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Higher twist corrections to the sum rule for semileptonic B decay

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Abstract

The sum rule for charmless inclusive semileptonic B-meson decays allows a theoretically clean and experimentally efficient determination of $|V_{ub}|$. The leading twist contribution to the sum rule is known in QCD. We compute higher twist corrections to the sum rule using the heavy-quark effective theory.

A new method to determine the Cabibbo–Kobayashi–Maskawa (CKM) matrix element $|V_{ub}|$ has been proposed [1] that takes advantage of the sum rule for charmless inclusive semileptonic B-meson decays $\overline{B} \to X_u \ell \bar{\nu}_\ell$ ($\ell = e$ or μ). The sum rule establishes a clean relationship between $|V_{ub}|$ and the observable

$$S \equiv \int_0^1 d\xi_u \, \frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u}(\overline{B} \to X_u \ell \bar{\nu}_\ell) \qquad (1)$$

with the kinematic variable $\xi_u = (q^0 + |\mathbf{q}|)/M_B$ in the B-meson rest frame, where q is the momentum transfer to the lepton pair and M_B denotes the B meson mass. Moreover, this method of determining $|V_{ub}|$ has experimental virtue too. The kinematic variable ξ_u is the most efficient discriminator between $\overline{B} \to X_u \ell \bar{\nu}_\ell$ signal and $\overline{B} \to X_c \ell \bar{\nu}_\ell$ background. A majority of $\overline{B} \to X_u \ell \bar{\nu}_\ell$ events have a value of ξ_u beyond the limit allowed for $\overline{B} \to X_c \ell \bar{\nu}_\ell$ decays with charm in the final state, $\xi_u > 1 - M_D/M_B = 0.65$ with M_D being the D-meson mass. Therefore, only a small extrapolation is needed to obtain S.

The charmless inclusive semileptonic decay of the B meson is a light-cone dominated process. The light-cone expansion allows a rigorous and systematic ordering of nonperturbative QCD effects, providing an effective technique for a separation and classification of higher twist (HT) effects [2]. The leading term in this expansion gives the leading twist contribution. HT contributions are contained in the light-cone expansion beyond the leading order. The sum rule at the leading twist order measures the bottomness carried by a B meson. There are no perturbative QCD corrections to the sum rule. Thus the primary hadronic uncertainty and the potential uncertainty of perturbative QCD are eliminated, dramatically reducing the theoretical error on $|V_{ub}|$. This inclusive method is to be contrasted with the determination of $|V_{ub}|$ from the charmless inclusive semileptonic branching fraction of B mesons where the calculation of the total semileptonic decay rate is model dependent or assumes quark–hadron duality, there are uncertainties due to perturbative QCD corrections and, in addition, a larger extrapolation is necessary to extract the total rate if the kinematic cut on a certain observable, such as the charged-lepton energy or the invariant mass of the lepton pair, is applied for the suppression of $b \to c$ background.

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Only uncertainties due to HT effects remain in the sum rule. Including the HT contribution ΔHT, the sum rule reads

$$S \equiv \int_0^1 d\xi_u \, \frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u} (\overline{B} \to X_u \ell \bar{\nu}_\ell) = |V_{ub}|^2 \frac{G_F^2 M_B^5}{192\pi^3} (1 + \Delta HT). \tag{2}$$

Although HT contributions are expected to be suppressed by powers of $\Lambda_{\text{QCD}}^2/M_B^2$ (Λ_{QCD} being the QCD scale), a quantitative estimate of them is indispensable for a complete understanding of remaining theoretical uncertainties in this determination of $|V_{ub}|$. In this Letter, we investigate HT effects on the sum rule for charmless inclusive semileptonic B decays using the heavy-quark effective theory (HQET) [3–5].

