An irreducible massive superspin one half action built from the chiral dotted spinor superfield

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A B S T R A C T

Although the chiral dotted spinor superfield should describe a Massive Superspin One Half multiplet, it has not been obvious how to derive this from an action. In this paper this is done by including a chiral undotted spinor superfield, finding the BRST transformations that govern both of these, and then finding the action as an invariant of the transformations. It turns out that both kinds of spinor superfields are needed. Moreover, the BRST transformations for the two kinds of chiral spinor superfields are generated from each other by a special involution that exchanges Grassmann odd (even) sources with Grassmann even (odd) fields.

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1. In some suitable limit for the superstring [21], the massive modes might be described by massive supersymmetric actions, coupled in some way to each other. As noted in [9] and as emphasized rather recently by Gates and Koutrolikos [7], massive supersymmetric theories possess a rich off shell structure and there is still much to learn about them. In [1] the authors posed the 'Off Shell Susy Problem' in a simple and general way, and pointed out that the answer is not likely to be simple, but that it is probably important. This has generated the adinkra approach which is making progress on this complicated problem [2].

Assembling interacting actions is a significant problem when one has only 'on-shell closure' since this necessarily implies some particular action of course, and that makes it tricky to generalize the action to include other couplings. It is usually taken for granted that the best way to approach the problem of generating actions and couplings is to look for auxiliary fields, and actions expressed in terms of superfields, so that the SUSY algebra closes on shell and the SUSY transformations are then obvious from superspace theory. Progress using these ideas has been reported in [3,4] and [6]. Of course, the general problem, as noted in [1], is complicated by the existence of other symmetries, such as those that must exist in the Standard Model of particle theory.

2. The intent in this paper, and its sequel, is to show that the BRST approach, using cohomology, offers a different approach to some of these problems. The present paper will illustrate some of the issues here by constructing a massive superspin $\frac{1}{2}$ action out of superfields with spin $\frac{1}{2}$. We will not use superfields here, except at the start, but the algebra is closed in the sense that the BRST operator is nilpotent. The nilpotence of the BRST operator arises as though the auxiliaries have been integrated out. This can happen even when no auxiliaries exist.[1] If there is a nilpotent BRST operator, one can use the spectral sequence to discover the cohomology [13]. The cohomology then points out where there are new invariants.

This BRST approach singles out the physical fields, and any remaining auxiliary fields get eliminated from consideration early on in the analysis [13]. So the BRST approach generates a different set of insights and problems, and it is not simple to sort out the relationship between the superfield approach and the BRST cohomology approach. They are complementary.

3. For a number of reasons to do with BRST cohomology,[2] it is of some interest to construct a well-behaved action starting with a chiral dotted spinor superfield $\widehat{\phi}_{\dot\alpha}$. Chirality means that $\overline{D}_{\dot\beta}\widehat{\phi}_{\dot\alpha} = 0$. It is well known [5] that this superfield can be

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[1] This is probably the case for the present action, and also for 10-D Super Yang–Mills theory [2,15], for example. This feature is related to the Batalin–Vilkovisky method, see for example [8] for a simple exposition of the latter.

[2] It has been evident for a long time [10,11,14,12] that the BRST cohomology of the chiral scalar superfield $\widehat{A}$ couples naturally to a chiral dotted spinor superfield $\widehat{\phi}^{\dot\alpha}$. The simplest example is $\int d^6 z \widehat{\phi}^{\dot\alpha}\widehat{A}\overline{C}_{\dot\alpha}$. Here $\overline{C}_{\dot\alpha}$ is a spacetime constant supersymmetry ghost. However the superfield $\widehat{\phi}^{\dot\alpha}$ here needs further constraints, which is the progress reported in this paper.

subjected to a 'reality constraint' in addition to chirality, and that on-shell it should represent superspin $\frac{1}{2}$. But the problem is that no action has been found, until now, that is consistent with both the chirality and the reality constraints and which then yields superspin $\frac{1}{2}$.

4. Trying to write down an action for a chiral dotted spinor superfield $\widehat{\overline{\phi}}_{\dot\alpha}$ meets a problem right at the start, because the most obvious action is

$$\mathcal{A} = \int d^8 z \widehat{\overline{\phi}}_{\dot\alpha} \partial^{\alpha\dot\alpha} \widehat{\overline{\phi}}_\alpha + m^2 \int d^6 z \widehat{\overline{\phi}}^{\dot\alpha} \widehat{\overline{\phi}}_{\dot\alpha} + m^2 \int d^6 \overline{z} \widehat{\overline{\phi}}^\alpha \widehat{\overline{\phi}}_\alpha \qquad (1)$$

but this immediately leads to higher derivative equations of motion, and there are tachyons in the spectrum too:

$$\left(\Box^2 - m^4\right) \widehat{\overline{\phi}}_{\dot\alpha} = \left(\Box - m^2\right)\left(\Box + m^2\right)\widehat{\overline{\phi}}_{\dot\alpha} = 0 \qquad (2)$$

This is certainly not a promising start for a model that is supposed to be phenomenologically viable.

5. The chiral undotted spinor superfield $\widehat{\overline{\chi}}_\alpha$ should also yield massive superspin $\frac{1}{2}$ on shell [5] and this is also puzzling. Here chirality means that $D_\beta \widehat{\chi}_{\dot\alpha} = 0$. $\widehat{\overline{\chi}}_{\dot\alpha}$ does appear in gauge theory [16], but as a massive matter representation it poses a difficulty, because the only known way to give it mass, until the present action, involves the spontaneous breaking of gauge symmetry together with the introduction of Higgs scalars. Such a method to construct a massive representation of superspin $\frac{1}{2}$ is clearly not irreducible, because it mixes $\widehat{\chi}_{\dot\alpha}$ with the components of chiral scalar Higgs multiplets.

