Modal parameter identification for a roof overflow powerhouse under ambient excitation

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Abstract

Modal parameter identification is a core issue in health monitoring and damage detection for hydraulic structures. For a roof overflow hydropower station with a bulb tubular unit under ambient excitation, a complex unit-powerhouse-dam coupling vibration system increases the difficulties of modal parameter identification. In this study, in view of the difficulties of modal order determination and the noise jamming caused by ambient excitation, along with false mode identification and elimination problems, the ensemble empirical mode decomposition (EEMD) method was used to decrease noise, the singular entropy increment spectrum was used to determine system order, and multiple criteria were used to eliminate false modes. The eigensystem realization algorithm (ERA) and stochastic subspace identification (SSI) method were then used to identify modal parameters. The results show that the relative errors of frequencies in the first four modes were within 10% for the ERA method, while those of SSI were over 10% in the second and third modes. Therefore, the ERA method is more appropriate for identifying the structural modal parameters for this particular powerhouse layout.

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Keywords: Hydraulic structure; Order determination; Ensemble empirical mode decomposition; Singular entropy; Eigensystem realization algorithm; Bulb tubular unit

1. Introduction

In general, the reduction of hydraulic structural strength and stiffness results from the effects of design loads, working conditions, and unexpected external factors such as earthquakes. Hence, structural damage will occur during the structure’s service life, affecting the safety and stability of its operation. Therefore, this type of damage must be considered in structural health monitoring. Recently, the topic of vibration-based structural health monitoring has attracted considerable interest. This type of monitoring offers the possibility of obtaining more accurate and objective information with respect to the deterioration and damage of instrumented structures. With damage, the structural dynamic properties will change. This can be reflected in the modal parameters. Therefore, obtaining modal parameters with accurate techniques is a key prerequisite for monitoring the structural operation conditions (Darbre et al., 2000).

Traditionally, modal parameter identification is carried out through the frequency domain- or time domain-based methods (Ibrahim and Pappa, 1982; James et al., 1996; Brinker et al., 2001; Schoukens et al., 1998). The latter can avoid errors caused by data conversion and increase the identification accuracy, because it deals directly with measured response signals, without going through the Fourier transform (FT) process.
In this study, two identification methods, ERA and SSI, were used to identify modal parameters of a roof overflow powerhouse under ambient excitation. In order to overcome the difficulties of practical application due to noise, system order, and false modes, we used the EEMD method to decrease the noise effect and effectively identify modal parameters. This method is versatile and used in a broad range of applications for signals with nonlinear components, singular points, and irregular transient parts. However, it produces mode mixing, end effects, and stopping criterion problems, which cause a loss in useful signal (Rato et al., 2008; Sweeney et al., 2013). EEMD can not only self-adaptively decompose both nonlinear and non-stationary data, but also effectively solve the mode mixing, end effects, and termination condition problems of EMD.

In this study, two identification methods, ERA and SSI were used to identify modal parameters of a roof overflow powerhouse under ambient excitation. In order to overcome the difficulties of practical application due to noise, system order, and false modes, we used the EEMD method to decrease the noise effect and effectively identify modal parameters. This method is versatile and used in a broad range of applications for signals with nonlinear components, singular points, and irregular transient parts. However, it produces mode mixing, end effects, and stopping criterion problems, which cause a loss in useful signal (Rato et al., 2008; Sweeney et al., 2013). EEMD can not only self-adaptively decompose both nonlinear and non-stationary data, but also effectively solve the mode mixing, end effects, and termination condition problems of EMD.
decrease noise, the random decrement technique (RDT) to extract free-decaying response data after de-noising, the singular entropy increment spectrum to determine system order, and multiple criteria to eliminate false modes. A finite element method was used to calculate the dynamic properties of a coupled bedrock-structure-water model. The simulated results and experimental results are compared. Finally, some conclusions are drawn.

2. Theoretical aspects

2.1. Random decrement technique

RDT is a data processing method that can be used to obtain the free vibration response of a system from one or more stationary random responses (Lee et al., 2002).