Charmless inclusive semileptonic decays of the B meson are induced by the weak interactions. The differential decay rate to lowest order in the weak interactions is

$$d\Gamma = \frac{G_F^2 |V_{ub}|^2}{(2\pi)^5 E} L^{\mu\nu} W_{\mu\nu} \frac{d^3 k_\ell}{2E_\ell} \frac{d^3 k_\nu}{2E_\nu}. \tag{3}$$

Here E (P), E_ℓ (k_ℓ), and E_ν (k_ν) denote the energies (four-momentums) of the B meson, the charged lepton, and the antineutrino, respectively. The leptonic tensor for the lepton pair is completely determined by the standard electroweak theory since leptons do not have strong interactions:

$$L^{\mu\nu} = 2\big(k_\ell^\mu k_\nu^\nu + k_\nu^\mu k_\ell^\nu - g^{\mu\nu} k_\ell \cdot k_\nu + i\varepsilon^{\mu\nu}{}_{\alpha\beta} k_\ell^\alpha k_\nu^\beta\big). \tag{4}$$

The hadronic tensor incorporates all nonperturbative QCD physics for the inclusive semileptonic B decay. It is summed over all hadronic final states and can be expressed in terms of a current commutator taken between the B meson states:

$$W_{\mu\nu} = -\frac{1}{2\pi} \int d^4 y \, e^{iq\cdot y} \langle B| \big[j_\mu(y), j_\nu^\dagger(0) \big] |B\rangle, \tag{5}$$

where $j_\mu(y) = \bar{u}(y)\gamma_\mu(1-\gamma_5)b(y)$ is the charged weak current for the $b \to u$ transition. We adopt a covariant normalization for one-particle states, i.e., $\langle B(P)|B(P')\rangle = (2\pi)^3 2P^0 \delta^{(3)}(\mathbf{P} - \mathbf{P}')$.

The most general hadronic tensor form that can be constructed is a linear combination of $P_\mu P_\nu$, $P_\mu q_\nu$, $q_\mu P_\nu$, $q_\mu q_\nu$, $\varepsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta$ and $g_{\mu\nu}$, with coefficients being scalar functions $W_a(\nu, q^2)$ of the two independent Lorentz invariants, $\nu \equiv q \cdot P/M_B$ and q^2. However, the combination $P_\mu q_\nu - q_\mu P_\nu$ does not contribute since $L^{\mu\nu}(P_\mu q_\nu - q_\mu P_\nu) = 0$. Thus the hadronic tensor must take the form

$$W_{\mu\nu} = -g_{\mu\nu} W_1 + \frac{P_\mu P_\nu}{M_B^2} W_2 - i\varepsilon_{\mu\nu\alpha\beta} \frac{P^\alpha q^\beta}{M_B^2} W_3 + \frac{q_\mu q_\nu}{M_B^2} W_4 + \frac{P_\mu q_\nu + q_\mu P_\nu}{M_B^2} W_5. \tag{6}$$

Eq. (5) shows that $W_{\mu\nu}^* = W_{\nu\mu}$, so W_a, $a = 1, \ldots, 5$ are real. The interesting physics describing the hadron structure and the strong interactions is wrapped up in the five dimensionless real structure functions $W_a(\nu, q^2)$, $a = 1, \ldots, 5$ for the unpolarized processes.

In the following we will neglect the masses of the charged lepton and the u-quark. From Eqs. (3) and (6), we obtain the double differential decay rate for $\overline{B} \to X_u \ell \bar{\nu}_\ell$ in the rest frame of the B meson

$$\frac{d^2\Gamma}{d\xi_u \, dq^2} = \frac{G_F^2 |V_{ub}|^2}{48\pi^3 M_B} \frac{|\mathbf{q}|^2}{\xi_u} \big(W_1 3q^2 + W_2 |\mathbf{q}|^2\big), \tag{7}$$

where

$$|\mathbf{q}| = \frac{1}{2} M_B \xi_u \left(1 - \frac{q^2}{M_B^2 \xi_u^2}\right). \tag{8}$$

By integrating Eq. (7) over q^2, one gets the decay distribution of the kinematic variable ξ_u

$$\frac{d\Gamma}{d\xi_u} = \int_0^{M_B^2 \xi_u^2} dq^2 \frac{d^2\Gamma}{d\xi_u \, dq^2}. \tag{9}$$

