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Fusion Model of Multi Monitoring Points on Dam Based on Bayes Theory

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Abstract

Aiming at the shortcomings of the monitoring mathematical model of single point, by adopting Bayes Theory and taking variance as characteristic parameter, this paper has effectively integrated the monitoring data of multi monitoring points, established the abnormal behavior fusion diagnosis model of multi monitoring points, and presented the standard for the model, so as to achieve fusion analysis and diagnosis of the abnormal behavior in data of multi monitoring points and provide a project case. As the study shows, the fusion model of multi monitoring points based on Bayes Theory serves as a new and effective approach for quantitative description of overall dam behavior as well as analysis diagnose of abnormal monitoring points.

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1. Introduction

Currently, the analysis and modeling on dam safety monitoring data mainly adopts single monitoring point as research object ^{[1][2]}, and substantial progress and results have been achieved. Single-point monitoring model is effective in reflecting the local structural behavior where the monitoring point is positioned, but it remains much to be desired in reflecting the overall structural behavior of dam. So it is

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in essence an analysis method characteristic of local fineness and overall extensiveness. It is hereby necessary to figure out a new research thinking mode and seek for new technical support, so as to organically connect the monitoring data of multi monitoring points. We study the fusion of monitoring data of multi monitoring points as well as the abnormal behavior diagnosis to reveal the relationship between the overall structural behavior and local abnormal behavior of dam and evaluate its overall safety condition.

In this paper, we adopt data fusion ^[3] based on Bayes Theory, take multi monitoring points of the same effect quantity as research object, and adopt deformation monitoring effect quantity as research focus. We have united, related and combined multi-point monitoring data of single effect quantity, established the abnormal behavior diagnosis model of multi monitoring points based on Bayes Theory, and effectively overcome the limitations of the existing single-point monitoring data analysis and modeling in analyzing and mastering the overall structural behavior of dam, so as to provide a more reasonable and effective approach to evaluate and monitor dam safety.

2. Abnormal Behavior Diagnosis System of Multi Monitoring Points

The way to establish the model can be generalized as follows:

- (1) Extract fusion parameters able to describe the consistence of multi monitoring points.
- (2) Determine the prior distribution of characteristic parameters according to their property.
- (3) Apply Bayes Theory to carry out recursive fusion of characteristic parameters, based on multi-point monitoring data.
- (4) Establish the standard for evaluating abnormal behavior of multi monitoring points, and carry out fusion diagnosis to their abnormal behaviors.

2.1. Selection of fusion characteristic parameters

According to the existing research results, the monitoring data sequence of single point on dam is generally subject to normal distribution. For the random variable X , if the probability density function is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, -\infty < x < +\infty \quad (1)$$

X follows the normal distribution of σ^2 (variance) and μ (mean value), kept as $X \sim N(\mu, \sigma^2)$; $\sigma > 0$; σ and μ are constants.

In normal distribution, μ determines the central position of distribution curve, referred to as position parameter; its change results in the translation of distribution curve, but the shape of the curve remains unchanged; σ determines the shape of the curve, referred to as shape parameter; it mainly reflects the effect of observation precision of the measured value; its change only results in the change of the shape, and the central position of the curve remains unchanged.

Different monitoring points of the same effect quantity are distributed on different parts of dam. Under comprehensive effect of environmental changes, dam structure aging and foundation condition variance, all the monitoring data at different points follow different normal distributions. The mean value μ of monitoring point is usually significantly different. However, once given the same observation instrument, method and conditions, the variance of monitoring points σ^2 can be deemed as consistent within a given confidence interval, as long as no abnormal phenomenon occurs to the effect quantity. Once the measured value of a point or some points appears abnormal, the sample mean value μ and variance σ^2 could change significantly. The paper hereby selects σ^2 as characteristic parameter of monitoring-point consistency as well as the evaluation indicator of abnormal diagnose of multi monitoring points.

2.2. Procedures of recursive fusion

We take the deformation monitoring effect quantity as example. Suppose a given dam is set with “ m ” deformation monitoring points. We adopt the following recursive method to carry out Bayes fusion to the monitoring data of “ m ” points.

(1) Initial fusion: We randomly select the monitoring data of one monitoring point (kept as “Point 1”) from the “ m ” points as initial priori information, and randomly select the monitoring data of another point (kept as “Point 2”) as initial scene information. Carry out Bayes fusion to “Point 1” and “Point 2”, and then posterior information is made available.

(2) Recursive fusion: We randomly select the monitoring data of one point (kept as “Point 3”) from the rest “ $m-2$ ” points, and take posterior information upon initial fusion as priori information. Carry out Bayes fusion to “Point 1”, “Point 2” and “Point 3” and then posterior information is made available.

(3) Likewise, consecutively carry out fusion to monitoring data of the “ m ” monitoring points, and obtain the final posterior information.