6. Here we also want to add another feature, which is phase invariance, corresponding to some conserved quantity like Lepton or Baryon number. So we add a chirality index L, R to the superfields. Then the chirality constraints have the form:

$$\overline{D}_{\dot\beta} \widehat{\phi}_{L\dot\alpha} = \overline{D}_{\dot\beta} \widehat{\phi}_{R\dot\alpha} = D_\beta \widehat{\chi}_{L\dot\alpha} = D_\beta \widehat{\chi}_{R\dot\alpha} = 0 \qquad (3)$$

So the Complex Conjugates satisfy:

$$D_\alpha \widehat{\overline{\phi}}_{L\beta} = D_\alpha \widehat{\overline{\phi}}_{R\beta} = \overline{D}_{\dot\alpha} \widehat{\overline{\chi}}_{L\beta} = \overline{D}_{\dot\alpha} \widehat{\overline{\chi}}_{R\beta} = 0 \qquad (4)$$

Next, in order to get an irreducible representation of supersymmetry, along with a phase invariance, along the lines of the textbook [5], we want to also impose the additional 'reality constraints':

$$D_\alpha \widehat{\phi}_{L\dot\alpha} = \overline{D}_{\dot\alpha} \widehat{\overline{\phi}}_{R\alpha}; \quad \overline{D}^{\dot\alpha} \widehat{\chi}_{R\dot\alpha} = D^\alpha \widehat{\overline{\chi}}_{L\alpha} \qquad (5)$$

These are designed so that there is a global U(1) phase invariance that is conserved by the action.

7. In the context of BRST [22,8], a theory is defined by its BRST transformations, which can be derived from its BRST Poisson Bracket. Here is the BRST Poisson Bracket of the present theory:

$$\mathcal{P}_{\text{Total}}[\mathcal{A}] = \mathcal{P}_\chi[\mathcal{A}] + \mathcal{P}_\phi[\mathcal{A}] + \mathcal{P}_{\text{SUSY}}[\mathcal{A}] \qquad (6)$$

$$\mathcal{P}_\chi[\mathcal{A}] = \int d^4 x \left\{ \frac{\delta\mathcal{A}}{\delta U_{R\dot\alpha}} \frac{\delta\mathcal{A}}{\delta\chi_L^{\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta U_{L\dot\alpha}} \frac{\delta\mathcal{A}}{\delta\chi_R^{\dot\alpha}} \right. \qquad (7)$$

$$+ \frac{\delta\mathcal{A}}{\delta\overline{\Omega}_{\alpha\dot\alpha}} \frac{\delta\mathcal{A}}{\delta V^{\alpha\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta\overline{\Xi}} \frac{\delta\mathcal{A}}{\delta B} \qquad (8)$$

$$\left. + \frac{\delta\mathcal{A}}{\delta\overline{K}} \frac{\delta\mathcal{A}}{\delta\omega} + \frac{\delta\mathcal{A}}{\delta\overline{J}} \frac{\delta\mathcal{A}}{\delta\eta} + \frac{\delta\mathcal{A}}{\delta\overline{\Delta}} \frac{\delta\mathcal{A}}{\delta L} + * \right\} \qquad (9)$$

$$\mathcal{P}_\phi[\mathcal{A}] = \int d^4 x \left\{ \frac{\delta\mathcal{A}}{\delta Z_L^{\dot\alpha}} \frac{\delta\mathcal{A}}{\delta\phi_{R\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta Z_R^{\dot\alpha}} \frac{\delta\mathcal{A}}{\delta\phi_{L\dot\alpha}} \right. \qquad (10)$$

$$+ \frac{\delta\mathcal{A}}{\delta\overline{\Sigma}^{\alpha\dot\alpha}} \frac{\delta\mathcal{A}}{\delta W_{\alpha\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta\overline{\Upsilon}} \frac{\delta\mathcal{A}}{\delta E} \qquad (11)$$

$$\left. + \frac{\delta\mathcal{A}}{\delta\overline{J}'} \frac{\delta\mathcal{A}}{\delta\eta'} + \frac{\delta\mathcal{A}}{\delta\overline{K}'} \frac{\delta\mathcal{A}}{\delta\omega'} + \frac{\delta\mathcal{A}}{\delta\overline{\Delta}'} \frac{\delta\mathcal{A}}{\delta L'} + * \right\} \qquad (12)$$

$$\mathcal{P}_{\text{SUSY}}[\mathcal{A}] = \frac{\partial\mathcal{A}}{\partial h_{\alpha\dot\alpha}} \frac{\partial\mathcal{A}}{\partial\xi^{\alpha\dot\alpha}} \qquad (13)$$

As emphasized above, in this paper we do not try to keep manifest supersymmetry. We decompose the superfields into components and look for nilpotent BRST transformations, which then generate the action. All the above fields and sources are components, not superfields.

We note that each term in the above, such as the first one $\frac{\delta\mathcal{A}}{\delta U_{R\dot\alpha}} \frac{\delta\mathcal{A}}{\delta\chi_L^{\dot\alpha}}$, contains one Zinn source derivative (here it is $U_{R\dot\alpha}$) and one Field derivative (here it is $\chi_L^{\dot\alpha}$), and one of them is Grassmann even ($U_{R\dot\alpha}$ here), and the other odd ($\chi_L^{\dot\alpha}$ here). We will discuss the meaning of the symbols more fully below after Eq. (24), where we write down the action.