Computing the current commutator one obtains from Eq. (5)

$$W_{\mu\nu} = -\frac{1}{\pi} (S_{\mu\alpha\nu\beta} - i\varepsilon_{\mu\alpha\nu\beta}) \times \int d^4 y \, e^{iq\cdot y} \big[\partial^\alpha \Delta_u(y)\big] \times \langle B| \bar{b}(0) \gamma^\beta U(0, y) b(y) |B\rangle, \tag{10}$$

where $S_{\mu\alpha\nu\beta} = g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} - g_{\mu\nu} g_{\alpha\beta}$. In the above we have used

$$\big\{u(x), \bar{u}(y)\big\} = i(\gamma \cdot \partial) i\Delta_u(x - y) U(x, y) \tag{11}$$

with the Wilson link

$$U(x,y)=\mathcal{P}\exp\left[ig_s\int_y^x dz^\mu\,A_\mu(z)\right], \tag{12}$$

$$\Delta_u(y)=-\frac{i}{(2\pi)^3}\int d^4k\,e^{-ik\cdot y}\varepsilon\big(k^0\big)\delta\big(k^2\big), \tag{13}$$

where A^μ is the background gluon field and $\varepsilon(x)$ satisfies $\varepsilon(|x|)=1$ and $\varepsilon(-|x|)=-1$.

The matrix element $\langle B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B\rangle$ is the basic building block of the description of inclusive B decays in QCD. In general one can decompose it in the following form:

$$\begin{aligned}&\langle B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B\rangle\\&\quad=2\Big[P^\beta F\big(y^2,y\cdot P\big)+y^\beta G\big(y^2,y\cdot P\big)\Big],\end{aligned} \tag{14}$$

where $F(y^2,y\cdot P)$ and $G(y^2,y\cdot P)$ are functions of the two independent Lorentz scalars, y^2 and $y\cdot P$. The dominant part of the integrand in the hadronic tensor (10) stems from the space–time region near the light cone, with the deviation from the light cone being of the order of the inverse large momentum $y^2\sim 1/q^2\sim 1/M_B^2\to 0$ [2]. The light-cone expansion of the functions $F(y^2,y\cdot P)$ and $G(y^2,y\cdot P)$ in powers of y^2 leads to

$$\begin{aligned}&\langle B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B\rangle\\&\quad=2\left[P^\beta\sum_{n=0}^\infty\big(y^2\big)^n\mathcal{F}^{(2n+2)}(y\cdot P)\right.\\&\qquad\left.+\,y^\beta\sum_{n=0}^\infty\big(y^2\big)^n\mathcal{G}^{(2n+4)}(y\cdot P)\right]\\&\quad=2\Big\{P^\beta\big[\mathcal{F}^{(2)}(y\cdot P)+y^2\mathcal{F}^{(4)}(y\cdot P)+\cdots\big]\\&\qquad+\,y^\beta\big[\mathcal{G}^{(4)}(y\cdot P)+y^2\mathcal{G}^{(6)}(y\cdot P)+\cdots\big]\Big\}.\end{aligned} \tag{15}$$

The coefficients $\mathcal{F}^{(2n+2)}(y\cdot P)$ and $\mathcal{G}^{(2n+4)}(y\cdot P)$ in the light-cone expansion can be classified by twist. Following the notion of twist introduced by Jaffe and Ji [6], $\mathcal{F}^{(2n+2)}(y\cdot P)$ has twist $2n+2$ and $\mathcal{G}^{(2n+4)}(y\cdot P)$ has twist $2n+4$, as from dimension analysis we know that the contribution of the former is suppressed by $(\Lambda_{\rm QCD}/M_B)^{2n}$, and the contribution of the latter is suppressed by $(\Lambda_{\rm QCD}/M_B)^{2n+2}$. We will further discuss the nonlocal light-cone expansion of matrix elements below.