For a powerhouse under any excitation function, the forced vibration response of a measuring point \(y(t)\) can be expressed as

\[
y(t) = y(0)D(t) + y(0)V(t) + \int_0^t h(t - \tau)f(\tau)d\tau
\]

where \(t\) is time, \(D(t)\) is a free vibration response with an initial displacement of 1 and initial velocity of 0, \(V(t)\) is a free vibration response with an initial displacement of 0 and initial velocity of 1, \(y(0)\) and \(y(0)\) are the initial displacement and initial velocity of system vibration, respectively, \(h(t)\) is the unit impulse response function of a system, and \(f(t)\) is external excitation (the dynamic loads under the powerhouse operating conditions).

Selecting an appropriate constant \(A\), there are a series of intersection points with time \(t_i\) \((i = 1, 2, \ldots, N)\) between \(y = A\) and \(y(t)\). The response \(y(t-t_i)\) with a starting time of \(t_i\) can be regarded as a linear superposition of three parts: a free vibration response caused by the initial displacement, a free vibration response caused by the initial velocity, and a forced vibration response caused by random excitation \(f(t)\), i.e.,

\[
y(t-t_i) = y(t_i)D(t-t_i) + \ddot{y}(t_i)V(t-t_i) + \int_0^t h(t - \tau)f(\tau)d\tau
\]

where \(\ddot{y}(t_i)\) is the acceleration of system vibration at time \(t_i\). Because \(f(t)\) is stationary random vibration, the starting time does not affect random characteristics. Thus, the starting time \(t_i\) of \(y(t-t_i)\) can be moved to the coordinate origin, allowing us to obtain a series of sub-sample functions \(x_i(t)\) of a random process of response \(y(t-t_i)\). \(x_i(t)\) can be expressed as

\[
x_i(t) = AD(t) + \dot{y}(t_i)V(t) + \int_0^t h(t - \tau)f(\tau)d\tau
\]

The statistical average value of \(x_i(t)\) can be expressed as

\[
\overline{x_i(t)} = \frac{1}{N} \sum_{i=1}^{N} x_i(t) = AD(t) + E[y(t_i)]V(t) + \int_0^t h(t - \tau)E[f(\tau)]d\tau
\]

where \(E\) is the mathematical expectation. Because \(f(t)\) and \(y(t)\) are stationary random vibration and the mean value of each of them is 0, that is, \(E[y(t)] = 0\) and \(E[y(t)] = 0\), we have

\[
\overline{x_i(t)} = AD(t)
\]

Thus, a free vibration response with an initial displacement of \(A\) and initial velocity of 0 can be obtained.

2.2. Eigensystem realization algorithm

The ERA is a multi-input multi-output (MIMO) modal parameter identification algorithm in the time domain. The algorithm consists of two major parts: the basic formulation of the minimum-order realization and the modal parameter identification. It uses the Hankel matrix and the singular value decomposition technique to analyze the impulse response test data and free response test data, and generates an input and output linear model to match the dynamic system. The linear model is then transformed into modal space for modal parameter identification.

An \(n\)-dimensional discrete-time, linear dynamic system with \(m\) inputs and \(p\) outputs has the following state-space equations:

\[
\begin{bmatrix}
X(k+1) \\
Y(k)
\end{bmatrix} =
\begin{bmatrix}
G & B \\
C & 0
\end{bmatrix}
\begin{bmatrix}
X(k) \\
Y(k-1)
\end{bmatrix}
\]

where \(X\) is an \(n\)-dimensional state vector; \(U\) is an \(m\)-dimensional control input vector; \(Y\) is a \(p\)-dimensional output or measurement vector; \(k\) is the sample indicator, where \(k = 0, 1, 2, \ldots\); and \(G, B,\) and \(C\) are the system matrix with \(n\) rows and \(n\) columns, control matrix with \(n\) rows and \(m\) columns, and observation matrix with \(p\) rows and \(n\) columns, respectively. Based on Eq. (6), \(Y\) can be expressed as

\[
Y(k) = CG^kB
\]

In the case of initial state response, \(B = [X_0 \quad X_1 \quad \ldots \quad X_{m-1}]\), where \(X_i (i = 0, 1, \ldots, m - 1)\) represents the \(i\)th initial state vector. ERA can use multiple initial condition response data to identify the closely spaced and repeated modes.