The Action $\mathcal{A}$ of the theory contains two parts:

$$\mathcal{A} = \mathcal{A}_{\text{Zinn}} + \mathcal{A}_{\text{Fields}} \qquad (14)$$

To start with, one must find an action $\mathcal{A}_{\text{Zinn}}$ such that the related BRST Poisson Bracket vanishes identically. This action generates the transformations. We can define a sort of square root of the BRST Poisson Bracket, by

$$\delta_{\text{First}} = \delta_{\text{Fields}} + \delta_{\text{Zinns}} \qquad (15)$$

where

$$\delta_{\text{Fields}} = \sum_i \int d^4 x \frac{\delta\mathcal{A}_{\text{Zinn}}}{\delta\text{Zinn}_i} \frac{\delta}{\delta\text{Field}_i} \qquad (16)$$

$$= \int d^4 x \left\{ \frac{\delta\mathcal{A}}{\delta U_{R\dot\alpha}} \frac{\delta}{\delta\chi_L^{\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta U_{L\dot\alpha}} \frac{\delta}{\delta\chi_R^{\dot\alpha}} + \cdots \right\} + * \qquad (17)$$

and where

$$\delta_{\text{Zinns}} = \sum_i \int d^4 x \frac{\delta\mathcal{A}_{\text{Zinn}}}{\delta\text{Field}_i} \frac{\delta}{\delta\text{Zinn}_i} \qquad (18)$$

$$= \int d^4 x \left\{ \frac{\delta\mathcal{A}_{\text{Zinn}}}{\delta\chi_L^{\dot\alpha}} \frac{\delta}{\delta U_{R\dot\alpha}} + \frac{\delta\mathcal{A}_{\text{Zinn}}}{\delta\chi_R^{\dot\alpha}} \frac{\delta}{\delta U_{L\dot\alpha}} + \cdots \right\} + * \qquad (19)$$

If it is true that

$$\mathcal{P}[\mathcal{A}_{\text{Zinn}}] = 0 \qquad (20)$$

then[3] it follows that

$$\delta_{\text{First}}^2 = 0. \qquad (21)$$

Next one looks for an action that satisfies the invariance identity

$$\delta_{\text{Fields}}\mathcal{A}_{\text{Fields}} = 0 \qquad (22)$$

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[3] In the present case this further reduces to suboperators: $\delta_{\text{Fields}} = \delta_{\chi\,\text{Fields}} + \delta_{\phi\,\text{Fields}}$ and $\delta_{\text{Zinns}} = \delta_{\chi\,\text{Zinns}} + \delta_{\phi\,\text{Zinns}}$ and all these suboperators are nilpotent or they anticommute: $\delta_{\text{Fields}}^2 = \delta_{\text{Zinns}}^2 = \{\delta_{\text{Fields}}, \delta_{\text{Zinns}}\} = \delta_{\chi\,\text{Fields}}^2 = \delta_{\phi\,\text{Fields}}^2 = \delta_{\chi\,\text{Zinns}}^2 = \delta_{\phi\,\text{Zinns}}^2 = 0$ etc.

where the expression $\mathcal{A}_{\text{Fields}}$ is in the cohomology space of δ_{Fields} and depends only on Fields and not on Zinns. Since $\mathcal{A}_{\text{Fields}}$ depends only on Fields, it follows that

$$\mathcal{P}[\mathcal{A}] = \mathcal{P}[\mathcal{A}_{\text{Fields}}] = 0 \tag{23}$$

The expression $\mathcal{P}[\mathcal{A}_{\text{Fields}}]$ is trivially zero, because it contains no Zinns, and each term of the BRST Poisson Bracket contains one Zinn.

8. As we pointed out above, a theory can be constructed from

1. A Form of Poisson Bracket, which describes the Fields, Zinns and their pairing;
2. A Zinn Action which describes the transformations of the Fields, and, through the BRST Poisson Bracket, also the transformations of the Zinns;
3. A field action which is invariant under the field transformations.

We will start with the field action, since it is shorter.

9. Here is the action [16] that arises from the chiral undotted χ-type superfields referred to above:

$$\mathcal{A}_{\text{Kinetic } \chi} = \int d^4x \left\{ \chi_L^{\dot\alpha} \partial_{\alpha\dot\alpha} \overline{\chi}_L^\alpha + \chi_R^{\dot\alpha} \partial_{\alpha\dot\alpha} \overline{\chi}_R^\alpha \right.$$
$$\left. + G_{\dot\alpha\dot\beta} \overline{G}^{\dot\alpha\dot\beta} - 2B\overline{B} \right\} \tag{24}$$

where

$$G^{\dot\alpha\dot\beta} = G^{(\dot\alpha\dot\beta)} = \frac{1}{2}\left(\partial_\gamma^{\dot\alpha} V^{\gamma\dot\beta} + \partial_\gamma^{\dot\beta} V^{\gamma\dot\alpha}\right)$$
$$\overline{G}^{\alpha\beta} = \overline{G}^{(\alpha\beta)} = \frac{1}{2}\left(\partial_{\dot\gamma}^\alpha \overline{V}^{\dot\gamma\beta} + \partial_{\dot\gamma}^\beta \overline{V}^{\dot\gamma\alpha}\right) \tag{25}$$

and also

$$\overline{G}^{(\dot\alpha\dot\beta)} = \frac{1}{2}\left(\partial_\gamma^{\dot\alpha} \overline{V}^{\gamma\dot\beta} + \partial_\gamma^{\dot\beta} \overline{V}^{\gamma\dot\alpha}\right)$$
$$G^{(\alpha\beta)} = \frac{1}{2}\left(\partial_{\dot\gamma}^\alpha V^{\dot\gamma\beta} + \partial_{\dot\gamma}^\beta V^{\dot\gamma\alpha}\right) \tag{26}$$

