# Determination of the structure of the $X(3872)$ in $\bar{p} A$ collisions 

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#### Abstract

Currently, the structure of the $X(3872)$ meson is unknown. Different competing models of the $c \bar{c}$ exotic state $X(3872)$ exist, including the possibilities that this state is either a mesonic molecule with dominating $D^{0} \bar{D}^{* 0}+c . c$. composition, a $c \bar{c} q \bar{q}$ tetraquark, or a $c \bar{c}$-gluon hybrid state. It is expected that the $X(3872)$ state is rather strongly coupled to the $\bar{p} p$ channel and, therefore, can be produced in $\bar{p} p$ and $\bar{p} A$ collisions at PANDA. We propose to test the hypothetical molecular structure of $X(3872)$ by studying the $D$ or $\bar{D}^{*}$ stripping reactions on a nuclear residue. © 2015 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license


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## 1. Introduction

The discovery of exotic $c \bar{c}$ mesons at B-factories and at the Tevatron stimulated interest to explore the possible existence of tetraquark and molecular meson states. The famous $X(3872)$ state has been originally found by BELLE [1] as a peak in $\pi^{+} \pi^{-} J / \psi$ invariant mass spectrum from exclusive $B^{ \pm} \rightarrow K^{ \pm} \pi^{+} \pi^{-} J / \psi$ decays. Nowadays the existence of the $X(3872)$ state and its quantum numbers $J^{P C}=1^{++}$are well established [2]. In particular, radiative decays $X(3872) \rightarrow J / \psi \gamma, X(3872) \rightarrow \psi^{\prime}(2 S) \gamma$ [3] point to the positive C-parity of the $X$ (3872). Probably the most intriguing feature is that the mass of the $X(3872)$ is within 1 MeV the sum of the $D^{0}$ and $D^{* 0}$ meson masses. This prompted the popular conception of the $X(3872)$ being a $D \bar{D}^{*}+\bar{D} D^{*}$ molecule.

To probe the molecular nature of the $X(3872)$ structure has been difficult. So far, most theoretical calculations have been focused on the description of radiative and isospin-violating decays of the $X(3872)$. For example, the $X(3872) \rightarrow J / \psi \gamma$ decay can be well understood within the $D \bar{D}^{*}+$ c.c. molecular hypothesis [4]. On the other hand, the measured large branching fraction $B\left(X(3872) \rightarrow \psi^{\prime}(2 S) \gamma\right) / B(X(3872) \rightarrow J / \psi \gamma)=3.4 \pm 1.4$ [3] seems to disfavor the molecular structure and requires a signifi-

[^0]cant pure $c \bar{c}$ admixture in the $X(3872)$ [5]. The theoretical predictions for the decay rates are, however, quite sensitive to the model details even within various approaches like charmonium or $D \bar{D}^{*}+$ c.c. molecular models.

In this letter we suggest to test the charm meson molecular hypothesis of the $X(3872)$ structure in $\bar{p} A$ collisions at PANDA. Assuming that the $X(3872)$ is coupled to the $p \bar{p}$ channel, we consider the stripping reaction of the $D$-meson on a nuclear target nucleon such that a $\bar{D}^{*}$ is produced and vice versa. We show that the distribution of the produced charmed meson in the light cone momentum fraction $\alpha$ with $z$-axis along $\bar{p}$ beam momentum,
$\alpha=\frac{2\left(\omega_{D^{*}}\left(\mathbf{k}_{D^{*}}\right)+k_{D^{*}}^{z}\right)}{E_{\bar{p}}+m_{N}+p_{\text {lab }}}$,
will be sharply peaked at $\alpha \simeq 1$ at small transverse momenta which allows to unambiguously identify the weakly coupled $D \bar{D}^{*}+$ c.c. molecule. Here, $\mathbf{k}_{D^{*}}$ and $\omega_{D^{*}}\left(\mathbf{k}_{D^{*}}\right)=\left(\mathbf{k}_{D^{*}}^{2}+m_{D^{*}}^{2}\right)^{1 / 2}$ are, respectively, the momentum and energy of the produced $\bar{D}^{*}$ meson in the target nucleus rest frame. Similar studies of hadron-, lepton-, and nucleus-deuteron interactions at high energy have been proposed long ago to test the deuteron structure at short distances as in the spectator kinematics the $n$ - or $p$-stripping cross sections are proportional to the square of the deuteron wave function. For the $X(3872)$ this idea is depicted in Fig. 1 (details follow below).


Fig. 1. Processes contributing to the forward scattering amplitude of a proton on the $D D^{*}$ molecule. Wavy lines denote the $p D$ and $p D^{*}$ elastic scattering amplitudes. Straight lines are labeled with particle's four-momenta. The blobs represent the wave function of the molecule.

## 2. $X$ (3872)-proton cross section

For brevity, the bar, which can be seen over the $D^{*}$ or $D$, will be dropped in many cases below. The charge conjugated states are implicitly included in the calculated cross sections.

The most important ingredients of our calculations are the total $X p$ cross section and the momentum differential cross section $X p \rightarrow D^{*}(D)+$ anything. In the molecular picture, the latter cross section is the $D\left(D^{*}\right)$-meson stripping cross section. To calculate the total $X p$ cross section within the Glauber theory, we start from the graphs shown in Fig. 1 which assume the $D D^{*}$ composition of $X(3872)$. It is convenient to perform calculations in the $D D^{*}$ molecule center-of-mass (c.m.) frame with proton momentum $\mathbf{p}_{\mathrm{p}}$ directed along $z$-axis. The invariant forward scattering amplitudes of the first two processes are
$i M^{(1)}(0)=\int d^{3} k \frac{m_{X}}{\omega_{D}}|\psi(\mathbf{k})|^{2} i M_{p D}(0)$,
$i M^{(2)}(0)=\int d^{3} k \frac{m_{X}}{\omega_{D^{*}}}|\psi(\mathbf{k})|^{2} i M_{p D^{*}}(0)$,
where $m_{X}=\omega_{D}+\omega_{D^{*}}$ is the mass of the molecule and $\omega_{D}\left(\omega_{D^{*}}\right)$ is the energy of $D\left(D^{*}\right)$-meson. (The different assumptions on the momentum dependence of meson energies discussed in the next section have practically no effect on the $X p$ cross section.) The molecule wave function in momentum space is defined as
$\psi(\mathbf{k})=\int \frac{d^{3} r}{(2 \pi)^{3 / 2}} \mathrm{e}^{-i \mathbf{k} \mathbf{r}} \psi(\mathbf{r})$,
where $\mathbf{k}$ is the $D^{*}$ momentum in the $D D^{*}$ c.m. frame, with the normalization condition $\int d^{3} k|\psi(\mathbf{k})|^{2}=1$.