The algorithm begins by forming a Hankel matrix \(H\):

\[
H(k) =
\begin{bmatrix}
Y(k) & Y(k+1) & \cdots & Y(k+s-1) \\
Y(k+1) & Y(k+2) & \cdots & Y(k+s) \\
\vdots & \vdots & \ddots & \vdots \\
Y(k+r-1) & Y(k+r) & \cdots & Y(k+r+s-2)
\end{bmatrix}
\]
A singular value decomposition of $H(0)$ is

$$H(0) = PDV^T$$  \hspace{1cm} (9)

where $P$ is the left singular vector matrix with $rp$ rows and $n$ columns; $V$ is the right singular vector matrix with $s$ rows and $n$ columns (the column vectors of $P$ and $V$ are orthonormal); and $D$ is a diagonal matrix, where $D = \text{diag}(d_1, d_2, \ldots, d_n)$.

The matrices $G$, $B$, and $C$ can be obtained from following formulas:

$$G = D^{-\frac{1}{2}}PH(1)VD^{-\frac{1}{2}}$$  \hspace{1cm} (10)

$$B = D^{-\frac{1}{2}}V^T E_m$$  \hspace{1cm} (11)

$$C = E_p^T PD^\dagger$$  \hspace{1cm} (12)

where $E_m$ is an $s \times m$ matrix, and $E_m^T = [I_m \ O_m \ \cdots \ O_m]$, with $I_m$ being a unit matrix with an order of $m$ and $O_m$ being a null matrix with an order of $m$; and $E_p$ is a $p \times rp$ matrix, where $E_p^T = [I_p \ O_p \ \cdots \ O_p]$, with $I_p$ being a unit matrix with an order of $p$ and $O_p$ being a null matrix with an order of $p$.

The last step is the eigenvalue decomposition of the matrix $G$ and the obtaining of system modal parameters.

### 2.3. Stochastic subspace identification

The SSI algorithm is based on the discrete-time state-space equation of a linear system, which is suitable for the process of steady excitation. It is an output-only time domain method that directly uses time data. The state model is

$$\begin{bmatrix} \dot{X}(k+1) \\ Y(k) \end{bmatrix} = \begin{bmatrix} GX(k) + w_k \\ CX(k) + v_k \end{bmatrix}$$  \hspace{1cm} (13)

where $w_k$ is the process noise vector, and $v_k$ is the measurement noise vector. They have a mean of zero and are not related. We define

$$Y_p = \frac{1}{\sqrt{J}} \begin{bmatrix} Y(0) & Y(1) & \cdots & Y(j-1) \\ Y(1) & Y(2) & \cdots & Y(j) \\ \vdots & \vdots & \ddots & \vdots \\ Y(i-1) & Y(i) & \cdots & Y(i+j-2) \end{bmatrix}$$

$$Y_t = \frac{1}{\sqrt{J}} \begin{bmatrix} Y(i) & Y(i+1) & \cdots & Y(i+j-1) \\ Y(i+2) & Y(i+3) & \cdots & Y(i+j+1) \\ \vdots & \vdots & \ddots & \vdots \\ Y(2i-1) & Y(2i) & \cdots & Y(2i+j-2) \end{bmatrix}$$

Then, the Hankel matrix can be expressed as $H = \begin{bmatrix} Y_p \\ Y_t \end{bmatrix}$.

The autocovariance matrix $R_i$ of $Y(k)$ is defined as

$$R_i = E[Y(k+i)Y^T(k)]$$  \hspace{1cm} (14)

The Toeplitz matrix $\tilde{H}_i$, which is composed by the covariance sequence, is

$$\tilde{H}_i = Y_i Y_p^T$$  \hspace{1cm} (15)

Using singular value decomposition of the Toeplitz matrix, the number of non-zero singular values is the system order, and we have

$$\tilde{H}_i = FSW^T = (F_1 \ F_2) \begin{pmatrix} S_1 & 0 \\ 0 & W_1^T \end{pmatrix} = F_1 S_1 W_1^T$$  \hspace{1cm} (16)

TheToeplitz matrix can also be decomposed as follows:

$$\tilde{H}_i = Q_i Z_i$$  \hspace{1cm} (17)

where $Q_i$ is an observability matrix, with $Q_i = [C \ CG \ \cdots \ CG^{-1}]^T$, and $Z_i$ is a reversed random controllable matrix, with $Z_i = [G^{-1} A G^{-2} A \ \cdots \ A]^T$, where $A = E[X(k+1)Y^T(k)]$.

