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Dynamic bidding analysis in power market based on the supply function ☆

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ABSTRACT

This paper presents a dynamic bidding model of the power market based on the Nash equilibrium and a supply function. The new model is composed of different dynamic systems and semismooth equations by means of the nonlinear complementarity method. Comparing with those existing bidding models, the remarkable characteristic of the new model is twofold: First, it adopts a dynamic bid so that the bidding limit point is the Nash equilibrium point of the market; Second, it considers the system requirement and the market property such as involving the transmission constraints in the network, and using a supply function which is suitable for the oligopolistic competitive power market. All of these imply that the new model is very close to the practical power market. The computation of the dynamic model is discussed by using the semismooth theory. A numerical simulation is presented to test the model behaviors in the uncogestion and the congestion cases, respectively. The numerical tests include the computing behavior of the dynamic model to reach Nash equilibrium points, the influence of the adjusted parameters and the system parameters to the Nash equilibrium, the local stability of the model, and the comparison of simulation effect between the proposed model and the Cournot model. The simulations show that the new bidding model is valid.

1. Introduction

A competitive electricity market includes different participants such as electric power producers, electric consumers and transmission network companies, etc. Under the supply-demand relationship and the unisonal operation of electrical price, generators adjust their own generation quantity, and users change their consumed quantity constantly. The objective of the market is to realize an optimal allocation of the power system resources so that each of the participants can obtain the maximal profit. The dynamic evolvement of an electricity market with the behavior of generation, consumed quantity and power price, includes various messages of the market and operates as the following processes:

- Each participant submits his/her bid to the Independent System Operations (ISO) at a period time t;
- The ISO, taking account of the security of transmission network, solves a social cost minimization (or a social welfare maximization) problem to get a dispatch and to announce participants for their production/consumption and prices;
- Each participant operates according to the dispatch of the ISO, and submits the bid of the next period time $t + 1$.

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According to the operation mechanism of the power market and the Nash equilibrium concept in economics, the maximal profit problems of the market participants consist of a set of correlative bi-level optimizations (see [1] and references therein). We call the solution set of bi-level optimization problems *Nash Equilibrium Points*. [1] studied the Nash equilibrium problem in the power market, including the existence of solutions and solution approaches. We know that in an actual competitive market environment, the ISO can not completely control the behavior of the market participants, and none of the participants has global information of the market. Both of these imply that in a competitive market it is impossible to obtain Nash equilibrium solutions by solving the Nash equilibrium problem unitedly or independently. Actual methods should be an adjusting process step-by-step with bounded rationality of the market participates so that the market reaches a Nash equilibrium point. This is called a game problem in economics. Our problem in this paper is to set a dynamic bidding manner and to model the power market.

The bidding manner has been extensively studied and various bidding models are presented in the power market (see [2–10] and reference therein). Among the various methods, the simplest way is to estimate the market clearing price of the next time and then present the bid with a lower price than the estimated one. This method is based on an assumption that the market clearing price is not affected by any bidding of the marketers, which is not suitable for the power market since it is controlled by a few big electrical companies and is an oligopolistic competition market. The second method is to estimate the behaviors of the rivals and to present the bid, including conjectured variations [7] and conjectured supply function [11]. The third method is based on the game theory [12] with oligopolistic strategy such as Bertrand model [6], the Cournot model [9], and supply function models [3].

Except for the study of the bidding manners, another key question is the stable property of the power market due to the dynamic action of the marketers. Alvarado et al. [13,14] first studied these issues. By establishing the first-order differential equations with the variables of generation quantities and consumption ones, the stable condition of the power market (also called the dynamic behavior approach to the market equilibrium) is analyzed. [15] also studied the stability of the power market by the controlled method.

We note that most of the strategy bidding methods in the power market rarely considered the transmission constraints since the constraints in the ISO optimization problems will increase the analysis difficulty. In order to set a strategy bid close to the practical systems and the market, based on the Cournot model, [9] proposed a dynamic bidding model by combining the transmission constraints, and the stability of the model is also studied. We know that the Cournot model is set on the basis of quantity. However, the power market is a competitive market of quantity-price; what's more, it does not suit the case of inelastic demand (or small inelasticity). These motivate us to study a new strategy bidding model of the power market.

Our objective in this paper is to design a new dynamic bidding model according to the characteristics of the power market and to make some analysis on the model. At the end, based on some theories and approaches of the optimization problems, we construct a different dynamic system with constraints of semismooth equations by using the supply function and the nonlinear complementarity problem (NCP) method. The specialties of the proposed dynamic model is twofold. First, it uses the Nash equilibrium theory as a basis. In other words, the final objective of the dynamic bidding is to arrive at the Nash equilibrium point with a suitable dynamic adjustment. Second, the proposed strategy bidding is considered close to the practical power market, including the transmission constraints, being true of the competitive behavior of markets by a supply function which can reflect the potential market power of participants. The model approach is also studied based on the semismooth theory. In order to test the validity of the proposed bidding model, a power network system with three buses/nodes is presented. Numerical simulations analyze the Nash equilibrium points for the two cases of transmission constraints (non-congestion and congestion). Furthermore, the infection of adjusted dynamic parameters and system parameters to Nash equilibrium points is also investigated. Some comparison results with the Cournot model are also presented in simulations. The local stability of the model is studied.

The paper is organized as follows. Section 2 presents the Nash equilibrium mathematical model of the power market. Section 3 constructs the dynamic bidding model based on the Nash equilibrium theory and the optimization theory. Section 4 discusses the approaches to the problems of the new model. In Section 5, numerical simulations for a power network is presented to test the new model. Some final remarks are made in the last Section.

Some notations are used in this paper. $q \equiv (q_0, q_1, \ldots, q_N)$ is the generation/consumption quantity at the nodes. Constant vector C is the line limits in the network. We use the index i to express the i-th node, where $a = (a_0, a_1, \ldots, a_N)^{\mathrm{T}} \equiv (a_i, a_{-i})^{\mathrm{T}}$ is the bidding vector with $a_{-i} = (a_0, \ldots, a_{i-1}, a_{i+1}, \ldots, a_N)^{\mathrm{T}}$. α_i is the adjusted parameter in the model of dynamic bidding. $\frac{\partial q_i}{\partial a_i}$ is the generalized derivative in the sense of Clarke [16]. a_{li} and a_{ui} are the bidding bounds. π_i means the profit function. p_i denotes the marginal price. F_i is called the dynamic function used in the dynamic model. The symbol $*$ expresses the corresponding value at the equilibrium point.

2. Nash equilibrium in the power market

This section is a basis on which we construct a dynamic bidding model. Consider an electric network of $N + 1$ nodes, where there is one generator ($i = 0, 1, \ldots, s$) and one consumer ($i = s + 1, \ldots, N$) at each node. A special node indexed $i = 0$ refers to the reference node (swing bus). The network has a set $\mathcal{L}$ of links, denoted by ij, which connect nodes i and j and have the distribution factor matrix $\Phi = (\phi_{ij})$ on each link ij. Suppose that the power flows are approximated by the DC model.

