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Fuzzy analytic hierarchy process: A logarithmic fuzzy preference programming methodology ☆

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ABSTRACT

Fuzzy analytic hierarchy process (AHP) proves to be a very useful methodology for multiple criteria decision-making in fuzzy environments, which has found substantial applications in recent years. The vast majority of the applications use a crisp point estimate method such as the extent analysis or the fuzzy preference programming (FPP) based nonlinear method for fuzzy AHP priority derivation. The extent analysis has been revealed to be invalid and the weights derived by this method do not represent the relative importance of decision criteria or alternatives. The FPP-based nonlinear priority method also turns out to be subject to significant drawbacks, one of which is that it may produce multiple, even conflict priority vectors for a fuzzy pairwise comparison matrix, leading to entirely different conclusions. To address these drawbacks and provide a valid yet practical priority method for fuzzy AHP, this paper proposes a logarithmic fuzzy preference programming (LFPP) based methodology for fuzzy AHP priority derivation, which formulates the priorities of a fuzzy pairwise comparison matrix as a logarithmic nonlinear programming and derives crisp priorities from fuzzy pairwise comparison matrices. Numerical examples are tested to show the advantages of the proposed methodology and its potential applications in fuzzy AHP decision-making.

1. Introduction

As a practical yet popular methodology for dealing with fuzziness and uncertainty in multiple criteria decision-making (MCDM), fuzzy analytic hierarchy process (AHP) has found huge applications in recent years. Since fuzzy judgments are easier to provide than crisp judgments, it can be concluded that fuzzy AHP will find more applications in the near future. The use of fuzzy AHP for multiple criteria decision-making requires scientific approaches for deriving the weights from fuzzy pairwise comparison matrices. Existing approaches for fuzzy AHP weight derivation can be classified into two categories, one of which is to derive a set of fuzzy weights from a fuzzy pairwise comparison matrix, while the other is to derive a set of crisp weights from a fuzzy pairwise comparison matrix. The approaches for deriving fuzzy weights from fuzzy pairwise comparison matrices mainly include the geometric mean method [7], fuzzy logarithmic least-squares methods (LLSM) [2,62,66], Lambda–Max methods [19,63] and the linear goal programming (LGP) method [64]. The approaches for deriving crisp weights from fuzzy pairwise comparison matrices include the extent analysis [18] and the fuzzy preference programming (FPP) based nonlinear method [45].

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Since fuzzy weights are not as easy to compute as crisp ones, our literature survey shows that the vast majority of the fuzzy AHP applications uses a simple extent analysis method proposed by Chang [18] for fuzzy AHP weight derivation for simplicity. However, such an extent analysis method has been revealed by Wang et al. [68] to be invalid and the weights derived by this method do not represent the relative importance of decision criteria or alternatives at all. It has led to a significantly large number of misapplications in the literature [1,3–6,8–17,19–44,47–49,50–61,67,69,70]. Apparently, its usage as a weight derivation method should be rejected. The FPP-based nonlinear priority method proposed by Mikhailov [45] has also found some applications in recent years [21,65]. Unfortunately, such a method also turns out to be subject to some significant drawbacks. For example, it may produce multiple, even conflict priority vectors for a fuzzy pairwise comparison matrix, leading to distinct conclusions. This non-uniqueness in solutions damages its applications as a priority method for fuzzy AHP.

To provide a valid yet practical priority method for fuzzy AHP, this paper proposes a logarithmic fuzzy preference programming (LFPP) based methodology for fuzzy AHP priority derivation, which formulates the priorities of a fuzzy pairwise comparison matrix as a logarithmic nonlinear programming and derives crisp priorities from fuzzy pairwise comparison matrices. The paper is organized as follows. Section 2 briefly reviews the FPP-based nonlinear priority method and illustrates its non-uniqueness in solutions. Section 3 proposes the LFPP-based methodology for fuzzy AHP weight derivation. Its validities are tested with numerical examples in Section 4. The paper concludes in Section 5.

2. The FPP-based nonlinear priority method and its non-uniqueness in solutions

Suppose the decision maker (DM) provides fuzzy judgments instead of precise judgments for a pairwise comparison matrix. For example, it could be judged that criterion i is between l_{ij} and u_{ij} times as important as criterion j with m_{ij} being the most likely times. Then, a fuzzy pairwise comparison matrix can be expressed as

$$\widetilde{A} = (\tilde{a}_{ij})_{n\times n} = \begin{bmatrix} 1 & (l_{12}, m_{12}, u_{12}) & \cdots & (l_{1n}, m_{1n}, u_{1n}) \\ (l_{21}, m_{21}, u_{21}) & 1 & \cdots & (l_{2n}, m_{2n}, u_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (l_{n1}, m_{n1}, u_{n1}) & (l_{n2}, m_{n2}, u_{n2}) & \cdots & 1 \end{bmatrix}, \tag{1}$$

where $l_{ij} = 1/u_{ji}$, $m_{ij} = 1/m_{ji}$, $u_{ij} = 1/l_{ji}$ and $0 < l_{ij} \leqslant m_{ij} \leqslant u_{ij}$ for all $i,j = 1,\ldots,n$; $j \neq i$. To find a crisp priority vector $W = (w_1,\ldots,w_n)^T > 0$ with $\sum_{i=1}^{n} w_i = 1$ for the fuzzy pairwise comparison matrix in (1), Mikhailov [45] introduces the following membership function for each fuzzy judgment in $\tilde{A}$:

$$\mu_{ij}\left(\frac{w_i}{w_j}\right) = \begin{cases} \frac{(w_i/w_j) - l_{ij}}{m_{ij} - l_{ij}}, & \frac{w_i}{w_j} \leqslant m_{ij}, \\ \frac{u_{ij} - (w_i/w_j)}{u_{ij} - m_{ij}}, & \frac{w_i}{w_j} \geqslant m_{ij}, \end{cases} \tag{2}$$

where $\mu_{ij}(w_i/w_j)$ is the membership degree of w_i/w_j belonging to the fuzzy judgment $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$. Let

$$\lambda = \min\left\{ \mu_{ij}(w_i/w_j) |\ i = 1,\ldots,n-1;\ j = i+1,\ldots,n \right\}. \tag{3}$$