The matrix pair $[G, C]$ is assumed to be observable, which implies that all the dynamic modes of the system can be observed in the output.

The matrices $G$ and $C$ can be obtained from Eqs. (16) and (17):

$$G = S_1^{-\frac{1}{2}} F_1^T W S_1^{-\frac{1}{2}}$$  \hspace{1cm} (18)

$$C = Q_i (1 : p)$$  \hspace{1cm} (19)

After the system matrix $G$ and the observation matrix $C$ are determined, the modal parameters can be identified through eigenvalue decomposition of the system matrix $G$:

$$\psi^{-1} A \psi = G$$  \hspace{1cm} (20)

where $A = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$, $\lambda_i$ is the $i$th discrete eigenvalue, and $\psi$ is the eigenvector of the system.

### 3. De-noising, system order determination, and false mode elimination

#### 3.1. De-noising based on EEMD

The vibration of the powerhouse experiencing ambient excitation is characterized by multiple vibration sources, broadband, and complex vibration modes. The vibration signal is a superposition of the signals excited by different excitation sources, and has instantaneous nonlinear and non-stationary characteristics, which cause significant difficulties for de-noising. Fig. 1 is the time-history curve and frequency spectrum of the displacement signal of a typical powerhouse at a measuring point. As can be seen, the different peak values in the frequency spectrum show free vibration frequencies of the structure, noise frequencies (yellow box), flow fluctuation frequencies (red box), and frequencies of excitation source vibration (green box). This study used the EEMD technique to conduct preprocessing of structural vibration response.

#### 3.1.1. Ensemble empirical mode decomposition

The core task of EMD is decomposing data into a small number of independent and nearly periodic intrinsic modes based on the local characteristic time scale of the data, and representing each intrinsic mode as an intrinsic mode function.
However, when using EMD, mode mixing, which is defined as a single IMF consisting either of signals of widely disparate scales, or of a signal of a similar scale residing in different IMF components, frequently occurs. In order to overcome the mode mixing problem, Wu and Huang (2009) proposed the EEMD method. The basic idea of EEMD is that observed data are amalgamations of the true time series and noise. Thus, even if data are collected through separate observations with different noise levels, the ensemble mean is close to the true time series.

In EEMD, low-level random noise is added to the input signal, and then the signal is decomposed through EMD. The process of adding noise to the signal and decomposing the signal through EMD is one trial. This procedure is repeated \( Q \) times to obtain the final IMF. The algorithm of EEMD is, therefore, organized into the following steps:

1. The ensemble size \( Q \) is initialized, and the number of trials \( q \) is set to 1.
2. The \( q \)th trial for the signal with the added white noise is implemented, a step that involves the following tasks: (a) A white noise series \( n_q(t) \) with amplitude \( A_q \) is added to the input signal \( x(t) \) to generate a modified signal:
   
   \[
   x_q(t) = x(t) + A_qn_q(t)
   \]  
   
   where \( x_q(t) \) is the noise-added signal of the \( q \)th trial. (b) The signal \( x_q(t) \) is decomposed into \( M \) IMFs and a remainder with the EMD:
   
   \[
   x_q(t) = \sum_{i=1}^{M} c_{iq}(t) + r_{Nq}(t)
   \]  
   
   where \( c_{iq}(t) \) is the \( i \)th IMF of the \( q \)th trial and \( r_{Nq}(t) \) is the remainder after removing \( M \) IMFs for the \( q \)th trial. (c) If \( q < Q \), then \( q = q + 1 \) and steps (1) and (2) are repeated until \( q = Q \, \) with different white noise series each time, and with \( Q \) usually being equal to or greater than 100.
3. The ensemble mean of the \( Q \) trials for the \( i \)th IMF, \( C_i(t) \), is calculated:
   
   \[
   C_i(t) = \lim_{Q \to \infty} \frac{1}{Q} \sum_{q=1}^{Q} c_{iq}(t)
   \]  
   
   (23)
4. \( C_i(t) \) \((i = 1, 2, \cdots, M)\) is considered the final \( i \)th IMF.