The twist decomposition for the decay rate thus takes the form

$$d\Gamma=\sum_{n=0}^\infty d\Gamma^{(2n+2)}, \tag{16}$$

where

$$d\Gamma^{(2n+2)}=\frac{G_F^2|V_{ub}|^2}{(2\pi)^5E}L^{\mu\nu}W_{\mu\nu}^{(2n+2)}\frac{d^3k_\ell}{2E_\ell}\frac{d^3k_\nu}{2E_\nu} \tag{17}$$

is the twist-$(2n+2)$ contribution to the decay rate with

$$\begin{aligned}&W_{\mu\nu}^{(2n+2)}\\&\quad=-g_{\mu\nu}W_1^{(2n+2)}+\frac{P_\mu P_\nu}{M_B^2}W_2^{(2n+2)}\\&\qquad-i\varepsilon_{\mu\nu\alpha\beta}\frac{P^\alpha q^\beta}{M_B^2}W_3^{(2n+2)}+\frac{q_\mu q_\nu}{M_B^2}W_4^{(2n+2)}\\&\qquad+\frac{P_\mu q_\nu+q_\mu P_\nu}{M_B^2}W_5^{(2n+2)}\end{aligned} \tag{18}$$

$$\begin{aligned}&\quad=-\frac{2}{\pi}(S_{\mu\alpha\nu\beta}-i\varepsilon_{\mu\alpha\nu\beta})\int d^4y\,e^{iq\cdot y}\big[\partial^\alpha\Delta_u(y)\big]\\&\qquad\times\Big[P^\beta\big(y^2\big)^n\mathcal{F}^{(2n+2)}(y\cdot P)\\&\qquad\quad+y^\beta\big(y^2\big)^{n-1}\mathcal{G}^{(2n+2)}(y\cdot P)\Big].\end{aligned} \tag{19}$$

The leading twist contribution to the sum rule (2) results from $\mathcal{F}^{(2)}(y\cdot P)$ of twist 2 and is known in QCD to be [1]

$$\int_0^1 d\xi_u\,\frac{1}{\xi_u^5}\frac{d\Gamma^{(2)}}{d\xi_u}(\overline{B}\to X_u\ell\bar{\nu}_\ell)=|V_{ub}|^2\frac{G_F^2M_B^5}{192\pi^3}, \tag{20}$$

which is a consequence of the conservation of the b-quark vector current by the strong interactions. The next-to-leading twist contribution to the sum rule arises from $\mathcal{F}^{(4)}(y\cdot P)$ and $\mathcal{G}^{(4)}(y\cdot P)$ of twist 4. It can be obtained by integrating Eq. (9) over ξ_u with the two relevant structure functions $W_1^{(4)}(\nu,q^2)$ and $W_2^{(4)}(\nu,q^2)$ of twist 4.

We use the operator product expansion and the heavy quark effective theory to compute the twist-4 structure functions. The Wilson link is a gauge dependent operator. It is convenient to use the Fock–Schwinger gauge such that $U(0,y)$ is unity. Since the b quark inside the B meson behaves as almost free due to its large mass, relative to which its binding to the light constituents is weak, one can extract the large space–time dependence

$$b(y)=e^{-im_b v\cdot y}b_v(y), \tag{21}$$

where m_b is the b-quark mass and $v=P/M_B$ is the four-velocity of the B meson. This factorization makes clear why the large scale in matrix elements does not affect the relative size of terms in the light-cone expansion (15). The large scale hidden in matrix elements of b-quark operators is contained in an overall factor $e^{-im_b v\cdot y}$, so reduced matrix elements of the operators containing the rescaled operator b_v involve only momenta of order $\Lambda_{\rm QCD}$, which determine the relative size of terms in the light-cone expansion (15), i.e., schematically

$$\begin{aligned}&\langle B|\bar b(0)\gamma^\beta b(y)|B\rangle\\&\quad=e^{-im_b v\cdot y}\langle B|\bar b_v(0)\gamma^\beta b_v(y)|B\rangle\\&\quad\sim e^{-im_b v\cdot y}\sum_{n=0}^{\infty}\left(\frac{\Lambda_{\rm QCD}^2}{M_B^2}\right)^n.\end{aligned} \tag{22}$$

The rescaled operator for a free b-quark no longer depends on the space–time, so $b(y)=e^{-im_b v\cdot y}b(0)$. In this case all the coefficients $\mathcal{F}^{(2n+2)}(y\cdot P)$ and $\mathcal{G}^{(2n+4)}(y\cdot P)$ in the light-cone expansion (15) vanish except that $\mathcal{F}^{(2)}(y\cdot P)=e^{-im_b v\cdot y}$, because the conservation of the b-quark vector current implies that $\langle B|\bar b(0)\gamma^\beta b(0)|B\rangle=2P^\beta$. The leading-twist sum rule (20) is consistently reproduced in the free quark decay $b\to u\ell\bar\nu_\ell$. The conserved vector current $\bar b\gamma^\beta b$ is not renormalized by the strong interactions. This explains why there are no perturbative QCD corrections to the sum rule.