Then, λ is the minimum membership degree to which the crisp priority vector satisfies each fuzzy pairwise comparison. It is hoped that the priority vector should be able to maximize the DM's satisfaction. For this hope, Mikhailov [45] established the following FPP-based nonlinear priority model, which is an extension of the FPP priority method for crisp pairwise comparison matrix [46] in fuzzy environments:

$$\begin{aligned} &\text{Maximize} \quad \lambda \\ &\text{Subject to} \begin{cases} \mu_{ij}(w_i/w_j) \geqslant \lambda,\ i = 1,\ldots,n-1;\ j = i+1,\ldots,n, \\ \sum_{i=1}^{n} w_i = 1, \\ w_i \geqslant 0,\ i = 1,\ldots,n, \end{cases} \end{aligned} \tag{4}$$

which can be equivalently expressed as

$$\begin{aligned} &\text{Maximize} \quad \lambda \\ &\text{Subject to} \begin{cases} -w_i + l_{ij}w_j + \lambda(m_{ij} - l_{ij})w_j \leqslant 0,\ i = 1,\ldots,n-1;\ j = i+1,\ldots,n, \\ w_i - u_{ij}w_j + \lambda(u_{ij} - m_{ij})w_j \leqslant 0,\ i = 1,\ldots,n-1;\ j = i+1,\ldots,n, \\ \sum_{i=1}^{n} w_i = 1, \\ w_i \geqslant 0,\ i = 1,\ldots,n. \end{cases} \end{aligned} \tag{5}$$

If the optimal objective value $\lambda^* > 0$, then the optimal solution $w_1^*,\ldots,w_n^*$ satisfy $l_{ij} \leqslant w_i/w_j \leqslant u_{ij}$; otherwise, there exists strong inconsistency among the fuzzy judgments and the optimal solutions only approximately satisfy the fuzzy pairwise comparison matrix.

For convenience, we refer to the method that uses the above model (5) for fuzzy AHP priority derivation as the FPP-based nonlinear priority method. With regard to this method, we have the following research findings:

1. Negative membership degree makes no sense.
2. Model (5) produces multiple optimal solutions when there exists strong inconsistency among the fuzzy judgments.
3. The priority vectors derived by using the upper or lower triangular elements of a fuzzy pairwise comparison matrix are not the same, even significantly different.

Consider the following fuzzy pairwise comparison matrix as an example:

$$\widetilde{B}=\begin{bmatrix} 1 & \left(\frac{3}{2},2,\frac{5}{2}\right) & \left(\frac{3}{2},2,\frac{5}{2}\right) & \left(\frac{2}{3},1,\frac{3}{2}\right) \\ \left(\frac{2}{5},\frac{1}{2},\frac{2}{3}\right) & 1 & (1,1,1) & \left(\frac{3}{2},2,\frac{5}{2}\right) \\ \left(\frac{2}{5},\frac{1}{2},\frac{2}{3}\right) & (1,1,1) & 1 & \left(\frac{2}{5},\frac{1}{2},\frac{2}{3}\right) \\ \left(\frac{2}{3},1,\frac{3}{2}\right) & \left(\frac{2}{5},\frac{1}{2},\frac{2}{3}\right) & \left(\frac{3}{2},2,\frac{5}{2}\right) & 1 \end{bmatrix}.$$

By using the upper triangular elements of $\tilde{B}$, model (5) can be written as

$$\text{Maximize}\quad \lambda$$

$$\text{Subject to}\begin{cases} -w_1+1.5w_2+0.5\lambda w_2\leqslant 0,\\ w_1-2.5w_2+0.5\lambda w_2\leqslant 0,\\ -w_1+1.5w_3+0.5\lambda w_3\leqslant 0,\\ w_1-2.5w_3+0.5\lambda w_3\leqslant 0,\\ -w_1+2w_4/3+\lambda w_4/3\leqslant 0,\\ w_1-1.5w_4+0.5\lambda w_4\leqslant 0,\\ -w_2+w_3\leqslant 0,\\ w_2-w_3\leqslant 0,\\ -w_2+1.5w_4+0.5\lambda w_4\leqslant 0,\\ w_2-2.5w_4+0.5\lambda w_4\leqslant 0,\\ -w_3+0.4w_4+0.1\lambda w_4\leqslant 0,\\ w_3-2w_4/3+\lambda w_4/6\leqslant 0,\\ w_1+w_2+w_3+w_4=1,\\ w_1,w_2,w_3,w_4\geqslant 0. \end{cases}$$

It is easy to find that $W^*_{\underset{\sim}{B}}$ =(0.2178, 0.2489, 0.2489, 0.2844) and $\hat{W}^*_{\underset{\sim}{B}}$ =(0.4359, 0.1795, 0.1795, 0.2051) are both the optimal solutions to the above model with the same optimal value $\lambda^*_{\underset{\sim}{B}}=-1.25$ for λ. Since $\lambda^*_{\underset{\sim}{B}}=-1.25<0$, it makes no sense as a membership degree. The two optimal solutions also produce conflict conclusions since the first one indicates that w_1^* is the smallest, whereas the second one shows that w_1^* is the biggest.

If we use the lower triangular elements of $\widetilde{B}$ for weight derivation, model (5) can then be written as

$$\text{Maximize}\quad \lambda$$

$$\text{Subject to}\begin{cases} -w_2+0.4w_1+0.1\lambda w_1\leqslant 0,\\ w_2-2w_1/3+\lambda w_1/6\leqslant 0,\\ -w_3+0.4w_1+0.1\lambda w_1\leqslant 0,\\ w_3-2w_1/3+\lambda w_1/6\leqslant 0,\\ -w_3+w_2\leqslant 0,\\ w_3-w_2\leqslant 0,\\ -w_4+2w_1/3+\lambda w_1/3\leqslant 0,\\ w_4-1.5w_1+0.5\lambda w_1\leqslant 0,\\ -w_4+0.4w_2+0.1\lambda w_2\leqslant 0,\\ w_4-2w_2/3+\lambda w_2/6\leqslant 0,\\ -w_4+1.5w_3+0.5\lambda w_3\leqslant 0,\\ w_4-2.5w_3+0.5\lambda w_3\leqslant 0,\\ w_1+w_2+w_3+w_4=1,\\ w_1,w_2,w_3,w_4\geqslant 0. \end{cases}$$

It is not difficult to verify that this model produces many optimal solutions. The followings are just three of them:

$$\overline{W}^*_{\underset{\sim}{B}} = (0.2866, 0.2482, 0.2482, 0.2171), \quad \lambda^* = -1.25,$$
$$\widehat{W}^*_{\underset{\sim}{B}} = (0.3174, 0.2374, 0.2374, 0.2078), \quad \lambda^* = -1.25,$$
$$\overline{\overline{W}}^*_{\underset{\sim}{B}} = (0.5490, 0.1569, 0.1569, 0.1373), \quad \lambda^* = -1.25,$$

which have significant difference in the magnitude of w_1^*.