For the calculation of the third and forth processes in Fig. 1 we apply the generalized eikonal approximation (GEA) [6,7] which assumes the nonrelativistic motion of $D$ and $D^{*}$ inside the molecule. In this approximation, the propagator of the intermediate proton depends only on the $z$-component of momentum transfer $\mathbf{q} \equiv \mathbf{k}_{D^{*}}-\mathbf{k}_{D^{*}}^{\prime}$, while the $p D$ and $p D^{*}$ elastic scattering amplitudes depend only on the momenta of incoming particles and on the transverse momentum transfer. Thus, we obtain

$$
\begin{align*}
i M^{(3)}(0)= & \int \frac{d^{3} k d^{3} q}{(2 \pi)^{3}} \frac{i m_{X}}{2 \omega_{D} \omega_{D^{*}}} \psi^{*}(\mathbf{k}-\mathbf{q}) \\
& \times \frac{i M_{p D^{*}}\left(\mathbf{q}_{t}\right) i M_{p D}\left(-\mathbf{q}_{t}\right)}{2 p_{\mathrm{p}}\left(q^{z}+i \varepsilon\right)} \psi(\mathbf{k}) \tag{5}
\end{align*}
$$

$$
\begin{align*}
i M^{(4)}(0)= & \int \frac{d^{3} k d^{3} q}{(2 \pi)^{3}} \frac{i m_{X}}{2 \omega_{D} \omega_{D^{*}}} \psi^{*}(\mathbf{k}-\mathbf{q}) \\
& \times \frac{i M_{p D^{*}}\left(\mathbf{q}_{t}\right) i M_{p D}\left(-\mathbf{q}_{t}\right)}{2 p_{\mathrm{p}}\left(-q^{z}+i \varepsilon\right)} \psi(\mathbf{k}) . \tag{6}
\end{align*}
$$

Therefore,

$$
\begin{align*}
& i M^{(3)}(0)+i M^{(4)}(0) \\
& =\int \frac{d^{3} k d^{2} q_{t}}{(2 \pi)^{2}} \frac{m_{X}}{4 \omega_{D} \omega_{D^{*}} p_{\mathrm{p}}} \psi^{*}\left(\mathbf{k}-\mathbf{q}_{t}\right) i M_{p D^{*}}\left(\mathbf{q}_{t}\right) \\
& \quad \times i M_{p D}\left(-\mathbf{q}_{t}\right) \psi(\mathbf{k}) . \tag{7}
\end{align*}
$$

The optical theorem for the proton-molecule forward scattering amplitude is

$$
\begin{equation*}
\operatorname{Im} M(0)=2 p_{\mathrm{p}} m_{X} \sigma_{p X}^{\mathrm{tot}} \tag{8}
\end{equation*}
$$

Substituting $M(0)=M^{(1)}(0)+M^{(2)}(0)+M^{(3)}(0)+M^{(4)}(0)$ and using the parameterization of the strong interaction scattering amplitudes in the usual form as
$M_{p D^{(*)}}\left(\mathbf{q}_{t}\right)=2 i I_{p D^{(*)}}\left(\mathbf{k}_{D^{(*)}}\right) \sigma_{p D^{(*)}}^{\text {tot }} \mathrm{e}^{-B_{p D^{(*)}} q_{t}^{2} / 2}$,
with $I_{p D^{(*)}}\left(\mathbf{k}_{D^{(*)}}\right)=\left[\left(E_{p} \omega_{D^{(*)}}-p_{\mathrm{p}} k_{D^{(*)}}^{z}\right)^{2}-\left(m_{p} m_{D^{(*)}}\right)^{2}\right]^{1 / 2}$ being the Moeller flux factor we obtain the following expression for the proton-molecule total cross section:

$$
\begin{align*}
\sigma_{p X}^{\mathrm{tot}}= & \int d^{3} k|\psi(\mathbf{k})|^{2}\left[\mathcal{I}_{p D}(-\mathbf{k}) \sigma_{p D}^{\mathrm{tot}}+\mathcal{I}_{p D^{*}}(\mathbf{k}) \sigma_{p D^{*}}^{\mathrm{tot}}\right] \\
& -\frac{1}{2} \int d^{3} k \psi(\mathbf{k}) \mathcal{I}_{p D}(-\mathbf{k}) \sigma_{p D}^{\mathrm{tot}} \mathcal{I}_{p D^{*}}(\mathbf{k}) \sigma_{p D^{*}}^{\mathrm{tot}} \\
& \times \int \frac{d^{2} q_{t}}{(2 \pi)^{2}} \psi^{*}\left(\mathbf{k}-\mathbf{q}_{t}\right) \mathrm{e}^{-\left(B_{p D^{*}}+B_{p D}\right) q_{t}^{2} / 2} \tag{10}
\end{align*}
$$

where the normalized flux factors are defined as $\mathcal{I}_{p D^{(*)}}(\mathbf{k}) \equiv$ $I_{p D^{(*)}}(\mathbf{k}) / p_{\mathrm{p}} \omega_{D^{(*)}}$. In the small binding energy limit the molecule wave function decreases rapidly with increasing momentum $k$ and becomes negligibly small at $k \ll B_{p D}^{-1 / 2}$. In this case one can set $B_{p D}=B_{p D^{*}}=0$ and perform the Taylor expansion of the flux factors in $k^{z}$ in Eq. (10). Then, for the S-state molecule with accuracy up to the linear terms in $k^{z} / m_{D}$ and assuming that $m_{D} \simeq m_{D}^{*}$, $\sigma_{p D^{*}}^{\text {tot }} \simeq \sigma_{p D}^{\text {tot }}$ we obtain the formula
$\sigma_{p X}^{\mathrm{tot}}=\sigma_{p D^{*}}^{\mathrm{tot}}+\sigma_{p D}^{\mathrm{tot}}-\frac{\sigma_{p D^{*}}^{\mathrm{tot}} \sigma_{p D}^{\mathrm{tot}}}{4 \pi}\left\langle r^{-2}\right\rangle_{D D^{*}}$,
in line with previous calculations of the proton-deuteron total cross section [8].

We choose the wave function of a $D D^{*}$ molecule as the asymptotic solution of the Schroedinger equation at large distances:
$\psi(\mathbf{r})=\sqrt{\frac{\kappa}{2 \pi}} \frac{\mathrm{e}^{-\kappa r}}{r}$,
where the range parameter $\kappa=\sqrt{2 \mu E_{b}}$ depends on the reduced mass $\mu=m_{D} m_{D^{*}} /\left(m_{D}+m_{D^{*}}\right)$ and on the binding energy $E_{B}$ of the molecule. The corresponding momentum space wave function is

$$
\begin{equation*}
\psi(\mathbf{k})=\frac{\kappa^{1 / 2} / \pi}{\kappa^{2}+\mathbf{k}^{2}} \tag{13}
\end{equation*}
$$

Let us now discuss the input parameters of our model. Since there is no experimental information on $D p$ and $D^{*} p$ interactions, we rely on simple estimates in the high-energy limit. For small-size $q \bar{q}$ configurations the color dipole model predicts the


Fig. 2. The amplitude for the process $X(3872)+p \rightarrow D^{*}+\mathcal{F}$ where $\mathcal{F} \equiv\left\{\mathcal{F}_{1}, \ldots, \mathcal{F}_{n}\right\}$ is an arbitrary final state in the $p D$ interaction. See also caption to Fig. 1 .
scaling of the total meson-nucleon cross section with the average square of the transverse distance between quark and antiquark in the meson, which is proportional to the square of the Bohr radius $r_{B}=3 / 4 \mu \alpha_{s}$. Here, $\mu=m_{q} m_{\bar{q}} /\left(m_{q}+m_{\bar{q}}\right)$ is the reduced mass with $m_{q}$ and $m_{\bar{q}}$ being the constituent quark and antiquark masses. The Bohr radii of pion, kaon, $D$-meson and $J / \psi$ are ordered as $r_{B \pi}>r_{B K}>r_{B D}>r_{B J / \psi}$. Hence, we expect that the total mesonnucleon cross sections follow the same order. At a beam momentum of $3.5 \mathrm{GeV} / \mathrm{c}$ ( $1 / 2$ of the momentum of $X(3872)$ formed in the $\bar{p} p \rightarrow X$ process on the proton at rest) the total $\pi^{+} p$ and $K^{+} p$ cross sections are about 28 mb and 17 mb , respectively [2]. The $J / \psi p$ cross section is expected to be much smaller, $3.5-6 \mathrm{mb}$ (cf. [9] and refs. therein.). We assume the total $D p$ cross section $\sigma_{p D}^{\text {tot }}=14 \mathrm{mb}$, i.e. slightly below the $K^{+} p$ total cross section. This choice is in reasonable agreement with effective field theory calculations [10].

