish, A. De Roeck, hep-ph/9712301.
[17] F.J. Yndurain, hep-ph/9604263;
F.J. Yndurain, hep-ph/9605265.