If we use both the upper and lower triangular elements of $\widetilde{B}$ simultaneously for weight derivation, we still get many optimal solutions, some of which are:

$$W^{*(1)}_{\underset{\sim}{B}} = (0.4545, 0.1818, 0.1818, 0.1818), \quad \lambda^* = -2,$$
$$W^{*(2)}_{\underset{\sim}{B}} = (0.25, 0.25, 0.25, 0.25), \quad \lambda^* = -2,$$
$$W^{*(3)}_{\underset{\sim}{B}} = (0.2855, 0.2382, 0.2382, 0.2382), \quad \lambda^* = -2.$$

These different priority vectors undoubtedly make fuzzy AHP decision-making tougher and more difficult and the non-uniqueness of the solutions also raises the questions about the validity of the FPP-based nonlinear priority method as a priority method. In the next section, we will develop a logarithmic fuzzy preference programming (LFPP) based methodology for fuzzy AHP priority derivation to overcome the above-mentioned drawbacks.

3. The LFPP-based nonlinear priority method

For the fuzzy pairwise comparison matrix in (1), we take its logarithm by the following approximate equation:

$$\ln \tilde{a}_{ij} \approx (\ln l_{ij}, \ln m_{ij}, \ln u_{ij}), \quad i,j = 1, \ldots, n. \tag{6}$$

That is, the logarithm of a triangular fuzzy judgment $\tilde{a}_{ij}$ can still be seen as an approximate triangular fuzzy number, whose membership function can accordingly be defined as

$$\mu_{ij}\left(\ln\left(\frac{w_i}{w_j}\right)\right) = \begin{cases} \frac{\ln\left(w_i/w_j\right) - \ln l_{ij}}{\ln m_{ij} - \ln l_{ij}}, & \ln\left(\frac{w_i}{w_j}\right) \leqslant \ln m_{ij}, \\ \frac{\ln u_{ij} - \ln\left(w_i/w_j\right)}{\ln u_{ij} - \ln m_{ij}}, & \ln\left(\frac{w_i}{w_j}\right) \geqslant \ln m_{ij}, \end{cases} \tag{7}$$

where $\mu_{ij}(\ln(w_i/w_j))$ is the membership degree of $\ln(w_i/w_j)$ belonging to the approximate triangular fuzzy judgment $\ln \tilde{a}_{ij} = (\ln l_{ij}, \ln m_{ij}, \ln u_{ij})$. It is very natural that we hope to find a crisp priority vector to maximize the minimum membership degree $\lambda = \min\{\mu_{ij}(\ln(w_i/w_j)) | i = 1, \ldots, n-1;\ j = i+1, \ldots, n\}$. The resultant model can be constructed as

$$\begin{aligned} &\text{Maximize} \quad \lambda \\ &\text{Subject to} \begin{cases} \mu_{ij}(\ln(w_i/w_j)) \geqslant \lambda, i = 1, \ldots, n-1; j = i+1, \ldots, n, \\ w_i \geqslant 0, i = 1, \ldots, n, \end{cases} \end{aligned} \tag{8}$$

or as

$$\begin{aligned} &\text{Maximize} \quad 1 - \lambda \\ &\text{Subject to} \begin{cases} \ln w_i - \ln w_j - \lambda \ln(m_{ij}/l_{ij}) \geqslant \ln l_{ij}, i = 1, \ldots, n-1; j = i+1, \ldots, n, \\ -\ln w_i + \ln w_j - \lambda \ln(u_{ij}/m_{ij}) \geqslant -\ln u_{ij}, i = 1, \ldots, n-1; j = i+1, \ldots, n, \\ w_i \geqslant 0, i = 1, \ldots, n. \end{cases} \end{aligned} \tag{9}$$

It is seen that the normalization constraint $\sum_{i=1}^{n} w_i = 1$ is not included in the above two equivalent models. This is because the models will become computationally complicated if the normalization constraint is included. As a matter of fact, normalization can be carried out after the priorities are obtained from model (9). Before normalization, without loss of generality, we can assume $w_i \geqslant 1$ for all $i = 1, \ldots, n$ such that $\ln w_i \geqslant 0$ for $i = 1, \ldots, n$. Note that the nonnegative assumption for $\ln w_i \geqslant 0$ ($i = 1, \ldots, n$) is not essential. We make this assumption is just for convenience in solution.

Generally, there is no guarantee that model (9) can always produce a positive value for the membership degree λ. The reason for producing a negative value for λ is that there are no weights that can meet all the fuzzy judgments in $\widetilde{A}$ within their support intervals. That is to say, not all the inequalities $\ln w_i - \ln w_j - \lambda \ln(m_{ij}/l_{ij}) \geqslant \ln l_{ij}$ or $-\ln w_i + \ln w_j - \lambda \ln(u_{ij}/m_{ij}) \geqslant -\ln u_{ij}$ can hold at the same time. To avoid λ from taking a negative value, we introduce nonnegative deviation variables δ_{ij} and η_{ij} for $i = 1, \ldots, n-1$ and $j = i+1, \ldots, n$ such that they meet the following inequalities:

$$\begin{aligned} &\ln w_i - \ln w_j - \lambda \ln(m_{ij}/l_{ij}) + \delta_{ij} \geqslant \ln l_{ij},\ i = 1, \ldots, n-1;\ j = i+1, \ldots, n, \\ &-\ln w_i + \ln w_j - \lambda \ln(u_{ij}/m_{ij}) + \eta_{ij} \geqslant -\ln u_{ij},\ i = 1, \ldots, n-1;\ j = i+1, \ldots, n. \end{aligned}$$