Therefore, the goal of de-noising through EEMD is decomposition of the signal into a series of IMF components whose characteristic time scales vary from small to large (or whose frequencies vary from high to low). For a signal mixed with random noise, the high-frequency IMF components are usually the noise. Then, these IMF components are removed, the signal is reconstructed with the rest of the IMF components, and the noise can be reduced.

3.1.2. Application of EEMD to de-noising

A simulated signal \( C \) was composed of a sinusoidal component with a frequency of 5 Hz and normally distributed white noise with a mean of 0 and a standard deviation of 0.1, as shown in Fig. 2.

The decomposition results of the signal with the EEMD are shown in Figs. 3 and 4. The signal was decomposed into seven IMF components and a remainder. According to the time-history curves and frequency spectrum distribution, IMF5, IMF6, and IMF7, with dominant frequencies of 5 Hz, were selected to reconstruct the signal. The reconstructed sinusoidal signal and noise signal from EEMD decomposition results are shown in Fig. 5. Fig. 6 shows the reconstructed sinusoidal signal and noise signal from EMD decomposition results, with obvious disturbance components (circle regions). This was due to the mode mixing of EMD.

Fig. 1. Time-history curve and frequency spectrum of displacement signal of typical powerhouse at measuring point.

Fig. 2. Timer-history curve and frequency spectrum of simulated signal \( C \).

Fig. 3. Time-history curve and frequency spectrum of the \( i \)th IMF.

Fig. 4. Frequency spectrum of the \( i \)th IMF.

Fig. 5. Time-history curve and frequency spectrum of simulated signal \( C \).
The signal-to-noise ratio (SNR) of the original signal was 16.831. After de-noising through the EMD and EEMD methods, the SNRs of the signal were 21.463 and 25.012, respectively. The EEMD method obtains a higher SNR. This proves that the EEMD method is feasible and more effective.

3.2. System order determination based on singular entropy

The system order is one of the most important parameters for modal identification in the time domain. For the ERA, it is necessary to construct a Hankel matrix and to determine the orders of the Hankel matrix and system matrix. Because the system is unknown when it is experiencing ambient excitation, and its order is unknown in advance, a tentative system order must be provided before modal parameter identification. Here we introduce the concept of entropy to overcome the difficulty of system order determination.

3.2.1. Singular entropy

Considering the structural dynamic response signal $x$, the original signal $x(t) = [x(t) \ x(t+\tau) \ x(t+2\tau) \cdots]$ is mapped into the phase space with a size of $m \times n$ by means of a time-lapse technique ($\tau$ is the time lapse), and the reconstructed attractor orbit matrix $L$ can be formed like a Hankel matrix as follows:

$$L = \begin{bmatrix}
    x(t) & x(t+\tau) & \cdots & x(t+(n-1)\tau) \\
    x(t+\tau) & x(t+2\tau) & \cdots & x(t+n\tau) \\
    \vdots & \vdots & \ddots & \vdots \\
    x(t+(m-1)\tau) & x(t+m\tau) & \cdots & x(t+(m+n-2)\tau)
\end{bmatrix}$$

(24)

Through singular value deposition of the matrix $L$, the singular spectrum $\sigma_i$ can be obtained based on the main diagonal element $f_i(i = 1, 2, \cdots, l)$ of the diagonal matrix and can be described as follows (Yang and Peter, 2003):

$$\sigma_i = \lg \frac{f_i}{\sum_{j=1} f_j}$$

(25)

The singular spectrum indicates energy proportions of the various state variables throughout the system. In order to investigate the changes of signal information with the singular spectrum order, the concept of the singular entropy increment is introduced. Its formula is
where $\Delta E_i$ is a increment of singular entropy on the order of $i$.