According to the operation pattern of the power market, on the one hand, the ISO considers the social cost minimization with *power balance*(powerflowingin = powerflowingout) and *line flow constraints* to decide generation and consumed quantity at each node, denoted by q_i ($i = 0, \ldots, N$). The behavior of the ISO can be expressed by the following quadratic programming:

$$\begin{aligned} &\text{minimize}_q && \sum_{i=0}^{N} c_i(q_i) \\ &\text{subject to} && q_0 + q_1 + \cdots + q_N = 0 \\ &&& -C_j \le \sum_{i=0}^{N} \phi_{ji} q_i \le C_j, \quad j \in \mathcal{L} = \{1, 2, \ldots, L\} \\ &&& q_i \ge 0, \quad i\text{: generator}, \qquad q_i \le 0, \quad i\text{: consumer}, \end{aligned} \tag{2.1}$$

where $c_i(q_i)$ is called a cost function and a benefit function for generator i or consumer i with form $c_i(q_i) = a_i q_i + b_i q_i^2$ when $q_i \ge 0$, and $c_i(q_i) = -a_i q_i - b_i q_i^2$ when $q_i \le 0$, respectively; $C_j > 0$ is the transmission limit of the line j. Here $a = (a_0, \ldots, a_N)^{\mathrm{T}}$ and $b = (b_0, \ldots, b_N)^{\mathrm{T}}$ are bidding parameters of the market participants. The power quantity $q = (q_0, \ldots, q_N)^{\mathrm{T}}$ is decided from the ISO problem (2.1) with the given bidding vectors a and b, which is denoted by $q = q(a, b)$. The power price is paid by quoted price, i.e., $p_i \equiv c_i'(q_i) = a_i + 2b_i q_i$, which is also called the local marginal price (LMP).

Let $A_i q + B_i q^2$ with $q \ge 0$ be the actual cost function of generator i, and let $-A_j q - B_j q^2 = A_j(-q) - B_j(-q)^2$ with $q \le 0$ be the actual benefit function of consumer j with constants $A_i \ge 0, B_i \ge 0$. On the other hand, generators and consumers take their optimal production decision according to the cost or the benefit functions with the aim of the maximum profit. Then participant i's profit maximization problem is:

$$\begin{aligned} &\text{maximize}_{(a_i, b_i)} && (a_i + 2b_i q_i) q_i - (A_i q_i + B_i q_i^2) \\ &\text{subject to} && a_{li} \le a_i \le a_{ui} \\ &&& b_{li} \le b_i \le b_{ui} \\ &&& q_i \quad \text{such that } q = (q_0, \ldots, q_N)^{\mathrm{T}} \text{ solves (2.1)} \\ &&& \text{given other participants' cost/benefit bids,} (a_{-i}, b_{-i}) \end{aligned} \tag{2.2}$$

where $a_{-i} = (a_0, \ldots, a_{i-1}, a_{i+1}, \ldots, a_N)^{\mathrm{T}}$, $b_{-i} = (b_0, \ldots, b_{i-1}, b_{i+1}, \ldots, b_N)^{\mathrm{T}}$, a_{li} (b_{li}) and a_{ui} (b_{ui}) are the lower bound and the upper bound on a_i (b_i), respectively.

In a completely competitive market, the market participates submit their bids in order to obtain the maximal benefit according to the real case, i.e., $a_i = A_i$, $b_i = B_i$, which is also the seeking aim of the market manager. However, a practical power market is oligopolistic competition, and the market participates may obtain a high benefit via strategy bidding. Hence, we consider a special case where the bidding variable is just for a_i with fixed $b_i = B_i$. Then (2.2) is reduced to the following version

$$\begin{aligned} &\text{maximize}_{a_i} && (a_i - A_i) q_i + B_i q_i^2 \\ &\text{subject to} && a_{li} \le a_i \le a_{ui} \\ &&& q_i \quad \text{such that } q = (q_0, \ldots, q_N) \text{ solves (2.1)} \\ &&& \text{given other participants' cost/benefit bids } a_{-i}. \end{aligned} \tag{2.3}$$

It needs to point out that our analysis throughout this paper is suitable for the case $b_i \ne B_i$.

According to the marginal price $p_i = c_i'(q_i) = a_i + 2b_i q_i$, the objective function of (2.3) is called a profit function and is denoted by

$$\pi_i(a) = (a_i - A_i) q_i(a) + B_i q_i^2(a). \tag{2.4}$$

(2.2) and (2.3) are bi-level optimization problems, and their solutions are called *Nash equilibrium* points. The exact definition of a Nash equilibrium is given as follows.

Definition 2.1. A pure strategy Nash equilibrium for (2.3) is a vector $a^* = (a_0^*, a_1^*, \ldots, a_i^*, \ldots, a_N^*)^{\mathrm{T}}$ such that for each participant i, given all other participants' bid $a_{-i}^* = (a_0^*, \ldots, a_{i-1}^*, a_{i+1}^*, \ldots, a_N^*)^{\mathrm{T}}$, a_i^* maximizers the i-th participant's profit, that is

$$a_i^* \in \operatorname*{argmax}_{a_{li} \le a_i \le a_{ui}} \pi_i(a_i, a_{-i}^*). \tag{2.5}$$

3. The dynamic bidding model of the power market

Throughout this paper, the bidding form is considered a special case (2.3). Denote the solution of the ISO as $q_i^*(a)$ or $q_i^*(a_i, a_{-i})$. According to the operation mechanism of a power market, the bidding process in a practical market is dynamic. This means that every participant of the market will present their bid via the dynamic adjustment.

This section will set a mathematical model for the dynamic process of the power market. Suppose that every participate is bounded rational, then the object of each participant is to reach the Nash equilibrium point as time tends to be infinite. Furthermore, we assume that each participant knows some messages at the period time t, especially for his/her own. Then the participant decides the next period bid, denoted by $a_i(t+1)$. Based on these assumptions and the necessary conditions of the optimization, we set the dynamic bidding model for the i-th participant as follows:

$$\begin{aligned} a_i(t+1) &= F_i(a(t)) = a_i(t) + \kappa_i(a_i(t))\frac{\partial \pi_i(a(t))}{\partial a_i(t)} \\ &= a_i(t) + \kappa_i(a_i(t))\left[q_i(a(t)) + (a_i(t) - A_i + 2B_i q_i(a(t)))\frac{\partial q_i(a(t))}{\partial a_i(t)}\right], \end{aligned} \tag{3.1}$$

where $a_i(t)$ and $a_i(t+1)$ are bids at node i during the periods t and $t+1$, respectively; $\kappa_i(a_i(t))$ is the extent of variation for the ith participant following a given profit signal satisfying $\kappa_i(a_i(t)) \geq 0$. If we assume $\kappa_i(a_i(t)) = \alpha_i a_i(t)$, i.e. a linear function, then the positive constant α_i is called the speed of adjustment. Some illustrations for the dynamic model (3.1) are expressed as follows.

Remark 3.1. (i) Related to the static game model (2.2), (3.1) is the dynamic description of a bidding manner; Moreover, the dynamic process (3.1) means $a_i(t+1) = a_i(t)$ to be an equilibrium point, which is correspondent to $\frac{\partial \pi_i(a(t))}{\partial a_i(t)} = 0$. This implies that, if we omit the bounded consideration in (2.3), the equilibrium point is correspondent to the necessary condition point of optimization problem (2.3). From this viewpoint, we say that the dynamic bidding model (3.1) is set on the Nash equilibrium.

(ii) $\frac{\partial \pi_i(a(t))}{\partial a_i(t)}$ is called marginal profit in economics. The model (3.1) is a typical adjustment pattern in various engineering and economic areas. Generally, α is chosen as a small constant so that the calculation can satisfy the requirement of the variable, such as the positive (or negative) property.