It is the most desirable that the values of the deviation variables are the smaller the better. We thus propose the following LFPP-based nonlinear priority model for fuzzy AHP weight derivation:

$$\text{Minimize} \quad J = (1-\lambda)^2 + M \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (\delta_{ij}^2 + \eta_{ij}^2)$$
$$\text{Subject to} \begin{cases} x_i - x_j - \lambda \ln(m_{ij}/l_{ij}) + \delta_{ij} \geqslant \ln l_{ij}, \ i = 1,\ldots,n-1; \ j = i+1,\ldots,n, \\ -x_i + x_j - \lambda \ln(u_{ij}/m_{ij}) + \eta_{ij} \geqslant -\ln u_{ij}, \ i = 1,\ldots,n-1; \ j = i+1,\ldots,n, \\ \lambda, x_i \geqslant 0, \ i = 1,\ldots,n, \\ \delta_{ij}, \eta_{ij} \geqslant 0, \ i = 1,\ldots,n-1; \ j = i+1,\ldots,n, \end{cases} \tag{10}$$

where $x_i = \ln w_i$ for $i = 1,\ldots,n$ and M is a specified sufficiently large constant such as $M = 10^3$. The main purpose of introducing a big constant M into the above model is to find the weights within the support intervals of fuzzy judgments without violations or with as little violations as possible. So, the first priority of model (10) is given to minimize the violations of the fuzzy judgments and the weights can then be optimized to maximize the DM's satisfaction, namely, the value of λ.

Theoretically, L_1 or L_∞ norm can also be used instead of the L_2 norm in the objective function of (10) to model the deviation variables and membership degree. If we do so, the model will be linear. However, the linear models based on these two norms may sometimes produce multiple optimal solutions. So, they are not considered.

Let x_i^* ($i = 1,\ldots,n$) be the optimal solution to model (10). The normalized priorities for fuzzy pairwise comparison matrix $\widetilde{A} = (\tilde{a}_{ij})_{n\times n}$ can then be obtained as

$$w_i^* = \frac{\exp(x_i^*)}{\sum_{j=1}^{n} \exp(x_j^*)}, \quad i = 1,\ldots,n, \tag{11}$$

where exp() is the exponential function, namely, $\exp(x_i^*) = e^{x_i^*}$ for $i = 1,\ldots,n$. We refer to the method that utilizes model (10) for fuzzy AHP priority derivation as the LFPP methodology and the resultant priorities as the LFPP priorities.

With regard to the LFPP methodology, we have the following theorems.

Theorem 1. *The priorities derived by the LFPP methodology from the upper triangular elements of a fuzzy pairwise comparison matrix are exactly the same as those derived from the lower triangular elements of the fuzzy pairwise comparison matrix.*

Proof. Consider a pair of fuzzy judgments $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$ and $\tilde{a}_{ji} = (1/u_{ij}, 1/m_{ij}, 1/l_{ij})$. The constraints of model (10) derived from $\tilde{a}_{ji}$ can be written as

$$x_j - x_i - \lambda \ln(m_{ji}/l_{ji}) + \delta_{ij} \geqslant \ln l_{ji},$$
$$-x_j + x_i - \lambda \ln(u_{ji}/m_{ji}) + \eta_{ij} \geqslant -\ln u_{ji},$$

which can be equivalently expressed as

$$-x_i + x_j - \lambda \ln(u_{ij}/m_{ij}) + \delta_{ij} \geqslant -\ln u_{ij},$$
$$x_i - x_j - \lambda \ln(m_{ij}/l_{ij}) + \eta_{ij} \geqslant \ln l_{ij}.$$

It is easy to see that these two inequalities are exactly the constraints of model (10) for $\tilde{a}_{ij}$. That is to say, the constraints of model (10) for $\tilde{a}_{ij}$ and $\tilde{a}_{ji}$ are always the same. So, the use of the upper or lower triangular elements of a fuzzy pairwise comparison matrix for weight derivation will always give the same priorities when the LFPP methodology is applied. □

Theorem 2. *The LFPP methodology produces the unique normalized optimal priority vector for any fuzzy pairwise comparison matrix.*

Proof. The objective function of model (10) is a strict convex function since its Hessian matrix is positively definite. The constraints of model (10) are all linear inequalities, which form a convex feasible region. Therefore, model (10) is a convex programming. From the theory of optimization, it is known that for a convex programming with a strict convex objective function, its local optimal solution is the only one global optimal solution. So, the optimal solution to model (10) is unique. As a result, the normalized priority vector determined by model (10) is also unique. This completes the proof. □

Generally speaking, it is desirable that a positive optimal value can be achieved for λ. If its optimal value turns out to be $\lambda^* = 0$, then there exists strong inconsistency among the fuzzy judgments unless $\delta^* = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(\delta_{ij}^{*2} + \eta_{ij}^{*2}) = 0$. The bigger the value of δ^*, the stronger the inconsistency among the fuzzy judgments. So, the value of δ^* can be treated as an inconsistency measure for fuzzy pairwise comparison matrices.

4. Numerical examples

In this section, we test three numerical examples using the proposed LFPP methodology to illustrate its advantages and potential applications in fuzzy AHP decision-making.

Example 1. Consider the 4×4 fuzzy pairwise comparison matrix $\widetilde{B}$ in Section 2. Model (10) for this fuzzy pairwise comparison matrix can be written as

$$\text{Minimize} \quad J = (1-\lambda)^2 + M \cdot \sum_{i=1}^{3} \sum_{j=i+1}^{4} (\delta_{ij}^2 + \eta_{ij}^2)$$

$$\text{Subject to} \begin{cases} x_1 - x_2 - \lambda \ln(4/3) + \delta_{12} \geqslant \ln(3/2), \\ -x_1 + x_2 - \lambda \ln(5/4) + \eta_{12} \geqslant -\ln(5/2), \\ x_1 - x_3 - \lambda \ln(4/3) + \delta_{13} \geqslant \ln(3/2), \\ -x_1 + x_3 - \lambda \ln(5/4) + \eta_{13} \geqslant -\ln(5/2), \\ x_1 - x_4 - \lambda \ln(3/2) + \delta_{14} \geqslant \ln(2/3), \\ -x_1 + x_4 - \lambda \ln(3/2) + \eta_{14} \geqslant -\ln(3/2), \\ x_2 - x_3 + \delta_{23} \geqslant 0, \\ -x_2 + x_3 + \eta_{23} \geqslant 0, \\ x_2 - x_4 - \lambda \ln(4/3) + \delta_{24} \geqslant \ln(3/2), \\ -x_2 + x_4 - \lambda \ln(5/4) + \eta_{24} \geqslant -\ln(5/2), \\ x_3 - x_4 - \lambda \ln(5/4) + \delta_{34} \geqslant \ln(2/5), \\ -x_3 + x_4 - \lambda \ln(4/3) + \eta_{34} \geqslant -\ln(2/3), \\ \lambda, x_1, x_2, x_3, x_4, \delta_{12}, \delta_{13}, \delta_{14}, \delta_{23}, \delta_{24}, \delta_{34}, \eta_{12}, \eta_{13}, \eta_{14}, \eta_{23}, \eta_{24}, \eta_{34} \geqslant 0. \end{cases}$$