2.3. Initial fusion

Preprocess the original data of “Point 1”, and obtain priori information $x_{11}, x_{12}, \dots, x_{1n_1}$. The data sequence follows normal distribution $N_1(\mu_1, \sigma_1^2)$. Preprocess the original data of “Point 2”, and obtain the initial scene information $x_{21}, x_{22}, \dots, x_{2n_2}$. The data sequence follows normal distribution $N_2(\mu_2, \sigma_2^2)$.

Inverse Gamma distribution is selected as prior distribution of σ^2 , kept as $\sigma^2 \sim IGa(\alpha_1, \beta_1)$. If $\theta = \sigma^2$, the prior distribution density of θ is kept as $\pi(\theta)$, and the probability density formula is:

$$\pi(\theta) \propto \frac{\alpha_1^{\beta_1}}{\Gamma(\beta_1)} \theta^{-(\beta_1+1)} e^{-\alpha_1/\theta}, \quad \alpha_1 > 0, \beta_1 > 0 \quad (2)$$

According to Bayes formula and conjugate distribution, we can derive the fusion posterior distribution:

$$\pi(\theta) \propto \frac{\alpha_2^{\beta_2}}{\Gamma(\beta_2)} \theta^{-(\beta_2+1)} e^{-\alpha_2/\theta} \quad (3)$$

In Formula (3), α_2 and β_2 are parameters of posterior distribution; α_1 and β_1 are parameters of prior distribution.

2.4. Recursive fusion

Upon initial fusion, further recursive fusion is to be done. Preprocess the original monitoring data of “Point 3”, and obtain the scene information $x_{31}, x_{32}, \dots, x_{3n_3}$. Then take the previous posterior distribution as this prior distribution. The fusion process is the same as previous procedure, and the fusion posterior distribution is:

$$\pi_3(\theta|x) \propto \frac{\alpha_3^{\beta_3}}{\Gamma(\beta_3)} \theta^{-(\beta_3+1)} e^{-\alpha_3/\theta} \quad (4)$$

Consecutively carry out fusion to the monitoring data of “Point 4”... “Point m ”, and obtain the final fusion posterior distribution:

$$\pi_m(\theta|x) \propto \frac{\alpha_m^{\beta_m}}{\Gamma(\beta_m)} \theta^{-(\beta_m+1)} e^{-\alpha_m/\theta} \quad (5)$$

See Formula (2) for the meaning of parameters in Formula (4) and (5).

2.5. Result of fusion estimation of multi monitoring points

Under squared error loss function, we find out mathematical expectation to the posterior distribution, and obtain the Bayes point estimation value of parameter after fusing as μ and σ^2 of monitoring points. For posterior distribution of initial fusion $\pi_2(\theta|x)$, we obtain Bayes estimation $\hat{\theta}_2$ of θ :

$$\hat{\theta}_2 = \frac{\alpha_2}{\beta_2 - 1} \tag{6}$$

Substitute it into the prior distribution parameters α_1 and β_1 as well as posterior distribution parameters α_2 and β_2 ; Formula (6) is expressed in the form of recursive fusion, namely:

$$\hat{\theta}_2 = \frac{n_1 - 2}{n_1 + n_2 - 2} E(\theta_1) + \frac{n_2 - 2}{n_1 + n_2 - 2} S_2^2 = \beta_2 E(\theta_1) + \zeta_2 S_2^2 \tag{7}$$

Here: $E(\theta_1)$ refers to the expectation of priori information, and $E(\theta_1) = \sigma_1^2$.

β_2 and ζ_2 are recursive parameters related with the sample number of priori information and scene information;

$$S_2^2 \text{ is the variance of scene information "Point 2", } S_2^2 = \frac{1}{n_2 - 1} \sum_{i=1}^{n_2} (x_{2i} - \bar{x}_2)^2.$$

According to Formula (7), the estimation value of the variance is related with the sample number and variance of priori information and scene information. So, after determining the data of priori information and scene information, we can carry out Bayes estimation to posterior variance. The estimation result makes full use of priori information and scene information.

According to the recursive estimation, we obtain Bayes estimation $\hat{\theta}_m$ of θ upon fusion of "Point m ":

$$\hat{\theta}_m = \frac{\alpha_m}{\beta_m - 1} \tag{8}$$

It is expressed in recursive formula:

$$\hat{\theta}_m = \rho_m \hat{\theta}_{m-1} + \zeta_m S_m^2 \tag{9}$$

See Formula (7) for the meaning of symbols in Formula (9).

Through the recursive formula (9), we can finally obtain the estimation value of variance upon fusion of " m " monitoring points.