(iii) In the iterative process of (3.1), the bidding variable a_i is constrained by $a_{li} \leq a_i \leq a_{ui}$ (see (2.3)). To this end, we will use the project approach in the practical implementation, i.e., the final bidding in $t+1$ time is defined by

$$\bar{a}_i(t+1) = \text{mid}\{a_{li}, a_{ui}, a_i(t+1)\}, \tag{3.2}$$

where $a_i(t+1)$ is obtained from (3.1), and the mid function is defined as

$$\text{mid}\{c, d, w\} = \begin{cases} c, & \text{if } w < c \\ w, & \text{if } c \leq w \leq d \\ d, & \text{if } w > d. \end{cases}$$

(iv) Since the solution $q_i(a(t))$ is a piecewise mapping due to the transmission limits (see Lemma 6.2.1 in [1]), the partial derivative $\frac{\partial q_i(a(t))}{\partial a_i(t)}$ is in the sense of Clarke generalized derivative (see [16]). Correspondingly, F_i is a multi-value function if one of the inequality constraints in (2.1) becomes active.

In the left part of this section, we reformulate the ISO problem (2.1) to a system of equations by means of nonlinear complementary methods. This will facilitate the calculation of (3.1). The KKT system of (2.1) can be written as

$$\begin{cases} a_i + 2b_i q_i + \lambda + \sum_{j=1}^{L} \phi_{j,i}(\overline{\mu}_j - \underline{\mu}_j) - \nu_i = 0, \quad (i = 0, 1, \ldots, N) \\ q_0 + q_1 + \cdots + q_N = 0 \\ C_j + \sum_{i=1}^{N} \phi_{j,i} q_i \geq 0,\ \underline{\mu}_j \geq 0,\ \underline{\mu}_j \left(C_j + \sum_{i=1}^{N} \phi_{j,i} q_i\right) = 0, \quad (j = 1, 2, \ldots, L) \\ C_j - \sum_{i=1}^{N} \phi_{j,i} q_i \geq 0,\ \overline{\mu}_j \geq 0,\ \overline{\mu}_j \left(C_j - \sum_{i=1}^{N} \phi_{j,i} q_i\right) = 0, \quad (j = 1, 2, \ldots, L) \\ q_i \geq 0,\ \nu_i \geq 0,\ q_i \nu_i = 0 \quad (i = 0, 1, \ldots, s) \\ q_i \leq 0,\ \nu_i \leq 0,\ q_i \nu_i = 0 \quad (i = s+1, \ldots, N) \end{cases} \tag{3.3}$$

where λ, $\underline{\mu}_j$, $\overline{\mu}_j$, ν_i are Lagrange multipliers of the optimization problem.

Consider the Fisher–Burmeister (FB) NCP function $\psi : R^2 \to R$ defined by

$$\psi(a, b) = a + b - \sqrt{a^2 + b^2},$$

which has a typical property as

$$\psi(a, b) = 0 \Longleftrightarrow a \geq 0, \quad b \geq 0,\ ab = 0$$

and is semismooth (see [17] for details).

Then the KKT system (3.3) is equivalent to the following semismooth equations.

$$\Psi(a,y)=\begin{pmatrix} a_i+2b_iq_i+\lambda+\sum_{j=1}^{L}\phi_{j,i}(\overline{\mu}_j-\underline{\mu}_j)-\nu_i \quad (i=0,1,\ldots,N)\\ q_0+q_1+\cdots,q_N\\ \underline{\mu}_j+\left(C_j+\sum_{i=1}^{N}\phi_{j,i}q_i\right)-\sqrt{\underline{\mu}_j^2+\left(C_j+\sum_{i=1}^{N}\phi_{j,i}q_i\right)^2} \quad (j=1,2,\ldots,L)\\ \overline{\mu}_j+\left(C_j-\sum_{i=1}^{N}\phi_{j,i}q_i\right)-\sqrt{\overline{\mu}_j^2+\left(C_j-\sum_{i=1}^{N}\phi_{j,i}q_i\right)^2} \quad (j=1,2,\ldots,L)\\ q_i+\nu_i-\sqrt{q_i^2+\nu_i^2} \quad (i=0,1,\ldots,s)\\ (-q_i)+(-\nu_i)-\sqrt{(-q_i)^2+(-\nu_i)^2} \quad (i=s+1,\ldots,N)\end{pmatrix}=0, \tag{3.4}$$

where $y=(q,\lambda,\underline{\mu},\overline{\mu},\nu)\in R^{(N+1)+1+2L+(N+1)}$, $L=|\mathcal{L}|$ is the cardinality of the set $\mathcal{L}$. From the semismoothness of the FB-NCP function, it follows that $\Psi(a,y):R^{(N+1)}\times R^{(N+1)+1+2L+(N+1)}\to R^{(N+1)+1+2L+(N+1)}$ is semismooth.

(3.1) and (3.4) consist of a difference dynamic system with constraints of semismooth equations, which is the new bidding model proposed in this paper. The optimization problem (2.1) is a strictly convex quadratic program. Hence, the solution of (3.4) is identical to the one of (2.1). Moreover, there are many algorithms can solve it effectively.

Note that in the bidding model, we use the KKT system (3.4) to replace the optimization problem (2.1). This replacement is based on two reasons. One is that to solve a system of nonlinear equations is much easier than for solving an optimization problem. Other is by using the KKT system, we can analyze the relationship of variables easily, which may be used in the dynamic bidding model, for example, $\frac{\partial q_i(a(t))}{\partial a_i(t)}$ in (3.1). The later is our main consideration for this replacement.

4. Computation of the dynamic bidding model

Denote (3.1) as

$$a_i(t+1)=F_i(a(t))=F_i(a_i(t),a_{-i}(t)), \tag{4.1}$$

where

$$F_i(a_i(t),a_{-i}(t))=a_i(t)+\kappa_i(a_i(t))\left[q_i(a(t))+(a_i(t)-A_i+2B_iq_i(a(t)))\frac{\partial q_i(a(t))}{\partial a_i(t)}\right]. \tag{4.2}$$

In the dynamic model (4.1), we assume that $\kappa_i(a_i(t))$ has the following linear version

$$\kappa_i(a_i)=\alpha_i a_i(t). \tag{4.3}$$

Then from Lemma 6.2.1 and Theorem 6.2.1 in [1] we know that $F_i(a(t))$ is a piecewise quadratic function.

By using the dynamic model (4.1), we need to compute $q_i(a(t))$ and $\frac{\partial q_i(a(t))}{\partial a_i(t)}$ in (4.1). To this end, we mainly consider the system of Eq. (3.4) in the follows.

Let $\pi_y\partial\Psi(a,y)$ be the set of all $[(N+1)+1+2L+(N+1)]\times[(N+1)+1+2L+(N+1)]$ matrices M_2 such that for some $[(N+1)+1+2L+(N+1)]\times(N+1)$ matrix M_1, the $[(N+1)+1+2L+(N+1)]\times\{[(N+1)+1+2L+(N+1)]+(N+1)\}$ matrix $[M_1,M_2]$ belongs to $\partial\Psi(a,y)$. Let $\pi_a\partial\Psi(a,y)$ be such that $[\pi_a\partial\Psi(a,y),\pi_y\partial\Psi(a,y)]=\partial\Psi(a,y)$. From Theorem 1.1 and 2.1 in [18], we have the following conclusion.