Taking a sufficiently large number for M, say $M = 1000$, to solve this model with the Microsoft Excel Solver, we get the optimal solution as

$$\begin{aligned} &x_1^* = 0.9775, \quad x_2^* = 0.6228, \quad x_3^* = 0.3693, \quad x_4^* = 0.5214, \quad \lambda^* = 0, \\ &\delta_{12}^* = 0.051, \quad \delta_{13}^* = \delta_{14}^* = \delta_{23}^* = 0, \quad \delta_{24}^* = 0.304, \quad \delta_{34}^* = 0, \\ &\eta_{12}^* = \eta_{13}^* = 0, \quad \eta_{14}^* = 0.051, \quad \eta_{23}^* = 0.253, \quad \eta_{24}^* = 0, \quad \eta_{34}^* = 0.253, \end{aligned}$$

based on which, we have normalized LFPP priorities as

$$w_1^* = EXP(x_1^*) / \sum_{i=1}^{4} EXP(x_i^*) = 0.3473, \quad w_2^* = EXP(x_2^*) / \sum_{i=1}^{4} EXP(x_i^*) = 0.2436,$$

$$w_3^* = EXP(x_3^*) / \sum_{i=1}^{4} EXP(x_i^*) = 0.1890, \quad \text{and} \quad w_4^* = EXP(x_4^*) / \sum_{i=1}^{4} EXP(x_i^*) = 0.2201.$$

Since $\delta^* = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (\delta_{ij}^{*2} + \eta_{ij}^{*2}) = 0.2271 \neq 0$, there exists strong inconsistency among the fuzzy judgments of $\tilde{B}$. It is more desirable that these fuzzy judgments can be rechecked to improve their qualities.

Example 2. Consider the following 4×4 fuzzy pairwise comparison matrix $\widetilde{C}$:

$$\widetilde{C} = \begin{bmatrix} 1 & \left(\frac{1}{5}, \frac{1}{2}, 1\right) & \left(\frac{1}{6}, \frac{2}{5}, \frac{3}{4}\right) & \left(\frac{1}{7}, \frac{1}{3}, \frac{3}{5}\right) \\ (1, 2, 5) & 1 & \left(\frac{1}{2}, \frac{4}{5}, \frac{5}{4}\right) & \left(\frac{3}{7}, \frac{2}{3}, 1\right) \\ \left(\frac{4}{3}, \frac{5}{2}, 6\right) & \left(\frac{4}{5}, \frac{5}{4}, 2\right) & 1 & \left(\frac{4}{7}, \frac{5}{6}, \frac{6}{5}\right) \\ \left(\frac{5}{3}, 3, 7\right) & \left(1, \frac{3}{2}, \frac{7}{3}\right) & \left(\frac{5}{6}, \frac{6}{5}, \frac{7}{4}\right) & 1 \end{bmatrix}.$$

By solving model (10) for this fuzzy pairwise comparison matrix, we obtain the optimal solution as

$$\begin{aligned} &.x_1^* = 0.0394, \quad x_2^* = 0.7325, \quad x_3^* = 0.9557, \quad x_4^* = 1.1380, \quad \lambda^* = 1, \\ &\delta_{12}^* = \delta_{13}^* = \delta_{14}^* = \delta_{23}^* = \delta_{24}^* = \delta_{34}^* = 0 \quad \text{and} \quad \eta_{12}^* = \eta_{13}^* = \eta_{14}^* = \eta_{23}^* = \eta_{24}^* = \eta_{34}^* = 0, \end{aligned}$$

based on which, we have normalized LFPP priorities as

$$w_1^* = EXP(x_1^*) / \sum_{i=1}^{4} EXP(x_i^*) = 0.1176, \quad w_2^* = EXP(x_2^*) / \sum_{i=1}^{4} EXP(x_i^*) = 0.2353,$$

$$w_3^* = EXP(x_3^*) / \sum_{i=1}^{4} EXP(x_i^*) = 0.2941 \quad \text{and} \quad w_4^* = EXP(x_4^*) / \sum_{i=1}^{4} EXP(x_i^*) = 0.3529.$$

Due to the fact that $\lambda^* = 1$, this set of priorities match the fuzzy pairwise comparison matrix $\widetilde{C}$ perfectly well. In other words, the modal values of all the fuzzy judgments can be precisely fitted by this set of normalized priorities. Apparently, this is the most desirable situation, but may rarely happen in the real-world fuzzy AHP decision-making.

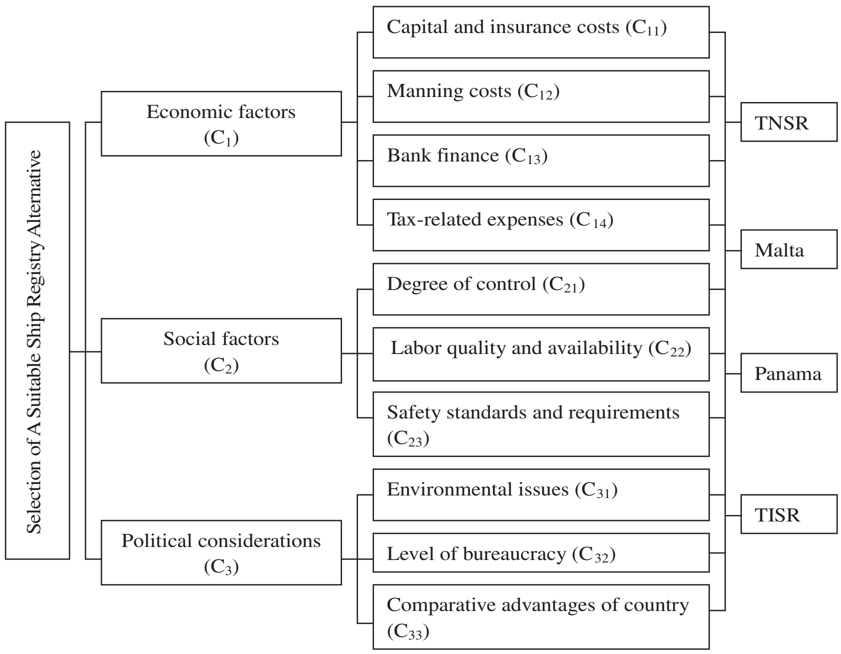

Fig. 1. Hierarchical structure for ship registry selection problem.