In the above, $\chi_L^{\dot\alpha}$ and $\chi_R^{\dot\alpha}$ are two-component Weyl spinors, B is a complex scalar (it turns out to be an auxiliary field), and G is a complex field strength made from a complex vector field $V^{\gamma\dot\beta}$. More explicitly we have

$$\int d^4x \left\{ G_{\dot\alpha\dot\beta} \overline{G}^{\dot\alpha\dot\beta} \right\} \tag{27}$$

$$= \int d^4x V^{\alpha\dot\alpha} \left(\square \overline{V}_{\alpha\dot\alpha} + \frac{1}{2} \partial_{\alpha\dot\alpha} \partial_{\gamma\dot\delta} \overline{V}^{\gamma\dot\delta} \right) \tag{28}$$

10. Here is the action that arises from the chiral dotted ϕ-type superfields referred to above:

$$\mathcal{A}_{\text{Kinetic } \phi} = \int d^4x \left\{ \phi_L^{\dot\alpha} \partial_{\alpha\dot\alpha} \overline{\phi}_L^\alpha + \phi_R^{\dot\alpha} \partial_{\alpha\dot\alpha} \overline{\phi}_R^\alpha \right.$$
$$+ W_{\alpha\dot\alpha} \overline{W}^{\alpha\dot\alpha} - \frac{1}{2} E \square \overline{E} + \frac{1}{2} \overline{\eta}' \left(\phi_L^\delta \overline{C}_{\dot\delta} + \overline{\phi}_R^\delta C_\delta \right)$$
$$\left. + \frac{1}{2} \eta' \left(\overline{\phi}_L^\delta C_\delta + \phi_R^\delta \overline{C}_{\dot\delta} \right) \right\} \tag{29}$$

In the above, $\phi_L^{\dot\alpha}$ and $\phi_R^{\dot\alpha}$ are two-component Weyl spinors, E is a complex scalar field, $W_{\alpha\dot\alpha}$ is a complex vector field (it turns out to be an auxiliary field); η' is a complex ghost antifield and C_α

is a constant Weyl spinor ghost corresponding to the rigid SUSY transformations.

The above kinetic actions are determined as invariants of the field transformations. Actually here these split into two:

$$\delta_{\chi \text{ Fields}} \mathcal{A}_{\text{Kinetic } \chi} = 0 \tag{30}$$

$$\delta_{\phi \text{ Fields}} \mathcal{A}_{\text{Kinetic } \phi} = 0 \tag{31}$$

As mentioned above, these transformations δ_{Fields} follow from the Zinn actions. Now we present these Zinn actions and explain where they come from.

11. First we have the Zinn Action for the χ sector, which follows from the textbook treatment of gauged supersymmetry [16]. We can write it so that the derivatives with respect to Zinns are easy to see, as follows:

$$\mathcal{A}_{\text{Zinn } \chi} \tag{32}$$

$$= \int d^4x \left\{ U_{R\dot\alpha} \left(B\overline{C}^{\dot\alpha} + G^{(\dot\alpha\dot\beta)} \overline{C}_{\dot\beta} + \xi \cdot \partial \, \chi_L^{\dot\alpha} \right) \right. \tag{33}$$

$$+ U_{L\dot\alpha} \left(-\overline{B}\overline{C}^{\dot\alpha} + \overline{G}^{(\dot\alpha\dot\beta)} \overline{C}_{\dot\beta} + \xi \cdot \partial \, \chi_R^{\dot\alpha} \right) \tag{34}$$

$$+ \Xi \left(\frac{1}{2} \partial^{\alpha\dot\alpha} \overline{\chi}_{L\alpha} \overline{C}_{\dot\alpha} - \frac{1}{2} \partial^{\alpha\dot\alpha} \chi_{R\dot\alpha} C_\alpha + \xi \cdot \partial \, \overline{B} \right) \tag{35}$$

$$+ \Omega_{\alpha\dot\alpha} \left(\partial^{\alpha\dot\alpha} \overline{\omega} + \chi_R^{\dot\alpha} C^\alpha + \overline{\chi}_L^\alpha \overline{C}^{\dot\alpha} + \xi \cdot \partial \, \overline{V}^{\alpha\dot\alpha} \right) \tag{36}$$

$$+ K \left(\overline{V}^{\beta\dot\beta} C_\beta \overline{C}_{\dot\beta} + \xi \cdot \partial \, \overline{\omega} \right) \tag{37}$$

$$\left. + \overline{J} \left(L + \xi \cdot \partial \, \eta \right) + \overline{\Delta} \left(C_\beta \overline{C}_{\dot\beta} \partial^{\beta\dot\beta} \eta + \xi \cdot \partial \, L \right) \right\} \tag{38}$$

In the above we have some more notation in addition to that noted after Eq. (26). In the above, ω is a complex anticommuting Faddeev–Popov Ghost, η is the corresponding complex Faddeev–Popov antighost, and L is a commuting scalar field which is auxiliary and useful for dealing with the gauge fixing term, as shown below. The Zinn sources $U_{R\dot\alpha}$ etc. are conjugate under the BRST Poisson Bracket to the fields.[4]