$$\Delta E_i = \frac{f_i}{i} \log \frac{f_i}{\sum_{i=1}^{i-1} f_i}$$

(26)

In view of this, we can use the singular entropy increment spectrum to determine system order. When the singular entropy increment tends toward stabilization, the corresponding order can be considered an approximated system order, or, based on the required accuracy for projects, when $\Delta E_i \leq \xi$, the smallest integer $i$ can be considered the system order, where $\xi$...
is a parameter. After the eigenvalue elimination of the non-mode items (non-conjugate roots) and the conjugate items (repetitive modes) of the system, the order of the system is \( i/2 \) (Lian et al., 2009).

3.2.2. Application of singular entropy to system order determination

A comparison study was made with two simulated signals: signal A was an impulse signal composed of three impulse functions, and signal B was a mixing signal composed of the impulse signal and normally distributed white noise with a mean of 0 and a standard deviation of 1. The time period was 0–6.28 s, and the time interval was 0.01 s. The time-history curves of signals A and B are shown in Fig. 7.

Fig. 8 shows the calculated singular entropy increment of the two simulated signals. It is obvious that, for an impulse response signal not affected by noise, a greater singular entropy increment value is correlated with a lower order. In addition, in the transition region between lower and higher orders, a smaller SNR value of the impulse response signal is correlated with a smaller decrease in the singular entropy. Thus, for the same signal, as the singular entropy increment begins to decrease toward the asymptotic value, the available characteristic information of the signal tends to become saturated (and approximately full). No matter how serious the interference in the signal, the order corresponding to the singular spectrum that extracts completely effective characteristic information is definite. The singular entropy increment corresponding to orders greater than the determined order is caused by noise with a wide frequency band and should be ignored. Thus, the order of this simulated signal A can be determined to be 7, and after the eigenvalue decomposition of matrix \( G \) and eigenvalue elimination of the non-mode items (non-conjugate roots) and the conjugate items (repetitive items), the effective order of the impulse response function is 3.

3.3. False mode elimination

The tentative order should include as much structure vibration information as possible in order to avoid losing modes, leading to the occurrence of false modes. Therefore, three criteria are proposed to eliminate false modes:

![Fig. 6. Reconstructed sinusoidal signal and noise signal from EMD decomposition results.](image)

![Fig. 7. Time-history curves of two signals.](image)

![Fig. 8. Relationship between order and singular entropy increment of simulated signals.](image)
The obvious non-structural natural frequency from identification results (such as the unit vibration frequency and flow fluctuation-induced vibration frequency) can be removed. The identified modes with a structural damping ratio beyond the range of $0.01 - 0.1$ are false and should be eliminated. After determination of the system order using the singular entropy increment spectrum, a modal parameter stabilization diagram is obtained by increasing the number of row spatial data of the Hankel matrix from $i_{\text{min}}$ to $i_{\text{max}}$ ($i_{\text{max}}$ is a relatively larger value that satisfies the size requirement of $j/i_{\text{max}}$). When the differences in the frequency and damping ratio between two adjacent points in the modal parameter stabilization diagram are within the limit of tolerance, the modes corresponding to both points are temporarily considered to be real modes. After investigation of all points, the stable and most frequent modes are considered the real modes.

4. Powerhouse layout and finite element model

4.1. Powerhouse layout

The powerhouse layout of a roof overflow hydropower station with bulb tubular units uses a combination of a spillway and powerhouse. The spillway overlaps all or part of the powerhouse in the plane projection. Both share the water-retaining wall (Xie and Huang, 1981). The Bingling Hydropower Project consists of a powerhouse in a river channel, roof overflow surface outlets, discharge bottom orifices, sand-flash outlets, and left and right assisting dams. The dam crest elevation is 1751.0 m. There are five bulb tubular units in the main channel. Each unit's capacity is 48 MW, with a rated speed of 107.1 r/min, rated flow of 335 m$^3$/s, and rated water head of 16.1 m. There is a sand-flash outlet on the left side of the 1st, 2nd, and 4th units. Five surface outlets with a width the same as the channel width are arranged on the units. Under the conditions of unit operation, surface outlet discharge, and surface outlet discharge in conjunction with unit operation, prototype observation was made for the 4th powerhouse dam section and unit, respectively. The 4th powerhouse dam section and unit are shown in Fig. 9.