It is well known that at incident energies of a few GeV , the amplitude of meson (nucleon) - nucleon elastic scattering is (to a good approximation) proportional to the product of the electric form factors of the colliding hadrons (see e.g. [11] and refs. therein). Thus, in the exponential approximation for the $t$-dependence of the form factors, the slope parameters $B_{p M}$ of the transverse momentum dependence of the meson-proton cross section at small $t$ should be proportional to $\left\langle r^{2}\right\rangle_{p}+\left\langle r^{2}\right\rangle_{M}$, where $\left\langle r^{2}\right\rangle_{p}$ and $\left\langle r^{2}\right\rangle_{M}$ are the mean-squared charge radii of the proton and meson, respectively. Since $\left\langle r^{2}\right\rangle_{M} \propto r_{B M}^{2}$, the slope parameters should be also ordered as the Bohr radii. Empirical values at $p_{\text {lab }}=3.65 \mathrm{GeV} / c$ are $B_{p \pi^{+}}=6.75 \pm 0.12 \mathrm{GeV}^{-2}$ and $B_{p K^{+}}=$ $4.12 \pm 0.12 \mathrm{GeV}^{-2}$ as fitted at $0.05 \leq-t \leq 0.44 \mathrm{GeV}^{2}$ [12]. On the other hand, $B_{p J / \psi}=3 \mathrm{GeV}^{-2}$ at the comparable beam momenta [11]. We will assume the value $B_{p D}=4 \mathrm{GeV}^{-2}$, since the Bohr radii of kaon and $D$-meson differ by $\sim 30 \%$ only. For the $p D^{*}$ interaction we assume for simplicity $\sigma_{p D^{*}}^{\text {tot }}=\sigma_{p D}^{\text {tot }}$ and $B_{p D^{*}}=B_{p D}$.

Our educated guess on the $D$ - and $D^{*}$-meson-nucleon cross sections and slope parameters should of course be checked experimentally. The empirical information on $\sigma_{p D}^{\text {tot }}$ can be obtained by measuring the $A$-dependence of the transparency ratio of $D$-meson production in $\bar{p} A$ reactions at beam momenta beyond the charmonium resonance peaks, where the background $\bar{p} p \rightarrow \bar{D} D$ channel dominates. The slope parameter $B_{p D}$ can be addressed by measuring the transverse momentum spread of $D$-meson production in $\bar{p} A$ reactions.

We will further assume that the $X$ (3872) wave function contains $86 \%$ of $D^{0} \bar{D}^{* 0}+c . c$. and $12 \%$ of the $D^{+} D^{*-}+$ c.c. component as predicted by the local hidden gauge approach [4]. The binding energy of $D^{0} \bar{D}^{* 0}$ is likely less than 1 MeV [13] and cannot be determined from existing data [2] accurately enough. We set $E_{b}^{D^{0} \bar{D}^{* 0}}=0.5 \mathrm{MeV}$ and $E_{b}^{D^{+} D^{*-}}=8 \mathrm{MeV}$ in numerical calculations. This corresponds to the range parameters $\kappa_{D^{0} \bar{D}^{* 0}}=0.16 \mathrm{fm}^{-1}$ and $\kappa_{D^{+} D^{*-}}=0.64 \mathrm{fm}^{-1}$. With these parameters the total $p X$ cross section (10) is $\sigma_{p X}^{\text {tot }}=26$ and 23 mb for $D^{0} \bar{D}^{* 0}$ and $D^{+} D^{*-}$ components, respectively, at the molecule momentum of $7 \mathrm{GeV} / c$ in the proton rest frame.

## 3. $D\left(\bar{D}^{*}\right)$ stripping cross section

In high energy hadron-deuteron reactions, the main contribution to the fast backward nucleon production (in the deuteron rest frame or equivalently - fast forward in the deuteron projectile case) is given by the inelastic interaction of the hadron with second nucleon of the deuteron [14]. For large nucleon momenta the spectrum is modified as compared to the impulse approximation (IA) due to the Glauber screening and antiscreening corrections [15] since the hadron may interact with both nucleons. In a similar way, in calculations of the cross section $X+p \rightarrow D^{*}+$ anything, we take into account the IA diagram (Fig. 2a) and the singlerescattering diagrams of the incoming proton (Fig. 2b) and of the outgoing proton or of the most energetic forward going baryon emerging from the inelastic $p D$ interaction (Fig. 2c). The expressions for the invariant matrix elements for the processes (a) and (b) in Fig. 2 are straightforward to obtain in the c.m. frame of the molecule state $X$ :

$$
\begin{align*}
M^{(a)}= & \sqrt{\frac{2 m_{X} \omega_{D^{*}}}{\omega_{D}}}(2 \pi)^{3 / 2} M_{\mathcal{F} ; p D} \psi(\mathbf{k})  \tag{14}\\
M^{(b)}= & \frac{i m_{X}^{1 / 2}}{2 p_{p} \sqrt{2 \omega_{D} \omega_{D^{*}}}} \int d^{3} r \psi(\mathbf{r}) \Theta(-z) \\
& \times \int \frac{d^{2} q_{t}}{(2 \pi)^{2}} \mathrm{e}^{-i\left(\mathbf{k}+\mathbf{q}_{t}\right) \mathbf{r}} M_{\mathcal{F} ; p^{\prime} D^{\prime}} M_{p D^{*}}\left(\mathbf{q}_{t}\right), \tag{15}
\end{align*}
$$

where $\mathbf{k} \equiv \mathbf{k}_{D^{*}}^{\prime}$. In the case of $M^{(b)}$ we applied the GEA by expressing the propagator of the intermediate proton in the eikonal form and using the coordinate representation with $\mathbf{r}=\mathbf{r}_{D^{*}}-\mathbf{r}_{D}$. The explicit form of the amplitude $M^{(c)}$ can be written only for specific outgoing states $\mathcal{F}$. However, for the diffractive states including the leading proton, the expression for $M^{(c)}$ can be obtained from the expression for $M^{(b)}$ by replacing $\Theta(-z) \rightarrow \Theta(z)$, which reflects the change of the time order of the $p D^{*}$ and $p D$ interactions. Thus, for the diffractive outgoing state $\mathcal{F}$ the expression for $M^{(b)}+M^{(c)}$ is given by Eq. (15) with replacement $\Theta(-z) \rightarrow 1$ (neglecting small differences in momenta of incoming and outgoing proton in elementary amplitudes). We assume that the same replacement can be done for any final state $\mathcal{F}$. By summing over all states $\mathcal{F}$ we then obtain the momentum differential $D^{*}$ production (i.e. $D$-stripping) cross section in the molecule rest frame:

$$
\begin{align*}
d \sigma_{p X \rightarrow D^{*}}= & \frac{d^{3} k_{D^{*}}^{\prime}}{(2 \pi)^{3} 2 \omega_{D^{*}} 4 p_{p} m_{X}} \\
& \times \sum_{\text {spins and sorts of } \mathcal{F}} \int\left|M^{(a)}+M^{(b)}+M^{(c)}\right|^{2} \\
& \quad \times(2 \pi)^{4} \delta^{(4)}\left(p_{\mathcal{F}}+k_{D^{*}}^{\prime}-p_{p}-p_{X}\right) \\
& \quad \times \frac{d^{3} p_{\mathcal{F}_{1}}}{(2 \pi)^{3} 2 E_{\mathcal{F}_{1}}} \cdots \frac{d^{3} p_{\mathcal{F}_{n}}}{(2 \pi)^{3} 2 E_{\mathcal{F}_{n}}} \tag{16}
\end{align*}
$$

where $p_{X}$ is the four momentum of the molecule $\left(p_{X}^{2}=m_{X}^{2}\right)$. With a help of the unitarity relation for the elementary amplitudes [16]


Fig. 3. The invariant differential cross section of $D^{* 0}$ production in $X(3872) p$ collisions at $p_{\text {lab }}=7 \mathrm{GeV} / c$. Thick solid line - full calculation according to Eqs. (17)-(19). Thin solid line - the calculation taking into account only IA and screening term of Eq. (18). Dashed line - the calculation with $\kappa=1$ in Eq. (17), i.e. only with the IA term. The inset at $k_{t}=0$ shows the behavior of the differential cross section for a smaller range of $\alpha$.
the sum over spin states and sorts of $\mathcal{F}$ and the integration over phase space volume can be reduced to the products of the imaginary parts of elastic scattering amplitudes. This leads to the following expression for the momentum differential cross section in the molecule rest frame:

$$
\begin{align*}
& \frac{d^{3} \sigma_{p X \rightarrow D^{*}}}{d^{3} k}=\sigma_{p D}^{\mathrm{tot}} \mathcal{I}_{p D}(-\mathbf{k})|\psi(\mathbf{k})|^{2} \kappa  \tag{17}\\
& \kappa= 1-\sigma_{p D^{*}}^{\mathrm{tot}} \mathcal{I}_{p D^{*}}(\mathbf{k}) \int \frac{d^{2} q_{t}}{(2 \pi)^{2}} \frac{\psi^{*}\left(\mathbf{k}+\mathbf{q}_{t}\right)}{\psi^{*}(\mathbf{k})} \mathrm{e}^{-\left(B_{p D}+B_{p D^{*}}\right) \mathbf{q}_{t}^{2} / 2} \\
&+\frac{\left(\sigma_{p D^{*}}^{\mathrm{tot}} \mathcal{I}_{p D^{*}}(\mathbf{k})\right)^{2}}{4} \int \frac{d^{2} q_{t} d^{2} q_{t}^{\prime}}{(2 \pi)^{4}} \frac{\psi\left(\mathbf{k}+\mathbf{q}_{t}\right) \psi^{*}\left(\mathbf{k}+\mathbf{q}_{t}^{\prime}\right)}{|\psi(\mathbf{k})|^{2}} \\
& \quad \times \mathrm{e}^{-\left[B_{p D^{*}}\left(\mathbf{q}_{t}^{2}+\mathbf{q}_{t}^{\prime 2}\right)+B_{p D}\left(\mathbf{q}_{t}^{\prime}-\mathbf{q}_{t}\right)^{2}\right] / 2} . \tag{18}
\end{align*}
$$

The first term in the r.h.s. of Eq. (18) is the pure IA contribution. The second and third terms are, respectively, the screening and antiscreening corrections (see Eqs. (8a) and (8b) in [15]). The $D^{*}$ meson is assumed to be on its vacuum mass shell, $\omega_{D^{*}}(\mathbf{k})=\sqrt{m_{D^{*}}^{2}+\mathbf{k}^{2}}$, while the energy of the $D$ meson is calculated from energy conservation, $\omega_{D}(-\mathbf{k})=m_{X}-\omega_{D^{*}}(\mathbf{k})$. (The condition $\omega_{D}>0$ constrains the maximum momentum of the emitted $D^{*}, k<3.3 \mathrm{GeV} / c$. Above this value our model looses its applicability.) In the case of the $D$-meson production one has to exchange $\mathcal{I}_{p D} \leftrightarrow \mathcal{I}_{p D^{*}}, \sigma_{p D}^{\text {tot }} \leftrightarrow \sigma_{p D^{*}}^{\text {tot }}$ and $B_{p D} \leftrightarrow B_{p D^{*}}$ in Eqs. (17), (18). In this case the on-shell condition is applied to the $D$-meson, while the $D^{*}$ energy is determined by energy conservation.

It is convenient to express the differential invariant $D^{*}$ production cross section (17) in terms of the relative fraction $\alpha$ of the light cone momentum of the $D D^{*}$ molecule carried by the $D^{*}$ :
$\omega_{D^{*}} \frac{d^{3} \sigma_{p X \rightarrow D^{*}}}{d^{3} k}=\alpha \frac{d^{3} \sigma_{p X \rightarrow D^{*}}}{d \alpha d^{2} k_{t}} \equiv G_{X}^{p \rightarrow D^{*}}\left(\alpha, \mathbf{k}_{t}\right)$,
where $\alpha=2\left(\omega_{D^{*}}(\mathbf{k})-k^{z}\right) / m_{X}$. Figs. 3 and 4 show the differential cross section of $D^{* 0}$ and $D^{* \pm}$ production from $X$ (3872) collisions at $7 \mathrm{GeV} / \mathrm{c}$ with proton at rest as a function of $\alpha$ for several values of transverse momentum $k_{t}$. At $k_{t}=0$, the cross section has a sharp maximum at $\alpha \simeq 2 m_{D^{*}} / m_{X} \simeq 1.04$ and is almost unaffected by the screening and antiscreening corrections. With increasing $k_{t}$, the width of $\alpha$-distribution increases while the screening and antiscreening corrections to the IA term become important. This is expected since the large- $k_{t}$ component of the molecule wave function corresponds to small transverse separation between $D$ and $D^{*}$. The corrections become large for $\alpha \simeq 1$ and large transverse momenta as can be directly seen from the structure of the integrands in Eq. (18). Indeed, $\alpha \simeq 1$ corresponds to $k^{z} \simeq 0$ in the molecule rest frame. Then at finite transverse momentum transfer $q_{t}$ the ratio $\psi^{*}\left(\mathbf{k}_{t}+\mathbf{q}_{t}\right) / \psi^{*}\left(\mathbf{k}_{t}\right)$ is less than unity at $k_{t}=0$ and asymptotically tends to unity with growing $k_{t}$. Due to the extremely narrow wave function of the $D^{0} D^{* 0}$ molecule in momentum space, the screening and antiscreening corrections are sharply peaked at $\alpha \simeq 1.1$ and develop structures in the $\alpha$-dependence of the cross section at large transverse momenta. In the case of $\bar{p} A$ reactions these structures are slightly smeared out due to the nucleon Fermi motion (see Fig. 6 below).