Theorem 4.1. *For any fixed a^*, let $y^*=(q^*,\lambda^*,\underline{\mu}^*,\overline{\mu}^*,\nu^*)$ be a solution of (3.4) (or q^* be a solution of (2.1)), i.e., $\Psi(a^*,y^*)=0$. If $\pi_y\partial\Psi(a^*,y^*)$ is of maximal rank, then there exist an open neighborhood Y of a^* and a function $G(\cdot):Y\to R^{(N+1)+1+2L+(N+1)}$ such that G is locally Lipschitz in Y, $G(a^*)=y^*$ and for every a in Y,*

$$\Psi(a,G(a))=0. \tag{4.4}$$

Moreover, if Ψ has a superlinear approximate property at (a^*,y^*), G is superlinearly approximate at a^*.

Since $\Psi(a,y)$ is semismooth, then it is H-differentiable with $\partial_B\Psi(a,y)$ as an H-differential (see [19] for the concept of H-differential). Denote the H-differential of G by $T_G(a)$ at point a. Then from Theorem 4 in [19], the H-differential of implicit function G has the following version:

$$T_G(a)=\{-M_1^{-1}M_2:[M_1,M_2]\in\partial_B\Psi(a,y)\}. \tag{4.5}$$

Theorem 4.1 shows that under the so-called *maximal rank* condition, from (3.4) there exists an implicit function relationship between variable a and y as

$$y = \begin{pmatrix} q \\ \lambda \\ \underline{\mu} \\ \overline{\mu} \\ \nu \end{pmatrix} = G(a) = \begin{pmatrix} q(a) \\ \lambda(a) \\ \underline{\mu}(a) \\ \overline{\mu}(a) \\ \nu(a) \end{pmatrix}. \tag{4.6}$$

Furthermore, we can compute the H-differential of G according to (4.5).

In the next part of this subsection, we compute the elements of $T_G(a)$. In order to express $\Psi(a, y)$ simply, we denote

$$\begin{aligned}
&\varphi_1 \equiv a_i + 2b_i q_i + \lambda + \sum_{j=1}^{L} \phi_{j,i}(\overline{\mu}_j - \underline{\mu}_j) - \nu_i \quad (i = 0, 1, \dots, N) \\
&\varphi_2 \equiv q_0 + q_1 + q_2 + \cdots + q_N \\
&\varphi_3 \equiv \underline{\mu}_j + \left(C_j + \sum_{i=1}^{N} \phi_{j,i} q_i\right) - \sqrt{\underline{\mu}_j^2 + \left(C_j + \sum_{i=1}^{N} \phi_{j,i} q_i\right)^2} \quad (j = 1, 2, \dots, L) \\
&\varphi_4 \equiv \overline{\mu}_j + \left(C_j + \sum_{i=1}^{N} \phi_{j,i} q_i\right) - \sqrt{\overline{\mu}_j^2 + \left(C_j + \sum_{i=1}^{N} \phi_{j,i} q_i\right)^2} \quad (j = 1, 2, \dots, L) \\
&\varphi_5 \equiv q_j + \nu_j - \sqrt{q_j^2 + \nu_j^2} \quad (j = 0, 1, \dots, s) \\
&\varphi_6 \equiv (-q_j) + (-\nu_j) - \sqrt{(-q_j)^2 + (-\nu_j)^2} \quad (j = s+1, s+2, \dots, N)
\end{aligned} \tag{4.7}$$

and for $j = 1, \dots, L$

$$\underline{g}_j(q) \equiv C_j + \sum_{i=1}^{N} \phi_{j,i} q_i, \qquad \overline{g}_j(q) \equiv C_j - \sum_{i=1}^{N} \phi_{j,i} q_i. \tag{4.8}$$

For each $M_2 \in \pi_y \partial \Psi(a, y)$, denote

$$M_2 = \begin{pmatrix} \frac{\partial \varphi_1}{\partial q} & \frac{\partial \varphi_1}{\partial \lambda} & \frac{\partial \varphi_1}{\partial \underline{\mu}} & \frac{\partial \varphi_1}{\partial \overline{\mu}} & \frac{\partial \varphi_1}{\partial \nu} \\ \frac{\partial \varphi_2}{\partial q} & \frac{\partial \varphi_2}{\partial \lambda} & \frac{\partial \varphi_2}{\partial \underline{\mu}} & \frac{\partial \varphi_2}{\partial \overline{\mu}} & \frac{\partial \varphi_2}{\partial \nu} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \varphi_6}{\partial q} & \frac{\partial \varphi_6}{\partial \lambda} & \frac{\partial \varphi_6}{\partial \underline{\mu}} & \frac{\partial \varphi_6}{\partial \overline{\mu}} & \frac{\partial \varphi_6}{\partial \nu}, \end{pmatrix}, \tag{4.9}$$

where each element of M_2 has the following versions:

$$\frac{\partial \varphi_1}{\partial q} = \begin{pmatrix} 2b_0 & 0 & \cdots & 0 \\ 0 & 2b_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 2b_N \end{pmatrix}, \qquad \frac{\partial \varphi_1}{\partial \lambda} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix},$$

$$\frac{\partial \varphi_1}{\partial \underline{\mu}} = -\left(\varphi_{j,i}\right), \qquad \frac{\partial \varphi_1}{\partial \overline{\mu}} = \left(\varphi_{j,i}\right), \quad (j = 1, \dots, L;\ i = 0, 1, \dots, N)$$

$$\frac{\partial \varphi_1}{\partial \nu} = -\mathbf{1}_{(N+1)\times(N+1)} \in R^{(N+1)\times(N+1)},$$

$$\frac{\partial \varphi_2}{\partial y} = \left(\frac{\partial \varphi_2}{\partial q}, 0, \dots, 0\right) = \left(\mathbf{1}_{1\times(N+1)}, \mathbf{0}_{1\times(1+2L+N+1)}\right).$$

$$\frac{\partial \varphi_3}{\partial q} = \mathrm{diag}(\beta_1(w), \beta_2(w), \dots, \beta_L(w)) * \frac{\partial \underline{g}(q)}{\partial q}, \qquad \frac{\partial \varphi_3}{\partial \underline{\mu}} = \mathrm{diag}(\gamma_1(w), \gamma_2(w), \dots, \gamma_L(w)),$$

$$\frac{\partial \varphi_3}{\partial \lambda} = \mathbf{0}_{L\times 1}; \qquad \frac{\partial \varphi_3}{\partial \overline{\mu}} = \mathbf{0}_{L\times L}; \qquad \frac{\partial \varphi_3}{\partial \nu} = \mathbf{0}_{L\times(N+1)}$$

with

$$\frac{\partial \underline{g}(q)}{\partial q} = \begin{pmatrix} 0 & \phi_{1,1} & \cdots & \phi_{1,N} \\ 0 & \phi_{2,1} & \cdots & \phi_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \phi_{L,1} & \cdots & \phi_{L,N} \end{pmatrix}$$

and

$$\begin{cases} \beta_j(w) = 1 - \dfrac{\underline{N}_j}{\underline{M}_j}, \quad \gamma_j(w) = 1 - \dfrac{\underline{\mu}_j}{\underline{M}_j}, & \text{if } (\underline{g}_j(x), \underline{\mu}_j) \neq 0, \\ \beta_j(w) = 1 - \xi_j, \quad \gamma_j(w) = 1 - \rho_j, & \text{if } (\underline{g}_j(x), \underline{\mu}_j) = 0, \end{cases}$$

where $\underline{N}_j = C_j + \sum_{i=1}^{N} \phi_{j,i} q_i$, $\underline{M}_j = \sqrt{\underline{\mu}_j^2 + \underline{N}_j^2}$, $\|(\xi_j, \rho_j)\| \leq 1$ for $j = 1, \ldots, L$.