4.2. Establishment of finite element model

In this study, the commercial finite element software (ANSYS) (Ji et al., 2006) was used to simulate the structure and calculate the main modal parameters. Based on the layout of the powerhouse, a three-dimensional model was established, and the flow passage of the turbine, a surface outlet floor, sand-flash outlets, and channels with a size larger than 1.0 m $\times$ 1.0 m $\times$ 1.0 m were simulated. The frame structure on the powerhouse was not included in the model. Accessory equipment such as cranes, treated as added mass, was exerted on corresponding nodes. The boundary conditions were set, including multiple constraints on the bottom of foundation, normal chain constraints at the four sides, and a free boundary all around the concrete structure. The meshes of the powerhouse model and the full dam model are shown in Fig. 10. The natural frequencies of the 1st through 10th modes of the whole powerhouse were 3.239, 5.069, 5.884, 8.472, 9.184, 9.467, 9.532, 9.919, 10.627, and 10.987 Hz, respectively. Fig. 11 shows the first three mode shapes of the powerhouse.

5. Modal identification

Various dynamic loads were used as the sources of environmental excitations. The response signals of the powerhouse...
were collected from a DP low-frequency vibration transducer with a sampling rate of 400 Hz. ERA was used to obtain the modal information from response data of the measuring point in section B with an elevation of 1732.0 m (⑤ in Fig. 9). SSI was used to obtain the dynamic displacement response from nine measuring points in section B, including a measuring point with an elevation of 1732.0 m, two measuring points on the ground of the left and right guide walls, and six measuring points 1.0 m, 2.5 m, and 5.0 m away from the ground of the left and right guide walls. The data collection was completed under discharge in conjunction with unit operation conditions.

5.1. De-noising and order determination

Taking a measurement point in section B as an example, response data (Fig. 12) were decomposed using the EEMD method. This process obtained seven IMF components and a remainder, as shown in Figs. 13 and 14. The time-domain waves of IMF1 and IMF2 show the characteristics of white noise, and the remainder is the harmonic component. IMF1, IMF2, and the remainder should be removed. The de-noised signal can be obtained by reconstructing the rest of IMF components. A comparison between the original signal and the de-noised signal is shown in Fig. 15.

The change of the singular entropy increment after de-noising is shown in Fig. 16. The singular entropy increment decreases slowly and tends to stabilize when the order is larger than 8. This means that the available characteristic information of the signal tends to become saturated (and approximately full). The singular entropy increment corresponding to orders larger than 8 can be neglected and considered the results of broadband noise. According to the complex mode theory, eliminating the non-mode items (non-conjugate roots) and the conjugate items (repetitive items) of the system, the modal order of the structure is 4.

5.2. Identification results of ERA

The damped-free vibration signals of the original and de-noised systems were extracted from the output signal through RDT (Fig. 17). After identification of modal parameters with ERA, according to the determined system order and false mode elimination principles, the frequencies of the unit vibration and flow fluctuation were clearly seen to be non-structural natural frequencies, including the frequency of flow fluctuation induced by discharge (≤2 Hz), the low-frequency vortex core (band) vibration frequency (0.297—0.595 Hz), the frequency of vibration due to incorrect connection relationships and its double frequency (8.925 Hz and 17.850 Hz), and the Karman vortex-induced vibration frequency (80—100 Hz) (Cao et al., 2007). The modes corresponding to these frequencies and the modes with damping ratios beyond the range of 0.01—0.1 were eliminated. The modal identification results are shown in Table 1.

It can be observed from Table 1 that, when ERA is used for the original signal, there are fairly large differences between simulated frequency values and identified results for the first three modes, and the damping ratios are also significantly larger. After the fifth mode, all of the frequencies are noise frequency, and the structural modal information is covered. Although the frequency value of the fourth mode is close to the simulated one, the validity cannot be determined due to the unknown order of the excitation system. In contrast, the de-noised identified results for the first four modes are very close to the simulated values, the
Fig. 13. Time-history curves of all components with EEMD method.