## 4. $D^{*}$ and $D$ production off nucleus

In antiproton-nucleus interactions, we focus on the $D^{*}$ (or $D$ ) meson production in the two-step process $\bar{p} p \rightarrow X, X N \rightarrow$ $D^{*}(D)+$ anything. Similar to the case of $X p$ interactions, we apply the Glauber theory to calculate the differential cross sections


Fig. 4. Same as Fig. 3 but for $D^{* \pm}$ production.


Fig. 5. The amplitude of the process $\bar{p} A \rightarrow D^{*} \mathcal{F}(A-2)^{*}$. The wave functions of the initial and final nuclei are denoted as $\psi_{A}$ and $\psi_{A-2}$, respectively. The mass numbers are shown as subscripts. Wavy lines represent elastic scattering amplitudes on nucleons. " $\mathcal{F}$ " stands for the arbitrary final state particles in the semi-inclusive process $X 2 \rightarrow D^{*} \mathcal{F}$. The summation is performed over all possible sets of nucleon scatterers $\left\{n_{1}\right\},\left\{n_{2}\right\}$ and $\left\{n_{3}\right\}$ for the $\bar{p}, X$ and $D^{*}$, respectively.
of the $D^{*}$ production in antiproton-nucleus interactions. We start from the multiple scattering diagram shown in Fig. 5 which can be evaluated within the GEA. We will assume that the nucleus can be described within the independent particle model disregarding the c.m. motion corrections (cf. [17]). The incoming antiproton, intermediate molecular state $X$ and outgoing $D^{*}$-meson are allowed to rescatter on nucleons elastically an arbitrary number of times. The $D^{*}$ production cross section is proportional to the product of the sum of the amplitudes of Fig. 5 and their conjugated. The $X$ state is formed on a proton 1 , while the $D^{*}$ is produced in the collision of $X$ with a nucleon 2 . The nucleons 1 and 2 are fixed in the direct and conjugated amplitudes while the sets of other nucleon scatterers are arbitrary. The leading order contribution is given by the product term without elastic rescatterings. Nuclear absorption corrections are accounted for by summing all possible product terms with non-overlapping sets of nucleon scatterers. This gives the fol-
lowing expression for the momentum differential cross section of $D^{*}$ production on the nucleus:

$$
\begin{align*}
\alpha \frac{d^{3} \sigma_{\bar{p} A \rightarrow D^{*}}}{d \alpha d^{2} k_{t}}= & v_{\bar{p}}^{-1} \int d^{3} r_{1} \mathcal{P}_{\bar{p}, \text { surv }}\left(\mathbf{b}_{1},-\infty, z_{1}\right) \\
& \times \int d^{2} p_{1 t} \frac{d^{2} \Gamma_{\bar{p}}^{1 \rightarrow X}}{d^{2} p_{1 t}} G_{X}^{p \rightarrow D^{*}}\left(\alpha, \mathbf{k}_{t}-\frac{\alpha}{2} \mathbf{p}_{1 t}\right) \\
& \times \int_{z_{1}}^{\infty} d z_{2} \mathcal{P}_{X, \text { surv }}\left(\mathbf{b}_{1}, z_{1}, z_{2}\right) \rho\left(\mathbf{b}_{1}, z_{2}\right) \\
& \times \mathcal{P}_{D^{*}, \operatorname{surv}}\left(\mathbf{b}_{1}, z_{2}, \infty\right) \tag{20}
\end{align*}
$$

where
$\frac{d^{2} \Gamma_{\bar{p}}^{1 \rightarrow X}}{d^{2} p_{1 t}}=\frac{\overline{\left|M_{X ; \bar{p} 1}\right|^{2}} v_{\bar{p}}}{(2 \pi)^{2} 4 p_{\mathrm{lab}}^{2} E_{1}} n_{p}\left(\mathbf{r}_{1} ; \mathbf{p}_{1 t}, \Delta_{m_{X}}^{0}\right)$
is the in-medium width of $\bar{p}$ with respect to production of $X$ with transverse momentum $\mathbf{p}_{1 t} ; v_{\bar{p}}=p_{\text {lab }} / E_{\bar{p}}$ is the antiproton velocity; $n_{p}\left(\mathbf{r}_{1} ; \mathbf{p}_{1 t}, \Delta_{m_{X}}^{0}\right)$ is the proton occupation number; $E_{1}=m_{N}-B$ with $B=8.6 \mathrm{MeV}$ being the nucleon binding energy in ${ }^{40} \mathrm{Ar}$ nucleus. The longitudinal momentum $\Delta_{m_{X}}^{0}$ of the proton 1 is obtained from the condition of on-shell production of the state $X$ in the process $\bar{p} 1 \rightarrow X$ :
$\Delta_{m_{X}}^{0}=\frac{m_{N}^{2}+E_{1}^{2}+2 E_{\bar{p}} E_{1}-m_{X}^{2}}{2 p_{\mathrm{lab}}}$.
The nucleon occupation numbers are taken as the depleted Fermi distributions supplemented by high-momentum tail due to shortrange quasideuteron correlations (SRCs) [18-20]:

$$
\begin{align*}
n_{q}(\mathbf{r} ; \mathbf{p})= & \left(1-P_{2, q}\right) \Theta\left(p_{F, q}-p\right) \\
& +\frac{\pi^{2} P_{2, q} \rho_{q}\left|\psi_{d}(p)\right|^{2} \Theta\left(p-p_{F, q}\right)}{\int_{p_{F, q}}^{\infty} d p^{\prime} p^{\prime 2}\left|\psi_{d}\left(p^{\prime}\right)\right|^{2}}, \quad q=p, n \tag{23}
\end{align*}
$$

where $p_{F, q(\mathbf{r})}=\left[3 \pi^{2} \rho_{q}(\mathbf{r})\right]^{1 / 3}$ are the nucleon Fermi momenta, $P_{2, p}=0.25$ and $P_{2, n}=P_{2, p} Z / N$ are the proton and neutron fractions above the Fermi surface, $\rho_{q}(\mathbf{r})$ are the nucleon densities, and $\psi_{d}(p)$ is the deuteron wave function. In Eq. (20), the nuclear absorption is given by the survival probabilities of the antiproton, the molecule, and the $D^{*}$ :

$$
\begin{gather*}
\mathcal{P}_{\bar{p}, \text { surv }}\left(\mathbf{b}_{1},-\infty, z_{1}\right)=\exp \left\{-\sigma_{p \bar{p}}^{\text {tot }} \int_{-\infty}^{z_{1}} d z \rho\left(\mathbf{b}_{1}, z\right)\right\},  \tag{24}\\
\mathcal{P}_{X, \operatorname{surv}}\left(\mathbf{b}_{1}, z_{1}, z_{2}\right)=\exp \left\{-\sigma_{p X}^{\text {tot }} \int_{z_{1}}^{z_{2}} d z \rho\left(\mathbf{b}_{1}, z\right)\right\},  \tag{25}\\
\mathcal{P}_{D^{*}, \text { surv }}\left(\mathbf{b}_{1}, z_{2}, \infty\right)=\exp \left\{-\sigma_{p D^{*}}^{\text {tot }} \int_{z_{2}}^{\infty} d z \rho\left(\mathbf{b}_{1}, z\right)\right\}, \tag{26}
\end{gather*}
$$

where $\rho=\rho_{p}+\rho_{n}$ is the total nucleon density. We use the twoparameter Fermi distributions of protons and neutrons [9]. As usual in the Glauber theory, Eqs. (24)-(26) neglect the Fermi motion of nucleon scatterers. In a similar way, in writing Eq. (20) we neglected the Fermi motion of nucleon 2 since the elementary cross section (19) depends only weakly on the proton momentum (via the flux factors, screening- and antiscreening contributions) and is in leading order proportional to the square of the molecule wave function. However, the transverse Fermi motion of proton 1 is taken into account in Eq. (20) in the high-energy approximation (cf. [15]). The latter implies that the light cone momentum fraction $\alpha$ can be expressed in the target nucleus rest frame according to Eq. (1) where the Fermi motion of the proton 1 is still neglected. (We have numerically checked that using the exact Lorentz transformation to the c.m. frame of $X$ to evaluate the invariant cross section $\omega_{D^{*}} \frac{d^{3} \sigma_{p X \rightarrow D^{*}}}{A^{d^{3}}}$ instead of using the infinite momentum frame in Eq. (20) ${ }^{d^{j}}$ which conserves $\alpha$ and assumes Galilean transformation for $\mathbf{k}_{t}$ produces indistinguishable results.)