A Taylor expansion of the field in a gauge-covariant form relates the bilocal and local operators. This leads to an operator product expansion

$$\begin{aligned}&\bar b(0)\gamma^\beta b(y)\\&\quad=e^{-im_b v\cdot y}\bar b_v(0)\gamma^\beta b_v(y)\\&\quad=e^{-im_b v\cdot y}\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}y_{\mu_1}\cdots y_{\mu_n}\\&\qquad\times\bar b_v(0)\gamma^\beta k^{\{\mu_1}\cdots k^{\mu_n\}}b_v(0),\end{aligned} \tag{23}$$

where $k_\mu=iD_\mu=i(\partial_\mu-ig_sA_\mu)$ and the symbol $\{\cdots\}$ means symmetrization with respect to the enclosed indices. Because of the weak dependence of the rescaled operator $b_v(y)$ on y, we attempt to estimate the matrix element of the bilocal operator sandwiched between the B meson states with the truncated y-expansion in Eq. (23). To obtain a twist-4 accuracy it suffices to keep only the first three terms

$$\begin{aligned}&\langle B|\bar b(0)\gamma^\beta b(y)|B\rangle\\&\quad=e^{-im_b v\cdot y}\Big[\langle B|\bar b_v(0)\gamma^\beta b_v(0)|B\rangle\\&\qquad+(-i)y_\mu\langle B|\bar b_v(0)\gamma^\beta iD^\mu b_v(0)|B\rangle\\&\qquad+\frac{(-i)^2}{2}y_\mu y_\nu\langle B|\bar b_v(0)\gamma^\beta\\&\qquad\times iD^{\{\mu}iD^{\nu\}}b_v(0)|B\rangle\Big].\end{aligned} \tag{24}$$

In the heavy quark effective theory the QCD b-quark field $b(y)$ is related to its HQET counterpart $h(y)$ by means of an expansion in powers of $1/m_b$:

$$b(y)=e^{-im_b v\cdot y}\left[1+\frac{i\not{D}}{2m_b}+\mathrm{O}\left(\frac{\Lambda_{\rm QCD}^2}{m_b^2}\right)\right]h(y). \tag{25}$$

The effective Lagrangian takes the form

$$\begin{aligned}\mathcal{L}_{\rm HQET}&=\bar h iv\cdot Dh+\bar h\frac{(iD)^2}{2m_b}h+\bar h\frac{g_sG_{\mu\nu}\sigma^{\mu\nu}}{4m_b}h\\&\quad+\mathrm{O}\left(\frac{1}{m_b^2}\right),\end{aligned} \tag{26}$$

where $g_sG^{\mu\nu}=i[D^\mu,D^\nu]$ is the gluon field-strength tensor. At the level of accuracy of the present discussion we take into account only the leading, $1/m_b$ correction to the heavy quark limit $m_b\to\infty$. Relating the matrix elements of the local operators in full QCD in Eq. (24) to those in HQET, it follows that

$$\begin{aligned}&\langle B|\bar b(0)\gamma^\beta b(y)|B\rangle\\&\quad=2e^{-im_b v\cdot y}\bigg\{P^\beta\bigg[1-y\cdot Pi\frac{5}{3}\frac{m_b}{M_B}E_b\\&\qquad-(y\cdot P)^2\frac{1}{3}\frac{m_b^2}{M_B^2}K_b+y^2\frac{1}{3}m_b^2K_b\bigg]\\&\qquad+y^\beta i\frac{2}{3}m_bM_BE_b\bigg\},\end{aligned} \tag{27}$$

where $E_b=K_b+G_b$ and K_b and G_b are the dimensionless HQET parameters of order $(\Lambda_{\rm QCD}/m_b)^2$, which are often referred to by the alternate names