Fig. 14. Frequency spectra of all components with EEMD method.
relative errors of frequencies are within 10%, and the identified damping ratio is within the range from 0.01 to 0.1. Even though the order number of operational modes of the powerhouse is determined to be 4, the relative error between the identification results of high-order (5th to 10th) natural frequencies obtained from ERA and the simulated values are less than 15%, and the identified damping ratios are within the range from 0.01 to 0.1. Generally, the inherent dynamic characteristics of a hydraulic structure are mainly reflected in the first few modes. Therefore, these results meet engineering precision requirements.

5.3. Identification results of SSI

The first task was analysis of the original and de-noised displacement response data from nine measurement points through the SSI method. Then, as in the ERA identification process described above, the frequencies of false modes were eliminated. The frequency stabilization diagram is shown in Fig. 18. After averaging all the frequencies and damping ratios of real modes, stable identification results were obtained. The identified results are shown in Table 2.

### Table 1

<table>
<thead>
<tr>
<th>Order</th>
<th>Simulated result</th>
<th>Identified result</th>
<th>De-noised signal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (Hz)</td>
<td>Frequency (Hz)</td>
<td>Damping ratio (%)</td>
</tr>
<tr>
<td>1</td>
<td>3.239</td>
<td>4.088</td>
<td>22.451</td>
</tr>
<tr>
<td>3</td>
<td>5.884</td>
<td>7.527</td>
<td>27.723</td>
</tr>
<tr>
<td>4</td>
<td>8.472</td>
<td>8.821</td>
<td>5.783</td>
</tr>
<tr>
<td>5</td>
<td>9.184</td>
<td>60.917</td>
<td>9.642</td>
</tr>
</tbody>
</table>

Fig. 15. Comparison between original signal and de-noised signal.

Fig. 16. Change of singular entropy increment with order.

Fig. 17. Damped-free vibration signals extracted through RDT.

Fig. 18. Frequency stabilization diagrams of original and de-noised signals.
It can be seen from Table 2 that, for the original signal, SSI can identify the first-order modal parameters, the frequency and damping ratio for the second and third modes are too large because of noise, and the modal information is completely covered by noise after the fourth mode. Meanwhile, the de-noised identification results for the first mode are very close to the simulated results. However, the relative errors of frequency for the rest of the modes are over 10%. Compared with ERA, the precision of identification results of SSI is poor, probably due to incomplete arrangement of measurement points or because SSI is unsuitable for identifying modal parameters of a roof overflow powerhouse.

### 6. Conclusions

A modal parameter identification technique was developed for determination of modal parameters of the powerhouse of a roof overflow hydropower station with bulb tubular units under ambient excitation. The EEMD method was used to decrease noise, RDT was used to extract free-decaying responses, the singular entropy was used to determine the modal order, multiple criteria were used to eliminate false modes, and the ERA and SSI were used to identify modal parameters. The following conclusions can be obtained:

1. The EEMD, singular entropy, and multiple criteria, combined with ERA, can precisely identify the low-order modal parameters of the structure under study. For high-order modal parameters, the identification precision decreases, but is still within the acceptable error limits.

2. Although the SSI method is convenient, compared with ERA, the precision of identification results is poor for this structure, probably due to incomplete arrangement of measurement points or because the SSI method is unsuitable for identifying modal parameters of a roof overflow powerhouse. Specific conclusions require further prototype observation for authentication.

3. The EEMD possesses attractive properties, such as a strong noise reduction ability, recovery of the entire useful vibration signal, and a high calculation efficiency. The singular entropy can determine the order of operational modes of structures when the order of the vibrated structure is unknown and the structure is experiencing unknown input excitation, and multiple criteria can help eliminate the false modes effectively.

4. The relative errors of frequencies in the first four modes were within 10% for the ERA method, and the method has demonstrated its robustness and reliability when applied to identification of the modal parameters and real-time monitoring of hydraulic structures with strong noise with complex ambient excitation.

### References


