The excitation of Guided-waves by underground point source: an investigation with theoretical seismograms

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Abstract

Near-Source scattering of Rg into S appears to be the primary contributor to the low-frequency Lg. The authors further suggest that the prominent low-frequency spectral null in Lg is due to Rg from a compensated linear vector dipole (CLVD) source, and the low-frequency null in Rg excitation is due to a zero-crossing of the horizontal displacement eigenfunctions. In this study, the mechanism of the excitation of Lg from explosions in layered earth structures are analyzed with theoretical seismograms. Our result shows that the CLVD source generates prominent Lg waves, and the null in the Lg spectra showing remarkably good agreement with those expected from Rg due to a CLVD source. We conclude that the derivative of displacement eigenfunction also takes a key role in the excitation of the null, only zero-crossing of the horizontal displacement eigenfunction can not fully explain it.

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1. Introduction

The Lg phase has a central role in nuclear test detection, discrimination, and yield estimation, as a result of its stable propagation property. However, the excitation mechanisms of the Lg phase are not fully understood. Regional Lg is a short-period guided wave, composed mainly of a sequence of multiple reflected post-critical S waves trapped in the crustal wave guide. The formation of Lg requires a significant amount of energy propagating as S waves, but an isotropic explosion is a poor S-wave source. Underground nuclear explosions usually generate much large Rg, and the influence of Rg scattering will generally be more distinct and observable on the spectra of S or Lg rather than on the various P phases.

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HE Yongfeng et al. / Procedia Engineering 37 (2012) 90 – 95

(such as Pn or Pg). Several recent studies\textsuperscript{[1-3]} have provided new insights into the generation of Lg from explosions by suggesting that the low-frequency part of the Lg spectra is mainly due to the near-source scattering of CLVD-generated Rg into S. Patton and Taylor\textsuperscript{[2]} also attributed the spectral null in CLVD generated Rg to the zero-crossing of the horizontal displacement eigenfunctions. Shot depth is important in defining the spectral null\textsuperscript{[4]}. This study however, differs from previous work in that it use a wavenumber integration algorithm based on generalized reflection and transmission coefficients method\textsuperscript{[5]}, which permit us to compute complete P-S wave field in an attenuating layered earth model in a more exact manner than the model summation or discrete wavenumber methods used by some earlier researchers\textsuperscript{[4]}.

2. The source model

For simplicity we restrict ourselves to the study of axisymmetric sources\textsuperscript{[2]}. We consider here two generic axisymmetric moment tensor sources: the pure explosion and the CLVD. The pure explosion moment tensor (\(M_e\)), and the CLVD moment tensor (\(M_c\)) can be expressed as,

\[
M_e = \begin{bmatrix} M_{XX} & M_{XY} & M_{ZX} \\ M_{YX} & M_{YY} & M_{YZ} \\ M_{ZX} & M_{ZY} & M_{ZZ} \end{bmatrix} = M_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

\(1\)

\[
M_c = \begin{bmatrix} M_{XX} & M_{XY} & M_{ZX} \\ M_{YX} & M_{YY} & M_{YZ} \\ M_{ZX} & M_{ZY} & M_{ZZ} \end{bmatrix} = M_{CLVD} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}
\]

\(2\)

where \(M_0\) is the isotropic explosion moment, and \(M_{CLVD}\) is the CLVD moment.

The velocity model used here is derived from model Steven\textsuperscript{[6]}. In this kind of high-near surface velocity model, the Lg energy is small theoretically, because most of the P-S energy shall leaks to the lower medium.

3. Theoretical seismograms

Fig.1 shows Green’s functions for a pure point explosion source at depths of 100,300,500,700,900,1100 and 1300 meters. We can see that, the Pg amplitude is roughly constant with depth, but the broadband Lg amplitude decreases with depth, because the primary source for S energy was the scattering of explosion-generated Rg into S near the free-surface, and this kind of Rg energy decreases dramatically with source depth.
At the same scale as Fig. 1, Fig. 2 shows Green’s functions of a CLVD source at the same depths. Note the strong energy of the Lg, showing that the deviatoric part of the CLVD source provides a direct source of SV energy to propagate at Lg phase velocities. The results shows that the CLVD produce more Lg than a pure explosion source over a broad bandwidth.