$\lambda_1 = -2m_b^2 K_b$ and $\lambda_2 = -2m_b^2 G_b/3$, defined as

$$\lambda_1 = \frac{1}{2M_B}\langle B|\bar{h}(iD)^2 h|B\rangle, \tag{28}$$

$$\lambda_2 = \frac{1}{12M_B}\langle B|\bar{h}g_s G_{\mu\nu}\sigma^{\mu\nu}h|B\rangle. \tag{29}$$

Comparing Eq. (27) with Eq. (15) yields

$$\mathcal{F}^{(4)}(y\cdot P) = \frac{1}{3}m_b^2 K_b e^{-im_b v\cdot y}, \tag{30}$$

$$\mathcal{G}^{(4)}(y\cdot P) = i\frac{2}{3}m_b M_B E_b e^{-im_b v\cdot y}. \tag{31}$$

We see that the coefficients $\mathcal{F}^{(4)}(y\cdot P)$ and $\mathcal{G}^{(4)}(y\cdot P)$ of the light-cone expansion (15) are indeed of order Λ_{QCD}^2 as expected. Substituting Eqs. (30) and (31) in Eq. (19) and integrating by parts, we arrive at

$$\begin{aligned}
W_{\mu\nu}^{(4)} = \frac{16m_b}{3M_B}\Big\{ & -g_{\mu\nu}M_B^2\Big[\tfrac{1}{4}m_b(m_b-\nu)K_b X \\
& + E_b\varepsilon\big(q^0 - m_b v^0\big)\delta\big(q^2 - 2m_b\nu + m_b^2\big) \\
& + m_b(m_b-\nu)K_b\varepsilon\big(q^0 - m_b v^0\big) \\
& \quad \times \delta'\big(q^2 - 2m_b\nu + m_b^2\big) \\
& + \big(q^2 - 2m_b\nu + m_b^2\big)E_b \\
& \quad \times \varepsilon\big(q^0 - m_b v^0\big)\delta'\big(q^2 - 2m_b\nu + m_b^2\big)\Big] \\
& + P_\mu P_\nu m_b^2\Big[\tfrac{1}{2}K_b X + 2(K_b + E_b) \\
& \quad \times \varepsilon\big(q^0 - m_b v^0\big)\delta'\big(q^2 - 2m_b\nu + m_b^2\big)\Big] \\
& + i\varepsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta m_b M_B K_b \\
& \quad \times \Big[\tfrac{1}{4}X + \varepsilon\big(q^0 - m_b v^0\big) \\
& \quad \times \delta'\big(q^2 - 2m_b\nu + m_b^2\big)\Big] \\
& + q_\mu q_\nu 2M_B^2 E_b\varepsilon\big(q^0 - m_b v^0\big) \\
& \quad \times \delta'\big(q^2 - 2m_b\nu + m_b^2\big) \\
& + (P_\mu q_\nu + q_\mu P_\nu)m_b M_B\Big[-\tfrac{1}{4}K_b X \\
& - K_b\varepsilon\big(q^0 - m_b v^0\big)\delta'\big(q^2 - 2m_b\nu + m_b^2\big) \\
& - 2E_b\varepsilon\big(q^0 - m_b v^0\big)\delta'\big(q^2 - 2m_b\nu + m_b^2\big)\Big]\Big\},
\end{aligned} \tag{32}$$

where $\delta'(x) = \frac{d}{dx}\delta(x)$ and

$$X = \frac{\partial^2}{\partial q^\mu \partial q_\mu}\big[\varepsilon\big(q^0 - m_b v^0\big)\delta\big(q^2 - 2m_b\nu + m_b^2\big)\big]. \tag{33}$$