12. The above Zinn action can usefully be rewritten so that the derivatives with respect to fields are easy to see:

$$\mathcal{A}_{\text{Zinn } \chi \text{ Form 2}} = \int d^4x \chi_R^{\dot\alpha} \left(-\frac{1}{2} \partial_{\alpha\dot\alpha} \Xi C^\alpha \right. \tag{39}$$

$$- \Omega_{\alpha\dot\alpha} C^\alpha + \xi \cdot \partial \, U_{L\dot\alpha} \right) \tag{40}$$

$$+ \chi_L^{\dot\alpha} \left(\frac{1}{2} \partial_{\alpha\dot\alpha} \overline{\Xi} C^\alpha - \overline{\Omega}_{\gamma\dot\alpha} C^\gamma + \xi \cdot \partial \, U_{R\dot\alpha} \right) \tag{41}$$

$$+ V^{\alpha\dot\alpha} \left(\overline{K} \overline{C}_{\dot\alpha} C_\alpha - \frac{1}{2} \partial_\alpha^{\dot\gamma} U_{R\dot\gamma} \overline{C}_{\dot\alpha} - \frac{1}{2} \partial_\alpha^{\dot\gamma} U_{R\dot\alpha} \overline{C}_{\dot\gamma} \right.$$

$$\tag{42}$$

$$\left. - \frac{1}{2} \partial_\alpha^\gamma \overline{U}_{L\gamma} C_\alpha - \frac{1}{2} \partial_\alpha^\gamma \overline{U}_{L\alpha} C_\gamma + \xi \cdot \partial \, \overline{\Omega}_{\alpha\dot\alpha} \right) \tag{43}$$

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[4] $\xi_{\alpha\dot\alpha}$ is an anticommuting constant ghost field for translations in spacetime. The expression $\xi \cdot \partial \equiv \xi_{\alpha\dot\alpha} \partial^{\alpha\dot\alpha}$ is needed to complete the transformations, since the anticommutator of two SUSY transformations is a translation. Note that there is a term linking each source and its field, by a translation, as in $U_{R\dot\alpha} \xi \cdot \partial \, \chi_L^{\dot\alpha}$. This yields exactly zero when we combine it with the variation which comes from (13): $\delta\xi_{\alpha\dot\alpha} = C_\alpha \overline{C}_{\dot\alpha}$ which comes from the action term $\mathcal{A}_{\text{SUSY}} = h^{\alpha\dot\alpha} C_\alpha \overline{C}_{\dot\alpha}$. Usually we can (and do) just ignore these kinds of terms, since they just compensate for total derivatives in the variations.

$$+ \overline{B}\left(\overline{U}_{R\beta}C^{\beta} - U_{L\dot{\beta}}\overline{C}^{\dot{\beta}} + \xi \cdot \partial \,\Xi\right) \tag{44}$$

$$+ \omega\left(\partial^{\gamma\dot{\delta}}\overline{\Omega}_{\gamma\dot{\delta}} + \xi \cdot \partial \,\overline{K}\right) \tag{45}$$

$$+ \eta\left(C_{\beta}\overline{C}_{\dot{\beta}}\partial^{\beta\dot{\beta}}\overline{\Delta} + \xi \cdot \partial \,\overline{J}\right) + L\left(\overline{J} + \xi \cdot \partial \,\Delta\right)\} \tag{46}$$

This form $\mathcal{A}_{\text{Zinn }\chi \text{ Form 2}}$ has another very nice use, explained in the next section.

13. The Zinn Action for the ϕ sector can be derived from the action $\mathcal{A}_{\text{Zinn }\chi \text{ Form 2}}$ in (39) to (46) above simply by changing the names of fields and Zinns. Here it is:

$$\mathcal{A}_{\text{Zinn }\phi} = \int d^4x\, Z_R^{\dot{\alpha}}\left(-\frac{1}{2}\partial_{\alpha\dot{\alpha}}E\,C^{\alpha} - W_{\alpha\dot{\alpha}}C^{\alpha}\right. \tag{47}$$

$$\left. + \xi \cdot \partial \,\phi_{L\dot{\alpha}}\right) \tag{48}$$

$$+ Z_L^{\dot{\alpha}}\left(\frac{1}{2}\partial_{\alpha\dot{\alpha}}\overline{E}\,C^{\alpha} - \overline{W}_{\alpha\dot{\alpha}}C^{\alpha} + \xi \cdot \partial \,\phi_{R\dot{\alpha}}\right) \tag{49}$$

$$+ \Sigma^{\alpha\dot{\alpha}}\left(\overline{\eta}'\overline{C}_{\dot{\alpha}}C_{\alpha} - \frac{1}{2}\partial_{\alpha}^{\dot{\gamma}}\phi_{R\dot{\gamma}}\overline{C}_{\dot{\alpha}}\right. \tag{50}$$

$$-\frac{1}{2}\partial_{\alpha}^{\dot{\gamma}}\phi_{R\dot{\alpha}}\overline{C}_{\dot{\gamma}} - \frac{1}{2}\partial_{\dot{\alpha}}^{\gamma}\overline{\phi}_{L\gamma}C_{\alpha} \tag{51}$$

$$\left. -\frac{1}{2}\partial_{\dot{\alpha}}^{\gamma}\overline{\phi}_{L\alpha}C_{\gamma} + \xi \cdot \partial \,\overline{W}_{\alpha\dot{\alpha}}\right) \tag{52}$$