$$\frac{\partial \varphi_4}{\partial q} = \mathrm{diag}(\beta_1(w), \beta_2(w), \ldots, \beta_L(w)) * \frac{\partial \overline{g}(q)}{\partial q}, \qquad \frac{\partial \varphi_4}{\partial \overline{\mu}} = \mathrm{diag}(\gamma_1(w), \gamma_2(w), \ldots, \gamma_L(w)),$$
$$\frac{\partial \varphi_4}{\partial \lambda} = \mathbf{0}_{L\times 1}, \qquad \frac{\partial \varphi_4}{\partial \underline{\mu}} = \mathbf{0}_{L\times L}, \qquad \frac{\partial \varphi_4}{\partial \nu} = \mathbf{0}_{L\times(N+1)},$$

where

$$\frac{\partial \overline{g}(q)}{\partial q} = -\frac{\partial \underline{g}(q)}{\partial q},$$

$\beta_j(w)$ and $\gamma_j(w)$ have the same version as before with $\underline{N}_j$, $\underline{M}_j$ being substituted by $\overline{N}_j = C_j - \sum_{i=1}^{N} \phi_{j,i} q_i$, $\overline{M}_j = \sqrt{\overline{\mu}_j^2 + \overline{N}_j^2}$ for $j = 1, \ldots, L$, respectively.

$$\frac{\partial \varphi_5}{\partial q} = \mathrm{diag}(\beta_0(w), \beta_1(w), \ldots, \beta_s(w)) * (\mathbf{I}_{s\times s}, \mathbf{0}_{s\times(N-s)}),$$
$$\frac{\partial \varphi_5}{\partial \nu} = \mathrm{diag}(\gamma_0(w), \gamma_1(w), \ldots, \gamma_s(w)) * (\mathbf{I}_{s\times s}, \mathbf{0}_{s\times(N-s)}),$$
$$\frac{\partial \varphi_5}{\partial \lambda} = \mathbf{0}_{(s+1)\times 1}, \qquad \frac{\partial \varphi_5}{\partial \underline{\mu}} = \mathbf{0}_{(s+1)\times L}, \qquad \frac{\partial \varphi_5}{\partial \overline{\mu}} = \mathbf{0}_{(s+1)\times L},$$

where

$$\begin{cases} \beta_i(w) = 1 - \dfrac{q_i}{Z_i}, \quad \gamma_i(w) = 1 - \dfrac{\nu_i}{Z_i}, & \text{if } (q_i, \nu_i) \neq 0, \\ \beta_i(w) = 1 - \xi_i, \quad \gamma_i(w) = 1 - \rho_i, & \text{if } (q_i, \nu_i) = 0, \end{cases}$$

with $Z_i = \sqrt{q_i^2 + \nu_i^2}$ and $\|(\xi_i, \rho_i)\| \leq 1$ for $i = 1, \ldots, s$.

$$\frac{\partial \varphi_6}{\partial q} = -\mathrm{diag}(\beta_{s+1}(w), \beta_{s+2}(w), \ldots, \beta_N(w)) * (\mathbf{0}_{(N-s)\times(s+1)}, \mathbf{I}_{(N-s)\times(N-s)}),$$
$$\frac{\partial \varphi_6}{\partial \nu} = -\mathrm{diag}(\gamma_{s+1}(w), \gamma_{s+2}(w), \ldots, \gamma_N(w)) * (\mathbf{0}_{(N-s)\times(s+1)}, \mathbf{I}_{(N-s)\times(N-s)}),$$
$$\frac{\partial \varphi_6}{\partial \lambda} = \mathbf{0}_{(N-s)\times 1}, \quad \frac{\partial \varphi_6}{\partial \underline{\mu}} = \mathbf{0}_{(N-s)\times L}, \qquad \frac{\partial \varphi_6}{\partial \overline{\mu}} = \mathbf{0}_{(N-s)\times L}$$

where

$$\begin{cases} \beta_i(w) = 1 + \dfrac{q_i}{Z_i}, \quad \gamma_i(w) = 1 + \dfrac{\nu_i}{Z_i}, & \text{if } (q_i, \nu_i) \neq 0, \\ \beta_i(w) = 1 - \xi_i, \quad \gamma_i(w) = 1 - \rho_i, & \text{if } (q_i, \nu_i) = 0, \end{cases}$$

with $Z_i = \sqrt{q_i^2 + \nu_i^2}$ and $\|(\xi_i, \rho_i)\| \leq 1$ for $i = (s + 1, \ldots, N)$.

To compute $\pi_a \partial \Psi(a, y)$, for each $M_1 \in \pi_a \partial \Psi(a, y)$, it has

$$M_1 = \left(\left(\frac{\partial \varphi_1}{\partial a}\right)^{\mathrm{T}}, \left(\frac{\partial \varphi_2}{\partial a}\right)^{\mathrm{T}}, \left(\frac{\partial \varphi_3}{\partial a}\right)^{\mathrm{T}}, \left(\frac{\partial \varphi_4}{\partial a}\right)^{\mathrm{T}}, \left(\frac{\partial \varphi_5}{\partial a}\right)^{\mathrm{T}}, \left(\frac{\partial \varphi_6}{\partial a}\right)^{\mathrm{T}} \right)^{\mathrm{T}} \tag{4.10}$$

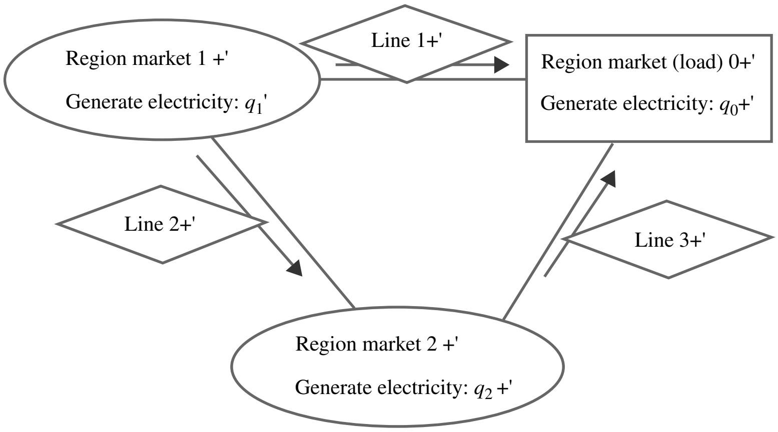

Fig. 1. Solution structure of power market.

Table 1
Parameters and initial values in case-I.