The $\bar{p} p \rightarrow X$ matrix element in Eq. (21) is one of the major uncertainties in our calculations. Its modulus squared can be formally expressed in terms of the partial decay width $\Gamma_{X \rightarrow \bar{p} p}$ as
$\overline{\left|M_{X ; \bar{p} 1}\right|^{2}}=\frac{4 \pi\left(2 J_{X}+1\right) m_{X}^{2} \Gamma_{X \rightarrow \bar{p} p}}{\sqrt{m_{X}^{2}-4 m_{N}^{2}}}$,
where the overline means averaging over antiproton and proton helicities and summation over the helicity of $X$. There is no experimental data on the partial decay width $\Gamma_{X(3872) \rightarrow \bar{p} p}$. (The recent LHCb data on $\bar{p} p$ invariant mass spectra from $B^{+} \rightarrow p \bar{p} K^{+}$ decays [21] do not allow to clearly identify $X$ (3872) in the $p \bar{p}$ decay channel due to statistical limitations.) In the present calculations, we will use the value $\Gamma_{X(3872) \rightarrow \bar{p} p} \simeq 30 \mathrm{eV}$ as suggested by theoretical estimates [22]. This value is about two times smaller than $\Gamma_{\chi_{c 1}(1 P) \rightarrow \bar{p} p}$. However, one should note that, in the molecular picture, the decay of the $X(3872)$ to the $p \bar{p}$ state requires the production of only two $q \bar{q}$ pairs, and not three $q \bar{q}$ pairs as in the ordinary charmonium decay to the $p \bar{p}$ channel. Thus, the partial decay width of the $X$ (3872) into the $p \bar{p}$ channel may be even larger than that of the $\chi_{c 1}(1 P)$ state [22].

Formula (20) has a simple physical interpretation if we express the integral $\int d^{3} r_{1}$ as $\int d^{2} b_{1} \int d z_{1}$. The factor $\mathcal{P}_{\bar{p}, \text { surv }}\left(\mathbf{b}_{1},-\infty, z_{1}\right)$ is the probability that the incoming from $z=-\infty$ antiproton with impact parameter $\mathbf{b}_{1}$ will reach the point $z=z_{1}$. The combination $\left(d z_{1} / v_{\bar{p}}\right) d^{2} p_{1 t} d^{2} \Gamma_{\bar{p}}^{1 \rightarrow X} / d^{2} p_{1 t}$ is the $X(3872)$ formation probability within the transverse momentum element $d^{2} p_{1 t}$ when the $\bar{p}$ is passing the longitudinal element $d z_{1}$. The factor $\mathcal{P}_{X, \text { surv }}\left(\mathbf{b}_{1}, z_{1}, z_{2}\right)$ is the probability that the molecule will reach the point $z=z_{2}$. The combination $d z_{2}(d \alpha / \alpha) d^{2} k_{t} G_{X}^{p \rightarrow D^{*}}\left(\alpha, \mathbf{k}_{t}-\frac{\alpha}{2} \mathbf{p}_{1 t}\right) \rho\left(\mathbf{b}_{1}, z_{2}\right)$ is the probability that a $D^{*}$ will be produced in the kinematical element $d \alpha d^{2} k_{t}$ when the $X(3872)$ is passing the longitudinal element $d z_{2}$. Finally, the factor $\mathcal{P}_{D^{*} \text {,surv }}\left(\mathbf{b}_{1}, z_{2}, \infty\right)$ is the probability that the $D^{*}$ will escape from the nucleus. In the spirit of the eikonal approach, all particles propagate parallel to the beam direction. (For example, we assumed that the transverse momentum of the molecule, $\mathbf{p}_{1 t}$, does not influence its trajectory.) The integration over $z_{2}$ can be taken with the explicit forms of the survival probabilities Eqs. (25), (26). As a result Eq. (20) takes the following simple form:

$$
\begin{align*}
\alpha \frac{d^{3} \sigma_{\bar{p} A \rightarrow D^{*}}}{d \alpha d^{2} k_{t}}= & \frac{1}{v_{\bar{p}}\left(\sigma_{p X}^{\text {tot }}-\sigma_{p D^{*}}^{\text {tot }}\right)} \int d^{3} r_{1} \mathcal{P}_{\bar{p}, \text { surv }}\left(\mathbf{b}_{1},-\infty, z_{1}\right) \\
& \times \int d^{2} p_{1 t} \frac{d^{2} \Gamma_{\bar{p}}^{1 \rightarrow X}}{d^{2} p_{1 t}} G_{X}^{p \rightarrow D^{*}}\left(\alpha, \mathbf{k}_{t}-\frac{\alpha}{2} \mathbf{p}_{1 t}\right) \\
& \times\left[\mathcal{P}_{D^{*}, \text { surv }}\left(\mathbf{b}_{1}, z_{1}, \infty\right)-\mathcal{P}_{X, \text { surv }}\left(\mathbf{b}_{1}, z_{1}, \infty\right)\right] \tag{28}
\end{align*}
$$

In Fig. 6 we display the differential cross sections of charmed meson production in antiproton collisions with argon nucleus at $7 \mathrm{GeV} / c$. The $D$ and $D^{*}$ cross sections are peaked at $\alpha \simeq$ $2 m_{D} / m_{X}=0.96$ and $\alpha \simeq 2 m_{D^{*}} / m_{X}=1.04$, respectively, and behave in similar way as a function of $\alpha$ and $k_{t}$. The widths of $\alpha$-dependence of the $D^{* 0}$ and $D^{0}$ cross sections are much smaller and the peak values are much larger as compared to the $D^{* \pm}$ and $D^{ \pm}$cross sections. The $\alpha$-dependence of $D^{*}$ and $D$ production in $\bar{p} A$ collisions is dominated by the elementary cross section (cf. Figs. 3, 4). However, a closer look reveals significant differences between $D^{*}$ production on a nucleus and on a proton due to the Fermi motion. These are better visible in the ratio of the two cross sections depicted in Fig. 7. At $k_{t}=0$ the ratio has a minimum at $\alpha \simeq 2 m_{D^{*}} / m_{X}$ because in this case the contribution from target protons with finite transverse momentum $p_{1 t}$ is suppressed by the factor $\left|\psi\left(\mathbf{k}_{t}-\frac{\alpha}{2} \mathbf{p}_{1 t}\right)\right|^{2} /\left|\psi\left(\mathbf{k}_{t}\right)\right|^{2}$. However, with increasing $k_{t}$ this factor becomes larger than unity for comoving proton 1 . This leads to the observed local maximum in the $\alpha$-dependence for $k_{t} \simeq 0.1-0.4 \mathrm{GeV} / c$. At large $k_{t}$ or for large deviations of $\alpha$ from unity the ratio tends to the constant value.