Comparing Eq. (32) with Eq. (18), we find

$$\begin{aligned}
& W_1^{(4)}\big(\nu, q^2\big) \\
& = \tfrac{16}{3}m_b M_B\Big\{\tfrac{1}{4}m_b(m_b-\nu)K_b X \\
& \quad + E_b\varepsilon\big(q^0 - m_b v^0\big)\delta\big(q^2 - 2m_b\nu + m_b^2\big) \\
& \quad + \Big[m_b(m_b-\nu)K_b + \big(q^2 - 2m_b\nu + m_b^2\big)E_b\Big] \\
& \quad \times \varepsilon\big(q^0 - m_b v^0\big)\delta'\big(q^2 - 2m_b\nu + m_b^2\big)\Big\},
\end{aligned} \tag{34}$$

$$\begin{aligned}
& W_2^{(4)}\big(\nu, q^2\big) \\
& = \tfrac{16}{3}m_b^3 M_B\Big[\tfrac{1}{2}K_b X + 2(K_b + E_b) \\
& \quad \times \varepsilon\big(q^0 - m_b v^0\big)\delta'\big(q^2 - 2m_b\nu + m_b^2\big)\Big],
\end{aligned} \tag{35}$$

$$\begin{aligned}
& W_3^{(4)}\big(\nu, q^2\big) \\
& = -\tfrac{16}{3}m_b^2 M_B^2 K_b\Big[\tfrac{1}{4}X + \varepsilon\big(q^0 - m_b v^0\big) \\
& \quad \times \delta'\big(q^2 - 2m_b\nu + m_b^2\big)\Big],
\end{aligned} \tag{36}$$

$$\begin{aligned}
& W_4^{(4)}\big(\nu, q^2\big) \\
& = \tfrac{32}{3}m_b M_B^3 E_b\varepsilon\big(q^0 - m_b v^0\big)\delta'\big(q^2 - 2m_b\nu + m_b^2\big),
\end{aligned} \tag{37}$$

$$\begin{aligned}
& W_5^{(4)}\big(\nu, q^2\big) \\
& = -\tfrac{16}{3}m_b^2 M_B^2\Big[\tfrac{1}{4}K_b X + (K_b + 2E_b) \\
& \quad \times \varepsilon\big(q^0 - m_b v^0\big)\delta'\big(q^2 - 2m_b\nu + m_b^2\big)\Big].
\end{aligned} \tag{38}$$

The twist-4 contribution to the sum rule can be obtained from Eqs. (9), (7), (34) and (35). The result is

$$\begin{aligned}
& \int_0^1 d\xi_u \frac{1}{\xi_u^5}\frac{d\Gamma^{(4)}}{d\xi_u}(\overline{B} \to X_u \ell \bar{\nu}_\ell) \\
& = |V_{ub}|^2 \frac{G_F^2 M_B^5}{192\pi^3}\Big[\frac{304}{45}K_b + \frac{76}{45}E_b + 2\frac{m_b^2}{M_B^2}K_b \\
& \quad - \frac{68}{9}\frac{m_b^3}{M_B^3}K_b - \frac{80}{9}\frac{m_b^3}{M_B^3}E_b - \frac{26}{3}\frac{m_b^4}{M_B^4}K_b \\
& \quad + \frac{28}{3}\frac{m_b^4}{M_B^4}E_b + \frac{112}{15}\frac{m_b^5}{M_B^5}K_b - \frac{32}{15}\frac{m_b^5}{M_B^5}E_b\Big].
\end{aligned} \tag{39}$$

This can serve as an estimate of HT contributions to the sum rule (2).

For the numerical analysis, we need to know the values for the parameters involved. The HQET parameter λ_2 can be extracted from the $B^* - B$ mass splitting: $\lambda_2 = (M_{B^*}^2 - M_B^2)/4 \simeq 0.12\ \text{GeV}^2$, while λ_1 and m_b are less determined. For the purpose of estimation, we take $\lambda_1 = -0.5\ \text{GeV}^2$ [7,8] and $m_b = 4.9$ GeV [9]. From Eq. (39), the HT correction to the sum rule (2) is then estimated to be $\Delta HT = 0.012$. This quantitative study shows that HT corrections are at the expected level of $\sim \Lambda_{\text{QCD}}^2/M_B^2$.