Node i	C_j (MW)	$a_i^{(0)}$ (y/MWh)	$b_i = B_i$ (y/MWh^2)	A_i (y/MWh)	α_i
0	5	2.57	0.07	2.83	0.05
1	5	1.23	0.05	1.52	0.03
2	5	1.22	0.04	1.46	0.02

where

$$\frac{\partial\varphi_1}{\partial a} = \mathbf{I}_{(N+1)\times(N+1)}, \qquad \frac{\partial\varphi_2}{\partial a} = 0, \qquad \frac{\partial\varphi_3}{\partial a} = \mathbf{0}_{L\times(N+1)},$$
$$\frac{\partial\varphi_4}{\partial a} = \mathbf{0}_{L\times(N+1)}, \qquad \frac{\partial\varphi_5}{\partial a} = \mathbf{0}_{(s+1)\times(N+1)}, \qquad \frac{\partial\varphi_6}{\partial a} = \mathbf{0}_{(N-s)\times(N+1)}.$$

Finally, from (4.5) we can obtain $\frac{\partial q_i(a)}{\partial a_i}$, as well as $F_i(a)$. Note that at the breakpoint, F_i is a multi-valued function.

Remark 4.1. The above formulas are used to compute terms $q_i(a(t))$ and $\frac{\partial q_i(a)}{\partial a_i}$ defined in the dynamic bidding model (4.1). In an actual market, they may be provided by the ISO.

5. Numerical simulation for the dynamic bidding model

Numerical simulations for (3.1) and (3.4) are tested in this section. In addition, since the system includes transmission constraints, the concepts of uncongestion and congestion are considered in the network of the power systems. If there is at least one of the transmission limits in the ISO optimization (2.1) to be active, we call the network system congestion; otherwise, we call it uncongestion. We will see that under the two cases, the system will take on different states of equilibrium and stability.

A power system of three-nodes (see Fig. 1) is considered. The direction of an arrow indicates the positive direction of the transmission power; indices i and j $(i, j = 0, 1, 2)$ express the node and line numbers, respectively. In details, $i = 0$ is the load node; the other two nodes $(i = 1, 2)$ are generators. The factor (distribution factor) matrix Φ is given by

$$\Phi = \begin{pmatrix} 0 & 1/3 & -1/3 \\ 0 & 1/3 & 2/3 \\ 0 & 2/3 & 1/3 \end{pmatrix}.$$

5.1. Numerical tests for equilibrium points

For the tested system, we consider two cases where the system chooses different system parameters and the same initial bidding. Note that an equilibrium point satisfies $a_i(t+1) = a_i(t)$ $(i = 0, 1, 2)$.

Case-I: Parameters and initial values used in model (3.1) and (3.4) are listed in Table 1. where $a_i^{(0)}$ $(i = 0, 1, 2)$ is the initial bidding value, and α_i $(i = 0, 1, 2)$ is the adjustment speed, respectively.

The calculating results show that in this case, the network system is uncongestion and the equilibrium point is

$$q^* = (q_0^*, q_1^*, q_2^*) = (-4.82, 2.11, 2.71), \qquad a^* = (a_0^*, a_1^*, a_2^*) = (2.62, 1.97, 1.83).$$

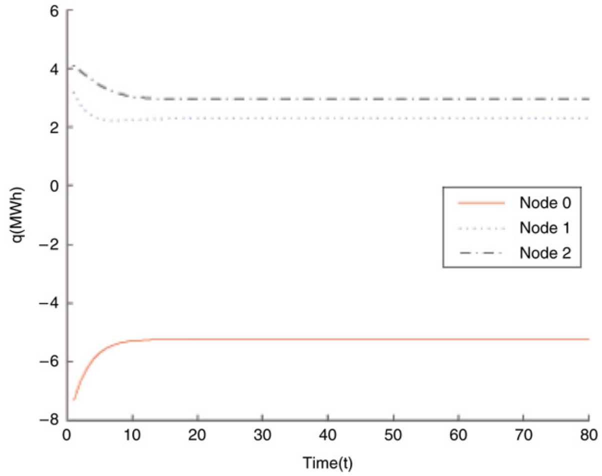

Fig. 2. Iterative process for uncongestion.

Table 2
Parameters and initial values in case-II.

Node i	C_j (MW)	$a_i^{(0)}$ (y/MWh)	$b_i = B_i$ (y/MWh^2)	A_i (y/MWh)	α_i
0	5	2.57	0.07	2.83	0.05
1	0.5	1.23	0.05	1.52	0.006
2	5	1.22	0.04	1.46	0.02

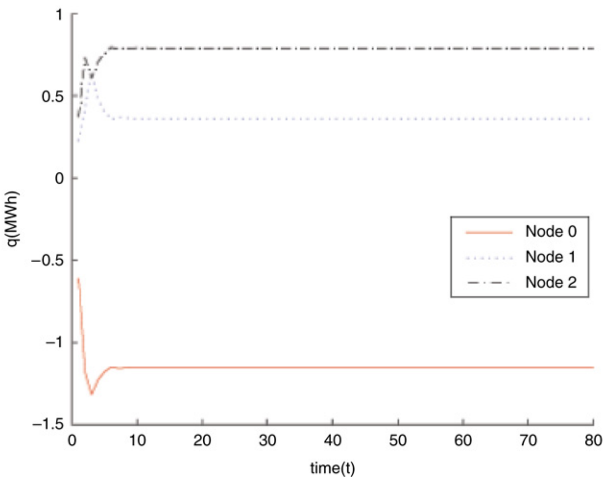

Fig. 3. Iterative process with the congestion in line 2.

The calculating tracks for the three nodes are drawn in Fig. 2 where the real-line (red), dotted-line (blue) and dashdotted-line (black) express the bidding process for nodes $i = 0, 1, 2$, respectively.

Case-II: Parameters and initial values used in model (3.1) and (3.4) are listed in Table 2.

From Table 2 we can see that when the limited value of line 2 is reduced, the possibility of congestion in the transmission lines well be increased. Indeed, the computing results show that in this case, when the congestion of the network system happens in line 2, we obtain the equilibrium point

$$q^* = (q_0^*, q_1^*, q_2^*) = (-1.15, 0.36, 0.79), \qquad a^* = (a_0^*, a_1^*, a_2^*) = (2.19, 1.68, 1.55).$$

The calculating tracks are shown in Fig. 3, where each line has the same meaning as that in Fig. 2.

In order to analyze the influence of the adjustment speed α_i to the equilibrium state, as an example, we change α_1 by step-up manner and fix $\alpha_0 = 0.05$, $\alpha_2 = 0.02$ for Case-II (see Table 2). The simulation shows that when $\alpha_1 \geq 0.04$, there is no equilibrium point in the system, and the dynamic evolution appears a periodical variation with a chaos phenomenon, see Fig. 4 for case $\alpha = (\alpha_0, \alpha_1, \alpha_2) = (0.05, 0.04, 0.02)$. Furthermore, the chaotic attractor process of $a_1(t)$ and $a_1(t + 1)$ is tracked in Fig. 5.

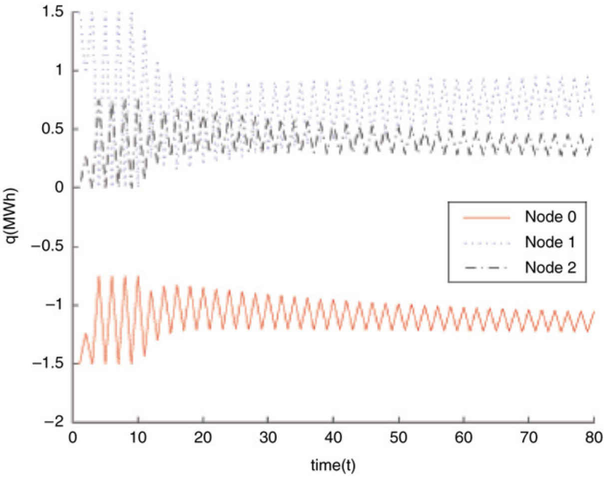

Fig. 4. Chaotic variation process with $\alpha = (0.05, 0.04, 0.02)$.