2.6. Standard for abnormal behavior diagnose of multi monitoring points

Here we derive $(1 - \alpha)$ confidence interval ^[5] of θ . Integrate the joint density function to μ , and we obtain the posterior density $\pi(\mu|\theta)$ of θ related to X :

$$\pi(\theta|x) = \int_{-\infty}^{+\infty} \pi(\mu, \theta) d\mu$$

It can be expressed according to the related nature of normal distribution as:

$$\pi(\theta|x) \propto \theta^{\frac{1}{2}} \cdot \theta^{(-\beta_m + 1)} e^{-\frac{\alpha_m}{\theta}} \tag{10}$$

Here θ follows the Gamma distribution. If $y = \frac{2\alpha_m}{\theta}$, then $d\theta = -\frac{2\alpha_m}{y^2} dy$, kept as $n = 2\beta_m + 2$, and

then:

$$P(Y < Y_0 | X) = C \cdot \int_{Y < Y_0} y^{\frac{n-2}{2}} \cdot e^{-\frac{y}{2}} dy \tag{11}$$

Namely, Y follows χ^2 distribution with the degree of freedom as $n-2$. And the $(1-\alpha)$ confidence interval of Y can be obtained:

$$[\chi_{\alpha/2}^2(n-2), \chi_{1-\alpha/2}^2(n-2)] \tag{12}$$

And the $(1-\alpha)$ confidence interval of θ is:

$$[2\alpha_m / \chi_{\alpha/2}^2(n-2), 2\alpha_m / \chi_{1-\alpha/2}^2(n-2)] \tag{13}$$

In case $n \gg 1$, then $\sqrt{2}\chi^2$ progression follows normal distribution $N(\sqrt{2(n-2)-1}, 1)$, namely $\sqrt{2}y \sim N(\sqrt{2n-5}, 1)$. And the $(1-\alpha)$ confidence interval of $\sqrt{2}y$ is:

$$[\sqrt{2n-5} - Z_{1-\alpha/2}, \sqrt{2n-5} + Z_{1-\alpha/2}] \tag{14}$$

And the $(1-\alpha)$ confidence interval of θ is:

$$[4\alpha_m / (\sqrt{2n-5} + Z_{1-\alpha/2})^2, 4\alpha_m / (\sqrt{2n-5} - Z_{1-\alpha/2})^2] \tag{15}$$

As mentioned above, the variance of monitoring data of multi points of the same effect quantity is consistent. When one or some points appear abnormal, the variance will significantly deviate from the aforesaid constant value. We hereby propose the judgment interval of abnormal points in multi monitoring points as:

$$[0, 4\alpha_m / (\sqrt{2n-5} + Z_{1-\alpha/2})^2] \cup [4\alpha_m / (\sqrt{2n-5} - Z_{1-\alpha/2})^2, +\infty] \tag{16}$$

Carry out abnormal behavior diagnose of multi monitoring points according to Formula (16): When the sample variance of the measured value is within the fusion interval, it is thought to have no abnormal conditions, and vice versa.

3. Empirical Analysis

In case study, we get 7 monitoring points with forward-intersection horizontal displacement (P1~P7) on a concrete high-arch dam as the object, and establish multi-point abnormal behavior diagnose model according to the observed data in 2001~2010.

Carry out posterior distribution to the 7 monitoring points according to Formula (5), and use Formula (9) and (15) to carry out Bayes estimation of parameters. See Table 1 for the calculation result.

Table 1 Fusion Parameters and Fusion Result of P1~P7 upon Horizontal Displacement

Point No.	α	β	Estimation result	Lower limit of interval estimation	Upper limit of interval estimation
P1	672.616	1113.5	0.605	0.570	0.641
P2	1454.812	2227.0	0.654	0.627	0.681
P3	2105.890	3322.0	0.634	0.613	0.656
P4	2805.877	4441.0	0.632	0.614	0.651
P5	3489.380	5561.5	0.628	0.611	0.644
P6	4162.733	6679.0	0.623	0.609	0.639
P7	4841.611	7799.0	0.621	0.607	0.635

According to Table 1, the optimum value of fusion parameter of seven monitoring points is $\hat{\theta}_7=0.621$. The 0.95 confidence interval of θ is [0.607, 0.635]. Among seven points, the fusion variance of P1 is 0.605, beyond the lower limit of confidence interval (0.607); the fusion variance of P2 is 0.654, beyond

the upper limit of confidence interval (0.635); the fusion variances of other points are within the confidence interval [0.607, 0.635]. It can be determined that the behaviors of P1 and P2 appear abnormal.

According to the above fusion result, we inspect P1 and P2 and find that the forced centering components at the top of observation pillars appear loosened, and P2 is even worse, with great error of centration.

4. Conclusion

Aiming at the shortcomings of single-point monitoring model in dam safety monitoring, the paper has introduced Bayes Theory into multi effect quantities model, and established single effect quantity fusion model of multi monitoring points and abnormal behavior diagnose standard, so as to propose a new thinking mode for similar modeling as well as safety monitoring data analysis and safety evaluation of dam in the days to come.

Fusion analysis of multi monitoring points is an advanced subject about data analysis on dam safety monitoring, wide-ranged and complicated. We herein have made preliminary discussion on the abnormal behavior diagnose of multi monitoring points from the perspective of Bayes fusion theory. There remains much to be done to further study the topic in theory and improve it in practice.

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