$$+ \overline{\Upsilon}\left(\overline{\phi}_{R\beta}C^{\beta} - \phi_{L\dot{\beta}}\overline{C}^{\dot{\beta}} + \xi \cdot \partial E\right) \tag{53}$$

$$+ J'\left(\partial_{\gamma\dot{\delta}}\overline{W}^{\gamma\dot{\delta}} + \xi \cdot \partial \overline{\eta}'\right) \tag{54}$$

$$+ \overline{K}'\left(C_{\beta}\overline{C}_{\dot{\beta}}\partial^{\beta\dot{\beta}}L' + \xi \cdot \partial \,\omega'\right) \tag{55}$$

$$+ \overline{\Delta}'\left(\omega' + \xi \cdot \partial \,L'\right) \tag{56}$$

The substitution, to go from (39) to (47) is

$$\chi_R^{\dot{\alpha}} \to Z_R^{\dot{\alpha}};\; \Xi \to E,\, \Omega_{\alpha\dot{\alpha}} \to W_{\alpha\dot{\alpha}};\, \cdots \text{etc.} \tag{57}$$

The new fields L', ω' in the above do not do much. The field η' plays an important role in Eq. (84) below. This generation of the ϕ Zinn action from the χ Zinn action is a kind of involution, since doing it twice will bring us back to the original χ Zinn action.

It is natural to ask why this happens. The author has no answer to that interesting question. But it works nicely as we will see when we look at the spectrum. It seems to be a form of 'BRST Recycling'. If one tries this with the chiral superfield and its Zinn sources, one gets nothing new, because for that case the Zinn sources are also in a chiral multiplet. But there are many situations where something new will arise. They need to be examined.

14. The Zinn Action for the ϕ sector can also be rewritten so that it is easy to take the derivatives by the fields. This yields:

$$\mathcal{A}_{\text{Zinn }\phi \text{ Form 2}} \tag{58}$$

$$= \int d^4x\, \phi_{R\dot{\alpha}}\left(-\Upsilon\overline{C}^{\dot{\alpha}} - \widetilde{\Sigma}^{(\dot{\alpha}\dot{\beta})}\overline{C}_{\dot{\beta}} + \xi \cdot \partial \, Z_L^{\dot{\alpha}}\right) \tag{59}$$

$$+ \int d^4x\, \phi_{L\dot{\alpha}}\left(+\overline{\Upsilon}\overline{C}^{\dot{\alpha}} - \widetilde{\overline{\Sigma}}^{(\dot{\alpha}\dot{\beta})}\overline{C}_{\dot{\beta}} + \xi \cdot \partial \, Z_R^{\dot{\alpha}}\right) \tag{60}$$

$$+ \int d^4x\, E\left(-\frac{1}{2}\partial^{\alpha\dot{\alpha}}\overline{Z}_{L\alpha}\overline{C}_{\dot{\alpha}} + \frac{1}{2}\partial^{\alpha\dot{\alpha}}Z_{R\dot{\alpha}}C_{\alpha} + \xi \cdot \partial \,\overline{\Upsilon}\right) \tag{61}$$

$$+ \int d^4x\, W_{\alpha\dot{\alpha}}\left(-\partial^{\alpha\dot{\alpha}}\overline{J}' - Z_R^{\dot{\alpha}}C^{\alpha} - \overline{Z}_L^{\alpha}\overline{C}^{\dot{\alpha}} + \xi \cdot \partial \,\overline{\Sigma}^{\alpha\dot{\alpha}}\right) \tag{62}$$

$$+ \int d^4x\, \eta'\left(-\overline{\Sigma}^{\beta\dot{\beta}}C_{\beta}\overline{C}_{\dot{\beta}} + \xi \cdot \partial \,\overline{J}'\right) \tag{63}$$

$$+ \int d^4x\, \overline{\omega}'\left(-\Delta' + \xi \cdot \partial \, K'\right) \tag{64}$$

$$+ \int d^4x\, \overline{L}'\left(-C_{\beta}\overline{C}_{\dot{\beta}}\partial^{\beta\dot{\beta}}K' + \xi \cdot \partial \,\Delta'\right) \tag{65}$$

In the above we define

$$\widetilde{\Sigma}^{(\dot{\alpha}\dot{\beta})} = \frac{1}{2}\left(\partial_{\gamma}^{\dot{\alpha}}\Sigma^{\gamma\dot{\beta}} + \partial_{\gamma}^{\dot{\beta}}\Sigma^{\gamma\dot{\alpha}}\right)$$

$$\widetilde{\overline{\Sigma}}^{(\alpha\beta)} = \frac{1}{2}\left(\partial_{\dot{\gamma}}^{\alpha}\overline{\Sigma}^{\dot{\gamma}\beta} + \partial_{\dot{\gamma}}^{\beta}\overline{\Sigma}^{\dot{\gamma}\alpha}\right) \tag{66}$$

$$\widetilde{\overline{\Sigma}}^{(\dot{\alpha}\dot{\beta})} = \frac{1}{2}\left(\partial_{\gamma}^{\dot{\alpha}}\overline{\Sigma}^{\gamma\dot{\beta}} + \partial_{\gamma}^{\dot{\beta}}\overline{\Sigma}^{\gamma\dot{\alpha}}\right)$$

$$\widetilde{\Sigma}^{(\alpha\beta)} = \frac{1}{2}\left(\partial_{\dot{\gamma}}^{\alpha}\Sigma^{\dot{\gamma}\beta} + \partial_{\dot{\gamma}}^{\beta}\Sigma^{\dot{\gamma}\alpha}\right) \tag{67}$$

Of course the above necessarily is the same as what we started with above in $\mathcal{A}_{\text{Zinn }\chi}$ in Eq. (32), provided one changes the names of the fields and Zinns appropriately.