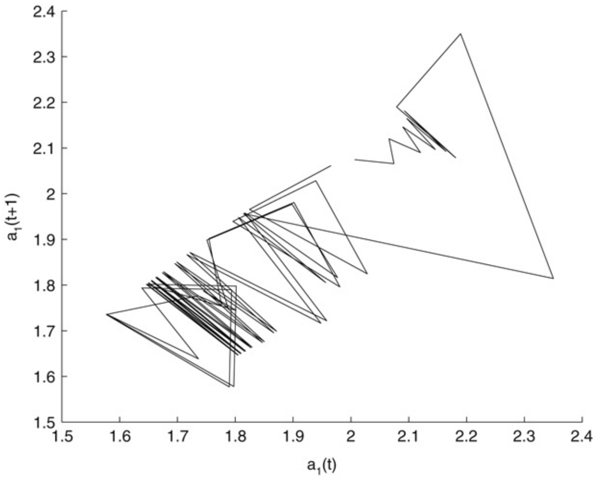

Fig. 5. Chaotic Attractors of bidding quantities for $a_1(t)$.

The simulation results show that the Nash equilibrium depends heavily on the choice of the adjusted parameters α_i $(i = 0, 1, 2)$. The variation of the state parameters can cause a remarkable difference in the moving track along with the dynamic bidding process. Moreover, when the market enters the chaotic state, the quantities of dynamic adjustments can not be decided effectively.

5.2. Computing comparison with the Cournot model

We use the Cournot model proposed in [9] to test the same example system in this subsection in order to analyze the effect of the new model. The two cases (uncongestion and congestion) are also studied in the comparison.

For the uncongestion case, the price bidding parameter is set $a = (2.62, 1.97, 1.83)$, and the other parameters are showed in Table 1. Both of these are the same as the ones of Case-I in Section 5.1. By using the dynamic Cournot model, the Nash equilibrium is solved as $q^* = (q_0^*, q_1^*, q_2^*) = (-4.37, 2.36, 2.01)$, and the calculating tracks for the tested system are drawn in Fig. 6.

We analyze the computing result for the dynamic Cournot model. On the one hand, the actual cost of the second generator (node-2) is lower than the first one (node-1); On the other hand, the bidding value a (quoted price) satisfies $a_2 < a_1$ at the equilibrium point. From the viewpoint of the competitive markets, both of the two observations should bear a result that the generation quantity of the second generator (node-2) is more than the first one (node-1). However, the computing result is $q_1^* = 2.36 > 2.01 = q_2^*$, which obtains an oppositive conclusion.

For the congestion case, as with the tests in the last subsection, we reduce the limit of line 2 and choose the same values in the Cournot model (see Table 2). Then line 2 happens congestion, and the bidding parameter is given according

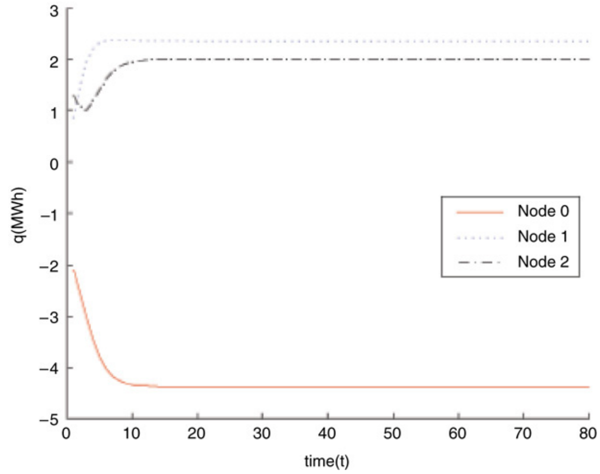

Fig. 6. Iterative process for uncongestion (Cournot).

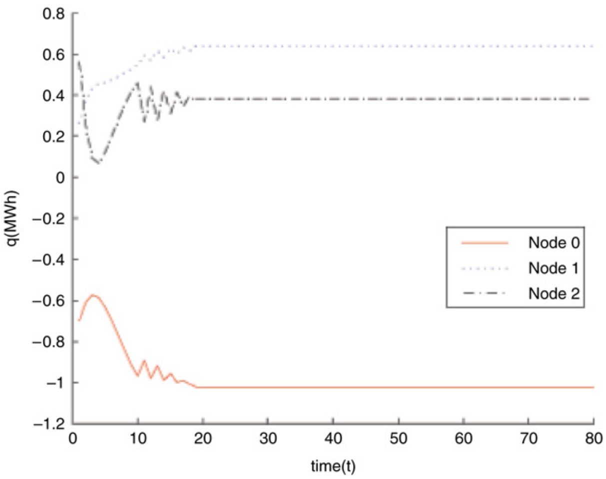

Fig. 7. Iterative process with the congestion in line 2 (Cournot).

to the computing result in Case-II as $a = (2.19, 1.68, 1.55)$. By using the dynamic Cournot bidding model, we get the Nash equilibrium solution $q^* = (q_0^*, q_1^*, q_2^*) = (-1.02, 0.64, 0.38)$. The calculating tracks for the three nodes are drawn in Fig. 7.

Similar to the uncongestion analysis, for the congestion case, the numerical computation via the Cournot model obtains an oppositive result relative to the competitive rules of markets.

Comparing the results of two models (i.e., the Cournot model and the supply function model used in this paper), we can see from Sections 5.1 and 5.2 that the dynamic bidding model based on the supply function expresses better market competition than the dynamic Cournot model. The reason is that the competitive behavior of markets is related to quantity and price. The shortcoming of the Cournot model is that it just considers the quantity of competition.

5.3. Local stability simulation for the dynamic bidding model

Another concerned question in the market is the stability at the equilibrium point. From the classical analysis method of smooth problems, the stability is set on eigenvalue analysis, i.e., to judge $|\lambda(\nabla F(a^*))| < 1$ for the dynamic model (4.1), where the dynamic function of the model (4.1)–(4.2) is

$$F(a) = \begin{pmatrix} a_0(t) + \alpha_0 a_0[q_0(a) + (a_0 - A_0 + 2B_0 q_0(a))]\dfrac{\partial q_0}{\partial a_0} \\ a_1(t) + \alpha_1 a_1[q_1(a) + (a_1 - A_1 + 2B_1 q_1(a))]\dfrac{\partial q_1}{\partial a_1} \\ a_2(t) + \alpha_2 a_2[q_2(a) + (a_2 - A_2 + 2B_2 q_2(a))]\dfrac{\partial q_2}{\partial a_2} \end{pmatrix}. \tag{5.1}$$

The dynamic stability is to study the relationship between the stability region and the dynamic adjusted parameters $\alpha = (\alpha_0, \alpha_1, \alpha_2)$. In order to facilitate the analysis and help the visualization, we fix $\alpha_2 = 0.02$ to the stability analysis in this subsection.