Fig. 6. The invariant differential cross sections of $D^{* 0}, D^{0}, D^{* \pm}$ and $D^{ \pm}$production in $\bar{p}^{40} \mathrm{Ar}$ collisions at $p_{\text {lab }}=7 \mathrm{GeV} / \mathrm{c}$. The calculations are done using full cross section $X(3872) p \rightarrow D^{*}(D)$ in Eq. (28) including the IA term as well as screening and antiscreening corrections (see Eqs. (17)-(19)). For $k_{t}=0$, the cross sections of $D^{* 0}$ and $D^{0}$ production are divided by a factor of 100 .



Fig. 7. The ratio of $D^{*}$ production cross sections in $\bar{p}^{40} \mathrm{Ar}$ and $X(3872) p$ collisions at $p_{\text {lab }}=7 \mathrm{GeV} / \mathrm{c}$ for several values of transverse momentum $k_{t}$ as a function of light cone momentum fraction $\alpha$. The ratio is normalized at unity for $\alpha=0.5$. Panel (a) $-D^{* 0}$. Panel (b) $-D^{* \pm}$.

## 5. Uncertainty and background

The uncertainty of our calculations can be read from Fig. 8. The unknown binding energy of the $D^{0} \bar{D}^{* 0}$ molecule is the main source of uncertainty as a weaker binding produces a narrower $\alpha$-distribution and vice versa. As a consequence of the partial cancellation between the survival probabilities of $D^{*}$ and molecule the reduction of the $p D$ and $p D^{*}$ cross sections from 14 mb to 7 mb leads to the reduction of the peak of $D^{* 0}$ production cross section by $\sim 15 \%$ only. Of course, on the top of these two effects there is an uncertainty due to the experimentally unknown width $\Gamma_{X \rightarrow \bar{p} p}$ which enters the cross section (28) as an overall multiplication factor. The qualitative behavior of the $\alpha$-distribution is not changed by varying the model parameters.

The major background is given by the direct process $\bar{p} N \rightarrow$ $D \bar{D}^{*}, \bar{D} D^{*}$ on the bound nucleon, because the thresholds of
$X$ (3872) and $D \bar{D}^{*}$ production in $\bar{p} p$ collisions are almost the same. The background cross section can be calculated as (derivation is similar to that of Eq. (28))

$$
\begin{align*}
\alpha \frac{d^{3} \sigma_{\bar{p} A \rightarrow D^{*}}^{\mathrm{bg}}}{d \alpha d^{2} k_{t}}= & \sum_{N_{1}=n, p} \frac{2}{(2 \pi)^{3} E_{1} p_{\mathrm{lab}} k_{D}^{z}} \\
& \times \int d^{2} p_{1 t} \frac{q_{\bar{p} 1}(\varepsilon) \varepsilon^{2}}{q_{D D^{*}}(\varepsilon)} \frac{d \sigma_{\bar{p} 1 \rightarrow D^{*} D}(\varepsilon)}{d \Omega} \\
& \times \int d^{3} r_{1} \mathcal{P}_{\bar{p}, \text { surv }}\left(\mathbf{b}_{1},-\infty, z_{1}\right) n_{1}\left(\mathbf{r}_{1} ; \mathbf{p}_{1 t}, \Delta_{\varepsilon}^{0}\right) \\
& \times \mathcal{P}_{D^{*}, \text { surv }}\left(\mathbf{b}_{1}, z_{1}, \infty\right)\left[1-\mathcal{P}_{D, \text { surv }}\left(\mathbf{b}_{1}, z_{1}, \infty\right)\right] \tag{29}
\end{align*}
$$



Fig. 8. The $\alpha$-dependence of $D^{* 0}$ production at $k_{t}=0$ in $\bar{p}^{40} \mathrm{Ar}$ collisions at $p_{\text {lab }}=$ $7 \mathrm{GeV} / c$. Panel (a) shows calculations with default cross section $\sigma_{p D}^{\text {tot }}=14 \mathrm{mb}$. The signal cross section (28) is shown for the different binding energies $E_{b}$ of the $D^{0} \bar{D}^{* 0}$ molecule. The background cross section (29) is shown with SRCs (default calculation) and without SRCs. Panel (b) shows calculations with default $E_{b}=0.5 \mathrm{MeV}$ for the different $\sigma_{p D}^{\text {tot }}$ as indicated. It is always assumed that $\sigma_{p D^{*}}^{\text {tot }}=\sigma_{p D}^{\text {tot }}$. Insets show the narrower region of $\alpha$. The background cross section is divided by a factor of 3 .
where
$\varepsilon \equiv \varepsilon\left(\alpha, \mathbf{k}_{t}^{\prime}=\mathbf{k}_{t}-\frac{\alpha}{2} \mathbf{p}_{1 t}\right)=\left(\frac{2\left[(2-\alpha) m_{D^{*}}^{2}+\alpha m_{D}^{2}+2 k_{t}^{\prime 2}\right]}{\alpha(2-\alpha)}\right)^{1 / 2}$
is the c.m. energy of $D$ and $D^{*} ; q_{\bar{p} 1}(\varepsilon)=\left(\varepsilon^{2} / 4-m_{p}^{2}\right)^{1 / 2}$ and $q_{D D^{*}}(\varepsilon)=\left[\left(\varepsilon^{2}+m_{D}^{2}-m_{D^{*}}^{2}\right)^{2} / 4 \varepsilon^{2}-m_{D}^{2}\right]^{1 / 2}$ are the c.m. momenta of the colliding $\bar{p} N$ pair and of the produced $D D^{*}$ pair, respectively. The longitudinal momentum $k_{D}^{z}$ of $D$-meson in the nucleus rest frame is calculated from relation
$2-\alpha=\frac{2\left(\omega_{D}+k_{D}^{z}\right)}{E_{\bar{p}}+m_{N}+p_{\text {lab }}}$,
with $\omega_{D}=\sqrt{k_{D}^{z}+m_{D}^{2}}$. The longitudinal momentum of the nucleon, $\Delta_{\varepsilon}^{0}$, is given by Eq. (22) with $m_{X}$ replaced by $\varepsilon$.