15. Now we have written down the kinetic terms and the Zinn terms. As noted above the two kinetic actions are invariant under separate field transformations. But now we want a mass term that mixes them. At this point it is not clear whether one exists. But it does.[5] For now, we can simply write it down:

$$\mathcal{A}_{\text{Mass }\chi\phi} = \int d^4x\, \left\{m\phi_{L\dot{\alpha}}\chi_R^{\dot{\alpha}} + m\overline{\phi}_{R\alpha}\overline{\chi}_L^{\alpha}\right.$$
$$\left. + mE\overline{B} + mW_{\alpha\dot{\alpha}}\overline{V}^{\alpha\dot{\alpha}} + m\eta'\overline{\omega}\right\} + * \tag{68}$$

and it is easy to verify that it satisfies

$$\left(\delta_{\chi \text{ Fields}} + \delta_{\phi \text{ Fields}}\right)\mathcal{A}_{\text{Mass }\chi\phi} = 0 \tag{69}$$

16. The χ action in Eq. (24) has gauge invariance, as is evident from the transformation of $V_{\alpha\dot{\alpha}}$ contained in line (36). This calls for a gauge-fixing and ghost action $\mathcal{A}_{\text{GGF}}$, and we choose:

$$\mathcal{A}_{\text{GGF}} = \int d^4x\, \delta_{\chi \text{ Fields}}\left(\overline{\eta}\left[\frac{1}{2}\partial_{\alpha\dot{\alpha}}V^{\alpha\dot{\alpha}} + \frac{g}{4}L\right]\right) + * \tag{70}$$

where $\delta_{\chi \text{ Fields}}$ is the BRST transformation of the theory that arises from the Zinn actions above. Here the gauge parameter g can be chosen to be real $g = \overline{g}$. We can integrate out the auxiliary field L by completing the quadratic and shifting, which leaves

$$\mathcal{A}_{\text{GF}} = -\frac{1}{2g}\int d^4x\, \left\{\left(\partial_{\alpha\dot{\alpha}}\overline{V}^{\alpha\dot{\alpha}}\right)\left(\partial_{\beta\dot{\beta}}V^{\beta\dot{\beta}}\right)\right\} \tag{71}$$

The other part is

$$\mathcal{A}_{\text{G}} = \int d^4x\, \left\{\overline{\eta}\Box\omega + \eta\Box\overline{\omega} - g\overline{\eta}C_{\beta}\overline{C}_{\dot{\beta}}\partial^{\beta\dot{\beta}}\eta\right\}$$
$$+ \int d^4x\, \left\{-\frac{1}{2}\overline{\eta}\partial_{\alpha\dot{\alpha}}\left(\chi_L^{\dot{\alpha}}C^{\alpha} + \overline{\chi}_R^{\alpha}\overline{C}^{\dot{\alpha}}\right)\right.$$
$$\left. -\frac{1}{2}\eta\partial_{\alpha\dot{\alpha}}\left(\chi_R^{\dot{\alpha}}C^{\alpha} + \overline{\chi}_L^{\alpha}\overline{C}^{\dot{\alpha}}\right)\right\} \tag{72}$$

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[5] The easiest way to find it is using the spectral sequence, which also shows the existence of other interesting terms. This will be the subject of a sequel paper.

17. So we have found a field action $\mathcal{A}_{\text{Fields}}$, which is the sum of (24), (29), (68), (71) and (72). We want to see what this free massive action says about the equations of motion of the various fields.

18. First we look at the functional derivatives with respect to the scalar fields.[6]

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta B} = -2\overline{B} + m\overline{E} = 0 \tag{73}$$

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta E} = -\frac{1}{2}\square \overline{E} + m\overline{B} = 0 \tag{74}$$

Putting these together yields

$$\left(\square - m^2\right) \overline{E} = 0 \tag{75}$$

19. For the vector bosons we have:

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta \overline{V}_{\alpha\dot\alpha}} = \frac{1}{2g}\partial^{\alpha\dot\alpha}\partial \cdot V + \left(\square V^{\alpha\dot\alpha} + \frac{1}{2}\partial^{\alpha\dot\alpha}\partial \cdot V\right) + mW^{\alpha\dot\alpha} = 0 \tag{76}$$

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta \overline{W}_{\alpha\dot\alpha}} = W^{\alpha\dot\alpha} + mV^{\alpha\dot\alpha} = 0 \tag{77}$$

where

$$\partial \cdot V \equiv \partial_{\gamma\dot\gamma}V^{\gamma\dot\gamma}; \, \partial \cdot W \equiv \partial_{\gamma\dot\gamma}W^{\gamma\dot\gamma} \tag{78}$$

If we choose the Feynman gauge $g = -1$ then this simplifies to

$$\left(\square - m^2\right) V^{\alpha\dot\alpha} = 0 \tag{79}$$

For other gauges things are more complicated in the longitudinal part of $V^{\alpha\dot\alpha}$. This would be more interesting in an interacting model of course.