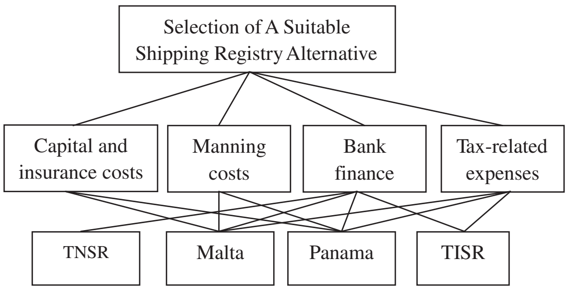

Fig. 2. The hierarchical structure considered by the extent analysis.

Table 1
Fuzzy pairwise comparison matrix of three selection criteria with respect to the decision goal and its priorities.

Criteria	C_1	C_2	C_3	LFPP priorities	EA priorities
C_1	(1,1,1)	(5/2,3,7/2)	(3/2,2,5/2)	0.5518	1
C_2	(2/7,1/3,2/5)	(1,1,1)	(2/3,1,3/2)	0.2015	0
C_3	(2/5,1/2,2/3)	(2/3,1,3/2)	(1,1,1)	0.2467	0

$\lambda^* = 0.5023$.

Table 2
Fuzzy pairwise comparison matrix of the four sub-criteria of economic factors (C_1) and its normalized LFPP priorities.

Criteria	C_{11}	C_{12}	C_{13}	C_{14}	LFPP priorities	EA priorities
C_{11}	(1,1,1)	(3/2,1,3/2)	(1,1,1)	(2/5,1/2,2/3)	0.2329	0.1413
C_{12}	(2/3,1,3/2)	(1,1,1)	(2/5,1/2,2/3)	(2/3,1,3/2)	0.1901	0.1797
C_{13}	(1,1,1)	(3/2,2,5/2)	(1,1,1)	(2/5,1/2,2/3)	0.2450	0.2610
C_{14}	(3/2,2,5/2)	(2/3,1,3/2)	(3/2,2,5/2)	(1,1,1)	0.3320	0.4179

$\lambda^* = 0$.

Table 3
Fuzzy pairwise comparison matrix of the three sub-criteria of social factors (C_2) and its normalized LFPP priorities.

Criteria	C_{21}	C_{22}	C_{23}	LFPP priorities	EA priorities
C_{21}	(1,1,1)	(2/5,1/2,2/3)	(1,1,1)	0.2225	0
C_{22}	(3/2,2,5/2)	(1,1,1)	(5/2,3,7/2)	0.5555	1
C_{23}	(1,1,1)	(2/7,1/3,2/5)	(1,1,1)	0.2220	0

$\lambda^* = 0.0179$.

Table 4
Fuzzy pairwise comparison matrix of the three sub-criteria of political considerations (C_3) and its normalized LFPP priorities.

Criteria	C_{31}	C_{32}	C_{33}	LFPP priorities	EA priorities
C_{31}	(1,1,1)	(2/3,1,3/2)	(2/5,1/2,2/3)	0.25	0.1461
C_{32}	(2/3,1,3/2)	(1,1,1)	(2/5,1/2,2/3)	0.25	0.1461
C_{33}	(3/2,2,5/2)	(3/2,2,5/2)	(1,1,1)	0.50	0.7078

$\lambda^* = 1$.

Table 5
Fuzzy pairwise comparison matrices of four selection alternatives with respect to the sub-criteria of C_1 and their normalized priorities.

	TNSR	Malta	Panama	TISR	LFPP priorities	EA priorities
A: Comparisons of the four alternatives with respect to the sub-criterion C_{11}						
TNSR	(1,1,1)	(2/7,1/3,2/5)	(2/9,1/4,2/7)	(2/3,1,3/2)	0.1051	0
Malta	(5/2,3,7/2)	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)	0.4143	0.5239
Panama	(7/2,4,9/2)	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)	0.3188	0.4761
TISR	(2/3,1,3/2)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1,1,1)	0.1617	0
$\lambda^* = 0$						
B: Comparisons of the four alternatives with respect to the sub-criterion C_{12}						
TNSR	(1,1,1)	(2/5,1/2,2/3)	(2/7,1/3,2/5)	(2/3,1,3/2)	0.1244	0
Malta	(3/2,2,5/2)	(1,1,1)	(2/5,1/2,2/3)	(3/2,2,5/2)	0.2736	0.3482
Panama	(5/2,3,7/2)	(3/2,2,5/2)	(1,1,1)	(3/2,2,5/2)	0.4264	0.6518
TISR	(2/3,1,3/2)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1,1,1)	0.1756	0
$\lambda^* = 0.136$						
C: Comparisons of the four alternatives with respect to the sub-criterion C_{13}						
TNSR	(1,1,1)	(2/3,1,3/2)	(2/3,1,3/2)	(2/5,1/2,2/3)	0.2	0.1645
Malta	(2/3,1,3/2)	(1,1,1)	(2/3,1,3/2)	(2/5,1/2,2/3)	0.2	0.1645
Panama	(2/3,1,3/2)	(2/3,1,3/2)	(1,1,1)	(2/5,1/2,2/3)	0.2	0.1645
TISR	(3/2,2,5/2)	(3/2,2,5/2)	(3/2,2,5/2)	(1,1,1)	0.4	0.5065
$\lambda^* = 1$						
D: Comparisons of the four alternatives with respect to the sub-criterion C_{14}						
TNSR	(1,1,1)	(2/9,1/4,2/7)	(2/7,1/3,2/5)	(2/5,1/2,2/3)	0.0968	0
Malta	(7/2,4,9/2)	(1,1,1)	(2/3,1,3/2)	(2/3,1,3/2)	0.3385	0.4076
Panama	(5/2,3,7/2)	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)	0.3388	0.4076
TISR	(3/2,2,5/2)	(2/3,1,3/2)	(2/5,1/2,2/3)	(1,1,1)	0.2258	0.1847
$\lambda^* = 0.0041$						

Example 3. Consider a ship registry selection problem investigated by Celik et al. [15]. The hierarchical structure for this selection problem is shown in Fig. 1, where C_1, C_2 and C_3 are three selection criteria, each involving some sub-criteria, and TNSR (Turkish National Ship Registry), Malta, Panama, and TISR (Turkish International Ship Registry) are four possible potential selection alternatives for Turkish ship owners.