The sum rule (2) requires measuring the ξ_u spectrum dividing by ξ_u^5, which emphasizes the contribution from very small ξ_u. In the free quark decay $b \to u\ell\bar{\nu}_\ell$, the weighted ξ_u spectrum at tree level is

$$\frac{1}{\xi_u^5}\frac{d\Gamma}{d\xi_u}(b \to u\ell\bar{\nu}_\ell) = |V_{ub}|^2 \frac{G_F^2 M_B^5}{192\pi^3}\delta\left(\xi_u - \frac{m_b}{M_B}\right), \tag{40}$$

because ξ_u is fixed by kinematics to be $\xi_u = m_b/M_B$. We see that in the free quark decay the ξ_u spectrum weighted with ξ_u^{-5} is just a discrete line at $\xi_u = m_b/M_B$, which is well above the charm threshold, and there are no contributions from small ξ_u. This is the reason why the kinematic variable ξ_u is the most efficient discriminator between $\overline{B} \to X_u\ell\bar{\nu}_\ell$ signal and $\overline{B} \to X_c\ell\bar{\nu}_\ell$ background. The inclusion of strong interactions amounts to a smearing of the spectrum. However, since in reality the b quark in the B meson is nearly free, we would expect the actual weighted ξ_u spectrum to remain small in the small ξ_u region from a general point of view independent of light-cone dominance.

Let us examine how strong interactions affect the weighted ξ_u spectrum at small values of ξ_u. The above calculation in the framework of HQET suggests that HT contributions to S in this region may be neglected. The weighted ξ_u spectrum to leading twist is given by [1]

$$\frac{1}{\xi_u^5}\frac{d\Gamma^{(2)}}{d\xi_u}(\overline{B} \to X_u\ell\bar{\nu}_\ell) = |V_{ub}|^2 \frac{G_F^2 M_B^5}{192\pi^3} f(\xi_u), \tag{41}$$

where the distribution function $f(\xi)$ is the Fourier transformation of $\mathcal{F}^{(2)}(y \cdot P)$ [2]. The distribution function $f(\xi)$ is known [2] in QCD to be sharply peaked around $\xi = m_b/M_B$ and vanish at $\xi = 0$. This result is consistent with the physical picture that the b quark in the B meson is nearly free. Eq. (41) shows that the weighted ξ_u spectrum has the same sharp shape as the distribution function, in particular, vanishing at $\xi_u = 0$. We also note that perturbative QCD corrections to the weighted spectrum are fairly small in the small ξ_u region and tend to vanish as $\xi_u \to 0$. The small correction is from the bremsstrahlung process of hard gluons. Together, these imply a small contribution to the sum rule value S from small ξ_u, despite the presence of the ξ_u^{-5} weighting. A detailed numerical study of the weighted spectrum has been presented in the second reference in [1], including both perturbative and nonperturbative QCD effects. It turns out that about 80% of the weighted spectrum, $\xi_u^{-5} d\Gamma/d\xi_u$, would lie above the charm threshold, $\xi_u > 1 - M_D/M_B$, as opposed to about 10% of the charged-lepton energy spectrum above the charm threshold, $E_\ell > (M_B^2 - M_D^2)/(2M_B)$. Thus a smaller extrapolation into an unmeasured ξ_u region is needed to obtain the integral S. Since the distribution function is universal, one can use the distribution function $f(\xi)$ measured from other observables, such as the photon energy spectrum in $B \to X_s\gamma$ (see the last reference in [2]), to extrapolate the weighted ξ_u spectrum, without making any model dependent assumptions. In practice, the sum rule (2) requires precision measurements over a wide range of ξ_u. It is important to have data with good accuracy down to ξ_u as small as possible.

In summary, we have elaborated a quantitative way of estimating HT contributions in inclusive B decays, which is based on the heavy quark effective theory. As an application we have calculated the twist-4 correction to the sum rule (2) for charmless inclusive semileptonic B decays. Using the sum rule, $|V_{ub}|$ can be determined from a measurement of the weighted integral S. The error on $|V_{ub}|$ due to HT corrections to the sum rule is estimated to be 1%. Combining theoretical cleanliness and experimental efficiency, together with the better understanding of remaining theoretical uncertainties, the sum rule holds high promise of a precise and model-independent determination of $|V_{ub}|$.

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