20. Next we turn to the ghost and fermion fields. We can easily evaluate the following functional derivatives, which yield the equations of motion for these fields:

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta \chi_L^{\dot\alpha}} = \partial_{\alpha\dot\alpha}\overline{\chi}_L^{\alpha} - \frac{1}{2}\partial_{\alpha\dot\alpha}\overline{\eta}C^{\alpha} - m\phi_{R\dot\alpha} = 0 \tag{80}$$

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta \overline{\chi}_R^{\alpha}} = \partial_{\alpha\dot\alpha}\chi_R^{\dot\alpha} - \frac{1}{2}\partial_{\alpha\dot\alpha}\overline{\eta}\overline{C}^{\dot\alpha} - m\overline{\phi}_{L\alpha} = 0 \tag{81}$$

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta \overline{\phi}_R^{\alpha}} = \partial_{\alpha\dot\alpha}\phi_R^{\dot\alpha} - \frac{1}{2}\overline{\eta}'C_{\alpha} - m\overline{\chi}_{L\alpha} = 0 \tag{82}$$

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta \phi_L^{\dot\alpha}} = \partial_{\alpha\dot\alpha}\overline{\phi}_L^{\alpha} - \frac{1}{2}\overline{\eta}'\overline{C}_{\dot\alpha} - m\chi_{R\dot\alpha} = 0 \tag{83}$$

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta \omega} = -\square \overline{\eta} - m\overline{\eta}' \tag{84}$$

Now define

$$m\phi_{R\dot\alpha}' = \frac{1}{2}\partial_{\alpha\dot\alpha}\overline{\eta}C^{\alpha} + m\phi_{R\dot\alpha} \tag{85}$$

$$m\phi_{L\dot\alpha}' = +\frac{1}{2}\partial_{\alpha\dot\alpha}\overline{\eta}C^{\alpha} + m\phi_{L\dot\alpha} \tag{86}$$

For nonzero m, we can write the above equations in the form

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta \chi_L^{\dot\alpha}} = \partial_{\alpha\dot\alpha}\overline{\chi}_L^{\alpha} - m\phi_{R\dot\alpha}' = 0 \tag{87}$$

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta \overline{\chi}_R^{\alpha}} = \partial_{\alpha\dot\alpha}\chi_R^{\dot\alpha} - m\overline{\phi}_{L\alpha}' = 0 \tag{88}$$

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta \overline{\phi}_R^{\alpha}} = \partial_{\alpha\dot\alpha}\phi_R^{'\dot\alpha} - m\overline{\chi}_{L\alpha} = 0 \tag{89}$$

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta \phi_L^{\dot\alpha}} = \partial_{\alpha\dot\alpha}\overline{\phi}_L^{'\alpha} - m\chi_{R\dot\alpha} = 0 \tag{90}$$

Then it is easy to derive that

$$\left(-\square + m^2\right)\overline{\chi}_L^{\alpha} = 0; \, \left(-\square + m^2\right)\chi_R^{\dot\alpha} = 0 \tag{91}$$

and

$$\left(-\square + m^2\right)\phi_R^{'\dot\alpha} = 0; \, \left(-\square + m^2\right)\overline{\phi}_L^{'\alpha} = 0 \tag{92}$$

These fermions $\overline{\phi}_L^{'\alpha}$ and $\phi_R^{'\dot\alpha}$ are made partly from the antighost, but the mass is not gauge dependent.

21. Finally we want to get the mass of the ghost ω. We have

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta \eta} \tag{93}$$

$$= \square \overline{\omega} - gC_\beta \overline{C}_{\dot\beta}\partial^{\beta\dot\beta}\overline{\eta} - \frac{1}{2}\partial_{\alpha\dot\alpha}\left(\chi_R^{\dot\alpha}C^\alpha + \overline{\chi}_L^\alpha \overline{C}^{\dot\alpha}\right) \tag{94}$$

$$\frac{\delta \mathcal{A}_{\text{Fields}}}{\delta \eta'} = \frac{1}{2}\left(\overline{\phi}_R^\delta C_\delta + \phi_L^{\dot\delta}\overline{C}_{\dot\delta}\right) + m\overline{\omega} \tag{95}$$

Adding these equations, if we choose the Feynman gauge $g = -1$, with the second multiplied by m, and using definitions (86) and (85) together with (87) and (88) yields:

$$\left(\square - m^2\right)\overline{\omega} = 0 \tag{96}$$

22. So, in the Feynman gauge, there are two Dirac fermions, one complex scalar boson, one complex vector boson and one complex ghost field with its antifield, all of them with mass m. These are the irreducible components for this supermultiplet. In other gauges, the longitudinal part of the vector boson and the ghosts are more complicated, which is the normal state of affairs for a vector boson. In an interacting model, we would expect to be able to show that the S-matrix is gauge independent.

Note that the gauge symmetry is not broken here, even though the gauge boson is massive. There are no 'Higgs' multiplets needed here, and no gauge symmetry breaking of the U(1) carried by the superspin $\frac{1}{2}$ multiplet here.

The transformations for the chiral dotted spinor superfield $\widehat{\phi}_{L\dot\alpha}$ were obtained by the trick of using the Zinn transformations of the χ sector, and the involution map, to convert them to field transformations of the ϕ sector. It was in this way that we discovered that the four original superfields resolve themselves into auxiliaries and fields in such a way as to provide an irreducible action formulation for the Dirac Irreducible Superspin $\frac{1}{2}$ Massive Multiplet. This action has interesting BRST cohomology, as will be discussed in a future paper, using spectral sequences to sort things out.

Acknowledgments

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[6] For the field equations we always set the Zinn sources to zero of course.

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