Celik et al. [15] conducted a decision analysis using the extent analysis, which has been revealed to be invalid and may result in a wrong decision being made. In particular, this invalid priority method assigns a zero weight to each of the selection criteria C_2 and C_3. These two zero weights fundamentally change the hierarchical structure of the selection problem in Fig. 1. If the zero weights for C_2 and C_3 were true, then these two selection criteria should not have been considered and included in the hierarchical structure in Fig. 1 from the very beginning of decision analysis. The DM should use only one selection criterion C_1 for the hierarchical decision analysis, as shown in Fig. 2, which is the actual hierarchical structure considered by the extent analysis. Now that the DM considers multiple selection criteria for ship registry selection, none of them should be given a zero weight. In other words, assigning a zero weight to any selection criterion or sub-criterion in the hierarchical structure in Fig. 1 makes no sense. Therefore, the extent analysis should be rejected.

Here, we reinvestigate this selection problem using the proposed LFPP methodology to provide a correct application of the fuzzy AHP. Tables 1–7 show the fuzzy pairwise comparison matrices taken from Celik et al. [15] with a very slight change

Table 6
Fuzzy pairwise comparison matrices of four selection alternatives with respect to the sub-criteria of C_2 and their normalized priorities.

	TNSR	Malta	Panama	TISR	LFPP priorities	EA priorities
A: Comparisons of the four alternatives with respect to the sub-criterion C_{21}						
TNSR	(1,1,1)	(2/3,1,3/2)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	0.1709	0.0717
Malta	(2/3,1,3/2)	(1,1,1)	(1,1,1)	(2/3,1,3/2)	0.2836	0.2164
Panama	(3/2,2,5/2)	(1,1,1)	(1,1,1)	(3/2,2,5/2)	0.3139	0.4305
TISR	(3/2,2,5/2)	(2/3,1,3/2)	(2/5,1/2,2/3)	(1,1,1)	0.2316	0.2815
$\lambda^* = 0$						
B: Comparisons of the four alternatives with respect to the sub-criterion C_{22}						
TNSR	(1,1,1)	(2/5,1/2,2/3)	(5/2,3,7/2)	(3/2,2,5/2)	0.3259	0.4199
Malta	(3/2,2,5/2)	(1,1,1)	(2/5,1/2,2/3)	(2/3,1,3/2)	0.2691	0.2349
Panama	(2/7,1/3,2/5)	(3/2,2,5/2)	(1,1,1)	(3/2,2,5/2)	0.2368	0.3136
TISR	(2/5,1/2,2/3)	(2/3,1,3/2)	(2/5,1/2,2/3)	(1,1,1)	0.1683	0.0316
$\lambda^* = 0$						
C: Comparisons of the four alternatives with respect to the sub-criterion C_{23}						
TNSR	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)	(3/2,2,5/2)	0.4055	0.5347
Malta	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)	0.2604	0.3850
Panama	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1,1,1)	(2/3,1,3/2)	0.1671	0.0401
TISR	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(2/3,1,3/2)	(1,1,1)	0.1671	0.0401
$\lambda^* = 0.1347$						

Table 7
Fuzzy pairwise comparison matrices of four selection alternatives with respect to the sub-criteria of C_3 and their normalized priorities.

	TNSR	Malta	Panama	TISR	LFPP priorities	EA priorities
A: Comparisons of the four alternatives with respect to the sub-criterion C_{31}						
TNSR	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)	(2/3,1,3/2)	0.3473	0.4313
Malta	(2/5,1/2,2/3)	(1,1,1)	(1,1,1)	(3/2,2,5/2)	0.2436	0.2633
Panama	(2/5,1/2,2/3)	(1,1,1)	(1,1,1)	(2/5,1/2,2/3)	0.1890	0.0194
TISR	(2/3,1,3/2)	(2/5,1/2,2/3)	(3/2,2,5/2)	(1,1,1)	0.2201	0.2860
$\lambda^* = 0$						
B: Comparisons of the four alternatives with respect to the sub-criterion C_{32}						
TNSR	(1,1,1)	(2/9,1/4,2/7)	(5/2,3,7/2)	(3/2,2,5/2)	0.2432	0.3637
Malta	(7/2,4,9/2)	(1,1,1)	(1,1,1)	(3/2,2,5/2)	0.4023	0.6363
Panama	(2/7,1/3,2/5)	(1,1,1)	(1,1,1)	(3/2,2,5/2)	0.2059	0
TISR	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1,1,1)	0.1486	0
$\lambda^* = 0$						
C: Comparisons of the four alternatives with respect to the sub-criterion C_{33}						
TNSR	(1,1,1)	(2/9,1/4,2/7)	(2/3,1,3/2)	(2/5,1/2,2/3)	0.1329	0
Malta	(7/2,4,9/2)	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)	0.4704	0.8621
Panama	(2/3,1,3/2)	(2/5,1/2,2/3)	(1,1,1)	(2/3,1,3/2)	0.1920	0
TISR	(3/2,2,5/2)	(2/5,1/2,2/3)	(2/3,1,3/2)	(1,1,1)	0.2048	0.1379
$\lambda^* = 0.0952$						

for the fuzzy judgment $\tilde{a}_{23}$ in Table 5 under the sub-criterion C_{14}. The priorities obtained by using the LFPP methodology are provided in the columns under the heading "*LFPP priorities*". To illustrate the fact that the extent analysis can make a wrong decision, the priorities obtained by the extent analysis, which are marked as "*EA priorities*", are provided in the last columns of these tables. The aggregated priorities are presented in Tables 8 and 9, from which it is seen that the LFPP methodology evaluates Malta as the best alternative, whereas the extent analysis draws a different conclusion which selects Panama as the best alternative. Without doubt, the decision conclusion made by the LFPP methodology takes account of all the selection criteria and sub-criteria, whereas the conclusion made by the extent analysis considers only the selection criterion C_1 without taking into account the other two criteria. From the point of view of multiple criteria decision-making, the conclusion made by the LFPP methodology is more convincing and more believable than that drawn by the extent analysis.