Note that the stability analysis of the dynamic system (4.1) is an implicit formula, which depends on the relationship of a and q. On the other hand, the uncongestion and congestion of the system will bring on different results. Hence, two cases are considered respectively. We have the following conclusion with respect to the relation of a and q.

Theorem 5.1. (i) *For the case of uncongestion, we have the relationship of a and q (also called optimal response curve) to be*

$$\begin{cases} q_0 = \dfrac{-a_0(b_2+b_1)+a_1b_2+a_2b_1}{2(b_1b_2+b_0b_2+b_1b_0)}, \\ q_1 = \dfrac{-a_1(b_2+b_0)+a_0b_2+a_2b_0}{2(b_1b_2+b_0b_2+b_1b_0)}, \\ q_2 = \dfrac{-a_2(b_0+b_1)+a_0b_1+a_1b_0}{2(b_1b_2+b_0b_2+b_1b_0)}. \end{cases} \tag{5.2}$$

(ii) *For the case of congestion where the congestion happens in Line 2 with $C_2 = 0.5$, the variables a and q satisfy*

$$\begin{cases} q_0 = \dfrac{2a_1-a_0-a_2-6C_2(b_2+2b_1)}{2(b_2+4b_1+b_0)}, \\ q_1 = \dfrac{a_0+a_2-2a_1-3C_2(b_0-b_2)}{b_2+4b_1+b_0}, \\ q_2 = \dfrac{3a_1-a_0-2a_2+3C_2(b_0+2b_1)}{b_2+4b_1+b_0}. \end{cases} \tag{5.3}$$

Proof. From the process of setting the bidding model, the variable q is a function of a determined from the KKT system (3.4) of the ISO optimization. We consider two cases as follows.

(i) For the uncongestion, we can derive the KKT system of the ISO to be

$$\begin{cases} a_0+2b_0q_0+\lambda = 0, \\ a_1+2b_1q_1+\lambda = 0, \\ a_2+2b_2q_2+\lambda = 0, \\ q_0+q_1+q_2 = 0. \end{cases} \tag{5.4}$$

To solve above system we obtain (5.2).

(ii) According to the calculation of the last subsection, the KKT system of the ISO for that case is

$$\begin{cases} a_0+2b_0q_0+\lambda = 0, \\ a_1+2b_1q_1+\lambda - \dfrac{1}{3}\underline{\mu}_3 = 0, \\ a_2+2b_2q_2+\lambda - \dfrac{2}{3}\underline{\mu}_3 = 0, \\ q_0+q_1+q_2 = 0, \\ C_2 - \dfrac{1}{3}q_1 - \dfrac{2}{3}q_2 = 0. \end{cases} \tag{5.5}$$

Then we follow the result (5.3). □

According to the result of Theorem 5.1, we can express the function F in the dynamic system (4.1) and its Jacobian, and then analyze the stability.

Case-I: Uncongestion case. From (5.2), the Jacobian of F has the following expression

$$\nabla F(a^*) = \begin{pmatrix} 1-15.57\alpha_0 & 1.24\alpha_0 & 1.55\alpha_0 \\ 1.23\alpha_1 & 1-14.35\alpha_1 & 2.47\alpha_1 \\ 0.04 & 0.06 & 0.67 \end{pmatrix}. \tag{5.6}$$

We change α_0 and α_1 with fixed $\alpha_2 = 0.02$, and draw the region of (α_0, α_1) which satisfies $|\lambda(\nabla F(a^*))| < 1$. The computing results are shown in Fig. 8 where the shady part is the stability region.

Case-II: Congestion case. Similar to the computation of equilibrium point, suppose that Line 2 happens congestion. Then F is a multi-valued function due to the piecewise smooth property of $q(a)$. For this case, the stability analysis is difficult due to the nonsmoothness of F. According to the approach of the generalized derivatives, we choose a special function $\bar{F}_i(a(t))$ as follows. Denote the solutions of (5.2) and (5.3) as $q^{un}(a)$ and $q^c(a)$, respectively, then a convex combination of them is defined as

$$\bar{q}(a(t)) = \beta q^{un}(a(t)) + (1-\beta)q^c(a(t)), \quad \beta \in [0, 1].$$

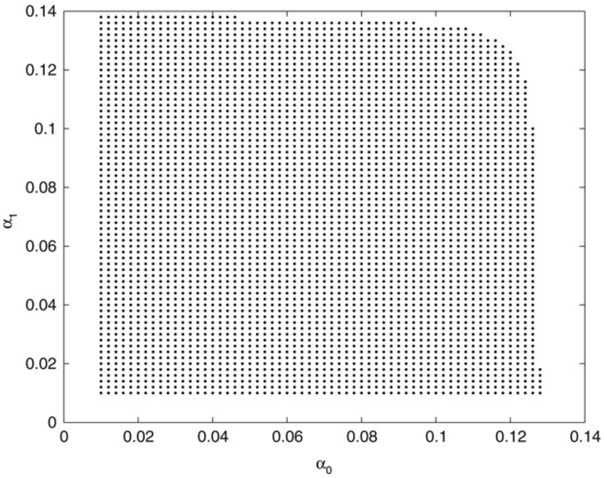

Fig. 8. Stability region with uncongestion.

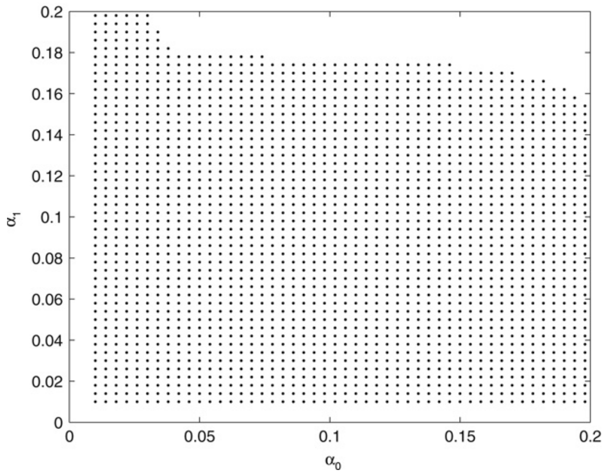

Fig. 9. Stability region with congestion.

From the above expression with the given β and (5.1), we can define the correspondent $\bar{F}_i(a(t))$ and compute $\nabla\bar{F}(a^*)$. Finally, we use a similar way as in Case-I to simulate the stability region.

From the numerical simulation shown in Fig. 9 with fixed $\beta = 0.2$, we find that the stability region changes with different β. But the shape of the stable region is not distinct in the region $0.1 < \beta < 0.9$.

From the stability analysis we can see that the stability is heavily related to the adjusted parameters of the dynamic model and the transmission limits.

6. Final remarks

This paper presents a dynamic bidding model of the power market based on the Nash equilibrium and the supply function. Numerical simulation studies the properties of the new model in the uncongestion and congestion cases, including the computing behaviors of the model, the inference of the adjusted parameter and system parameters to the market operation, as well as the stability of the model. Comparison between computation and the Cournot model is also presented. All the results show that the new dynamic model can be used to simulate the competitive behavior of the power market. We also note that for the congestion case, the function defined in the dynamic model loses some general properties, even the continuity of functions. This adds to the difficulties of the analysis and calculation to the models. How to set the stability analysis in theory and control the parameters so that the dynamic process of the market is stable are our further research.

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