For the $D^{* 0}$ production on the proton, the near-threshold $S$-wave cross section is $\sigma_{\bar{p} p \rightarrow D^{*} D}(\varepsilon)=2 \sigma_{\bar{p} p \rightarrow D^{* 0} \bar{D}^{0}}(\varepsilon)$, where the direct (non-resonant) cross section $\sigma_{\bar{p} p \rightarrow D^{* 0} \bar{D}^{0}}(\varepsilon)$ has been taken from Ref. [22] which is the only estimate of the discussed cross section available in the literature. (We included the factor of 2 since our $D^{* 0}$ includes both physical states, $D^{* 0}$ and $\bar{D}^{* 0}$.) The estimate of [22] was obtained by using dimensional counting considerations to express the cross section of $\bar{p} p \rightarrow D^{* 0} \bar{D}^{0}$ at high energies in terms of the cross section of $\bar{p} p \rightarrow K^{*-} K^{+}$which is known in the limited energy range. As the next step, the $\bar{p} p \rightarrow D^{* 0} \bar{D}^{0}$ cross section was extrapolated in [22] towards the threshold and multiplied by the $S$-wave fraction $f_{L=0} \simeq 9.3 \%$ which is regarded by the author of [22] himself as "a crude extrapolation". Thus, we feel that the near-threshold estimate of [22] can be considered as an order of magnitude estimate. In the case of the $D^{* 0}$ production on the neutron, the only possible channel is $\bar{p} n \rightarrow D^{* 0} D^{-}$. Thus, we assume $\sigma_{\bar{p} n \rightarrow D^{*} D}(\varepsilon)=\sigma_{\bar{p} p \rightarrow D^{* 0} \bar{D}^{0}}(\varepsilon)$.

The result of calculation using Eq. (29) is shown in Fig. 8. The dependence of the background cross section on the $p D^{*}$ and $p D$ cross sections is quite modest and follows the tendency of the signal cross section. Thus, at $k_{t}=0$, the sharp peak of $D^{*}$ production at $\alpha=1.04$ due to the stripping reaction is clearly visible on the smooth background. The peak is almost not influenced by intramolecular screening and antiscreening effects. Moreover, we expect that the elastic rescattering of antiproton and produced particles on the nucleons will practically not change the $\bar{D}^{*}$ and $D$ spectra at small $k_{t}$ [17]. Finally, the influence of the SRCs - which are always included in default calculations - is mainly in the reduction of the nucleon occupancies at small momenta. This results in $15 \%$ reduction of the background at $\alpha \simeq 1$ (Fig. 8a) and the corresponding rescaling of the signal cross section (not shown).

The signal-to-background ratio at the peak stays almost constant as the mass number of the target nucleus varies between 20 and 208 . The shape of the $\alpha$-dependence of the signal cross section practically does not vary with mass number in that region. The mass dependence of the total $D^{* 0}$ production cross section due to the stripping reaction can be well approximated by formula $\sigma_{D * 0}=16 \mathrm{pb} \cdot A^{0.46}$. With the high luminosity mode at PANDA, $L=2 \cdot 10^{32} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$, the estimated production rate due to the stripping reaction is about $60 D^{* 0}$ events per hour.

## 6. Discussion and conclusions

We proposed the idea of $D\left(\bar{D}^{*}\right)$ stripping from the $X(3872)$ state, to investigate if $X$ (3872) has a molecular structure from the narrow peaks in $\alpha$-distribution of $\bar{D}^{*}$ and $D$ at $\alpha \simeq 1$. Other microscopic models of $X(3872)$, e.g. tetraquark or $c \bar{c}$-gluon hybrid, would lead to the flat $\alpha$-spectrum of $\bar{D}^{*}(D)$. In such models, there are no primordial hadronic components in $X(3872)$ to be "released" or "knocked-out". Thus, the momentum distribution of $\bar{D}^{*}(D)$ in the process $X N \rightarrow \bar{D}^{*}(D)$ would be dominated by phase space of the final state particles.

There is, however, another possible source of narrow peaks in $\alpha$-distributions of $\bar{D}^{*}$ and $D$. The BELLE Collaboration [23] has found a significant near-threshold enhancement in the $D^{* 0} \bar{D}^{0}$ invariant mass spectrum from $B \rightarrow D^{* 0} \bar{D}^{0} K$ decays. We note that this does not exclude the existence of the $D^{* 0} \bar{D}^{0}$ bound state. (One similar example is $\Lambda(1405)$ which lies about 30 MeV below $K^{-} p$ threshold and can be treated as a $K^{-} p$ quasibound state although it strongly influences the $K^{-} p \rightarrow \Sigma^{ \pm} \pi^{\mp}$ and $K^{-} p \rightarrow \Sigma^{0} \pi^{0}$ cross sections at small beam momenta $[24,25]$.) But it is also possible that $X(3872)$ is a resonance coupled to the $D^{* 0} \bar{D}^{0}+$ c.c. channel. If such a resonance state is produced in peripheral $\bar{p} A$ collisions, it
will decay far away from the nucleus, since the width of $X$ (3872) is less than 1 MeV . The resulting $\alpha$-distributions of $D^{* 0}$ and $\bar{D}^{0}$ will be also sharply peaked near $\alpha \simeq 1$ at small $k_{t}$. However, in this case both decay products can in principle be detected. This gives a clear experimental signature for distinguishing such decay events. In contrast, the stripping events would contain only one meson, $D^{* 0}$ or $\bar{D}^{0}$, in the same kinematical region.

The stripping reaction can be also considered for other production channels of $X(3872)$, e.g. in proton-, electron- and photoninduced reactions on nuclei. Other exotic $X, Y, Z$ states, such as the $X(3940)$ [26], $Y(4140)$ [27], $X(4160)$ [28] (cf. recent reviews [29,13] for a more complete list), may be interpreted as molecular states of $D^{*} \bar{D}^{*}$ or $D_{s}^{*} \bar{D}_{s}^{*}$. These hypothetical molecular structures may also be tested by using stripping reactions, similar to $X(3872)$. Experimentally, such studies could be performed at J-PARC, FAIR, SPS@CERN, and EIC.

Apart from $c \bar{c}$ exotic states, there are other mesons which possibly have molecular structures. The $a_{0}(980)$ and $f_{0}(980)$ states viewed as a $K \bar{K}$ bound state " $f$ " can be produced in $\gamma(\pi) N \rightarrow f N$ reaction on the bound nucleon followed by the stripping process $f N \rightarrow \bar{K}(K)+$ anything on another nucleon of the nuclear target residue. The $D_{s 0}^{*}(2317)$ viewed as $D K$ and $D_{s 1}(2460)$ viewed as $D^{*} K$ can be produced in $\bar{p} p \rightarrow D_{s}^{ \pm} D_{s 0}^{* \mp}\left(D_{s 1}^{\mp}\right)$ reaction on the bound proton followed by the stripping process $D_{s 0}^{* F}\left(D_{s 1}^{\mp}\right) N \rightarrow$ $D\left(D^{*}\right)+$ anything or $D_{s 0}^{* \mp}\left(D_{s 1}^{\mp}\right) N \rightarrow K+$ anything on another nucleon. The antiproton-nucleus interactions open another unique opportunity to produce fast antibaryons. In this way, e.g. the molecular $K^{+} \bar{p}$ hypothesis for the $\bar{\Lambda}(1405)$ can be tested by using the two-step processes $\bar{p} p \rightarrow \bar{\Lambda}(1405) \Lambda, \bar{\Lambda}(1405) N \rightarrow K^{+}+$ anything. Such kind of processes can be studied at PANDA and possibly at J-PARC.

In conclusion, we have demonstrated that the spectra of $\bar{D}^{*}$ and $D$ in the light cone momentum fraction at small transverse momenta allow to test the hypothetical $D \bar{D}^{*}$ molecular structure of the $X$ (3872) produced in $\bar{p} A$ collisions at threshold. We propose to search the narrow peak in $\bar{D}^{*}$ or $D$ production at $\alpha \simeq 1$ and small $k_{t}$ as an unambiguous signal of the $D \bar{D}^{*}$ molecular state formation in $\bar{p} A$ collisions in PANDA experiment at FAIR.

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