![Graph of Green's functions](image1)

**Fig. 1.** Explosion Green’s functions with different depths at a distance of 300 km

4. The characteristic of nulls

In order to test that the prominent low-frequency null in Lg is due to Rg from a CLVD source, the method of Patton’s spectral ratios is used here, and the variation of the Lg’ nulls with depths is compared with that of near source Rg’s. First, the theoretical seismograms of near source Rg from CLVD source at a distance of 20km for the same source depths are provided, and their spectrums are compared with those calculated from Lg as shown in Fig. 2. The results are shown in Fig. 3. Just as expected, the variations of the positions of the nulls with depths are extremely in the same way, showing systematic increase in spectral null frequency with decreasing shot depth. These results indicate that the prominent low-frequency spectral null in Lg is due to Rg from a compensated linear vector dipole (CLVD) source, and the CLVD source may be the primary contributor to the low-frequency Lg from nuclear explosions.

![Graph of Green's functions](image2)

**Fig. 2.** CLVD Green’s functions at a distance of 300 km
5. The mechanism of the excitation of null

From the basic principle that the Rg normal modes are non-trivial solutions of the free elastodynamic equation under appropriate boundary conditions, we naturally derive the phase velocities and eigenfunctions. Here the normal modes are calculated by the method proposed by Chen [7]. If surface waves are excited by a point source described by its moment tensor, by using the modal summation method, we can finally write the Rg wave displacement in the following form [8]. Let us first consider the case of a pure explosion source.

\[ u_z(r, \omega) = M_0 \sum_n r_2(z)M_0 \left( \frac{2}{\pi k_n(\omega) r} \right)^{1/2} \exp \left[ ik_n(\omega)r + \frac{\pi}{4} \right] \times \left\{ k_n(\omega)r_1(h) + \frac{dr_2}{dz_h} \right\} \]  

(3)

Where \( r_1 \) and \( r_2 \) are horizontal and vertical displacement eigenfunctions. Then consider the case of CLVD source

\[ u_z(r, \omega) = M_{CLVD} \sum_n r_2(z)M_{CLVD} \left( \frac{2}{\pi k_n(\omega) r} \right)^{1/2} \exp \left[ ik_n(\omega)r + \frac{\pi}{4} \right] \times \left\{ 2 \times \frac{dr_2}{dz_h} - k_n(\omega)r_1(h) \right\} \]  

(4)
We define (only the fundamental normal mode is considered, since for very shallow events these dominate the seismograms at teleseismic distances. We introduce the notation

\[ P_{\text{EXP}} = k_0(\omega) r_1(h) + \frac{dr_2}{dz}_h \]

\[ P_{\text{CLVD}} = 2 \times \frac{dr_2}{dz}_h - k_0(\omega) r_1(h) \]

We can see that the main difference between eq.(3) and eq.(4) comes from the difference between \( P_{\text{EXP}} \) and \( P_{\text{CLVD}} \). The fundamental normal-mode rayleigh wave displacement were shown in Fig.4 (0.4-2.5Hz). From Fig.4, we can see that the obvious zero-crossing in the horizontal displacement eigenfunctions and the derivative of the vertical displacement eigenfunctions. no zero-crossing was observed in the vertical displacement eigenfunctions, and this zero-crossing was considered to be the source of the excitation of the spectral null in Rg wave by several authors[9].

Inserting \( r_1 \) and \( \frac{dr_2}{dz} \) into eq.(5) and eq.(6), \( P_{\text{EXP}} \) and \( P_{\text{CLVD}} \) can be calculated, and the zero-crossing can be observed obviously in \( P_{\text{CLVD}} \), but no such zero-crossing in \( P_{\text{EXP}} \) will be observed. We know that the fundamental mode contributes the most energy to the final summation near the surface, so the zero-crossing of the \( P_{\text{CLVD}} \) will result in nulls in the Rayleigh wave spectrum. While the summation of a pure explosion source has no null.

![Fig. 4. The displacement eigenfunctions of fundamental normal-mode Rayleigh wave: (a) horizontal component and (b) derivative of vertical displacement eigenfunction of fundamental normal-mode](image-url)
6. Discussion and Conclusions

Our study of low-frequency Lg synthetics from CLVD source with considerable variation in shot depth, compared with that of Rg synthetics, clearly indicates that the Lg spectra is mainly due to the near-source scattering of CLVD-generated Rg into S. Our results fully support Patton and Taylor’s[2] argument that the spectral null in Lg from underground explosions is primarily due to a CLVD source. In explaining the mechanism of the excitation of the null in CLVD generated Rg, our result is more reasonable and more convincing: the spectral null is due to a combination of displacement eigenfunction and its derivative of the fundamental-mode Rayleigh wave controlled by CLVD source.

References