To verify the conclusion made by the LFPP methodology, we conduct an analysis using the modified logarithmic least-squares method (LLSM) proposed in [66], which derives normalized fuzzy weights for fuzzy pairwise comparison matrices. The final global fuzzy priorities for the four selection alternatives produced by the modified LLSM are pictured in Fig. 3, which also reveals that Malta is the best decision alternative. So, we have reason to reject the conclusion arrived at by the extent analysis.

It is worth pointing out that the FPP-based nonlinear priority method is not tested for this application example because it produces too many priority vectors for the fuzzy pairwise comparison matrix (i.e. the numerical example illustrated in Section 2) of the four selection alternatives with respect to the sub-criteria C_{31} in Table 7, leading to the final conclusion lack of persuasiveness.

Table 8
Aggregation of the local priorities obtained by the LFPP methodology.

	C_{11}	C_{12}	C_{13}	C_{14}	Local priorities
Local priorities of the four selection alternatives with respect to C_1					
Weight	0.2329	0.1901	0.2450	0.3320	
TNSR	0.1051	0.1244	0.2000	0.0968	0.1293
Malta	0.4143	0.2736	0.2000	0.3385	0.3099
Panama	0.3188	0.4264	0.2000	0.3388	0.3168
TISR	0.1617	0.1756	0.4000	0.2258	0.2440

	C_{21}	C_{22}	C_{23}	Local priorities
Local priorities of the four selection alternatives with respect to C_2				
Weight	0.2225	0.5555	0.2220	
TNSR	0.1709	0.3259	0.4055	0.3091
Malta	0.2836	0.2691	0.2604	0.2704
Panama	0.3139	0.2368	0.1671	0.2385
TISR	0.2316	0.1683	0.1671	0.1821

	C_{31}	C_{32}	C_{33}	Local priorities
Local priorities of the four selection alternatives with respect to C_3				
Weight	0.2500	0.2500	0.5000	
TNSR	0.3473	0.2432	0.1329	0.2141
Malta	0.2436	0.4023	0.4704	0.3967
Panama	0.1890	0.2059	0.1920	0.1947
TISR	0.2201	0.1486	0.2048	0.1946

	C_1	C_2	C_3	Global priorities
Global priorities of the four selection alternatives with respect to the decision goal				
Priorities	0.5518	0.2015	0.2467	
TNSR	0.1293	0.3091	0.2141	0.1864
Malta	0.3099	0.2704	0.3967	**0.3233**
Panama	0.3168	0.2385	0.1947	0.2709
TISR	0.2440	0.1821	0.1946	0.2193

Table 9
Aggregation of the local priorities obtained by the extent analysis

	C_{11}	C_{12}	C_{13}	C_{14}	Local priorities
Local priorities of the four selection alternatives with respect to C_1					
Weight	0.1413	0.1797	0.2610	0.4179	
TNSR	0	0	0.1645	0	0.0429
Malta	0.5239	0.3482	0.1645	0.4076	0.3499
Panama	0.4761	0.6518	0.1645	0.4076	0.3977
TISR	0	0	0.5065	0.1847	0.2094

	C_{21}	C_{22}	C_{23}	Local priorities
Local priorities of the four selection alternatives with respect to C_2				
Weight	0	1	0	
TNSR	0.0717	0.4199	0.5347	0.4199
Malta	0.2164	0.2349	0.3850	0.2349
Panama	0.4305	0.3136	0.0401	0.3136
TISR	0.2815	0.0316	0.0401	0.0316

	C_{31}	C_{32}	C_{33}	Local priorities
Local priorities of the four selection alternatives with respect to C_3				
Weight	0.1461	0.1461	0.7078	
TNSR	0.4313	0.3637	0	0.1161
Malta	0.2633	0.6363	0.8621	0.7416
Panama	0.0194	0	0	0.0028
TISR	0.2860	0	0.1379	0.1394

	C_1	C_2	C_3	Global priorities
Global priorities of the four selection alternatives with respect to the decision goal				
Priorities	1	0	0	
TNSR	0.0429	0.4199	0.1161	0.0429
Malta	0.3499	0.2349	0.7416	0.3499
Panama	0.3977	0.3136	0.0028	**0.3977**
TISR	0.2094	0.0316	0.1394	0.2094

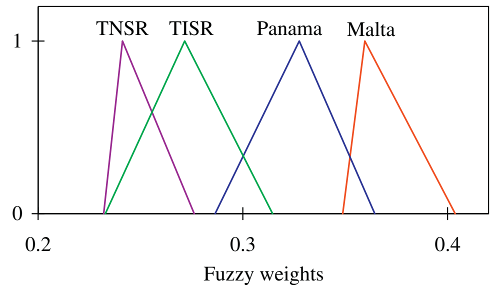

Fig. 3. Global fuzzy priorities of the four alternatives obtained by the modified LLSM.

5. Conclusions

Fuzzy AHP has been playing an increasingly important role in multiple criteria decision-making under uncertainty and has found extensive applications in a wide variety of areas such as supplier selection, customer requirements assessment and the like. The use of fuzzy AHP for multiple criteria decision-making requires scientific weight derivation from fuzzy pairwise comparison matrices. Existing approaches for deriving fuzzy weights from fuzzy pairwise comparison matrices turn out to be too sophisticated and rare to be applied, while the approaches for deriving crisp weights from fuzzy pairwise comparison matrices prove to be either invalid or subject to significant drawbacks such as producing multiple even conflict priority vectors for a fuzzy pairwise comparison matrix, leading to distinct conclusions. To address these drawbacks and provide a valid yet practical priority method for fuzzy AHP, we have proposed in this paper a logarithmic fuzzy preference programming based methodology for fuzzy AHP priority derivation, which we refer to as the LFPP methodology. It formulates the priorities of a fuzzy pairwise comparison matrix as a logarithmic nonlinear programming and derives crisp priorities from fuzzy pairwise comparison matrices. Theoretical analysis has revealed that the LFPP methodology can produce a unique optimal priority vector for any fuzzy pairwise comparison matrix. It overcomes the significant drawbacks suffered by a so-called fuzzy preference programming based nonlinear priority method in the literature. Three numerical examples examined using the LFPP methodology have illustrated its advantages and potential applications. It is expected that the LFPP methodology can arouse more research interests and applications of the fuzzy AHP in the near future.

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