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Covariant description of D-branes in IIA plane-wave background

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Dedicated to the memory of Youngjai Kiem

Abstract

We work out boundary conditions for the covariant open string in the type IIA plane wave background, which corresponds to the D-branes in the type IIA theory. We use the kappa symmetric string action and see what kind of boundary conditions should be imposed to retain kappa symmetry. We find half BPS as well as quarter BPS branes and the analysis agrees with the previous work in the light cone gauge if the result is available. Finally we find that D0-brane is non-supersymmetric.

1. Introduction

Recently the string theory on the plane wave background has attracted much attention in relation to the correspondence to the $N = 4$ Supersymmetric Yang–Mills (SYM) theory [1,2]. It is now well known that for the usual AdS/CFT correspondence, the various checks have been done essentially on the supergravity states on the string side. In the seminal paper by Berenstein et al. [2], this major obstacle was overcome, thereby showing the explicit correspondence between more general string states and the suitable Yang–Mills operators in the plane wave background [3], which is the Penrose limit of the AdS [4]. After their paper, more progress was made on how the string Hamiltonian is mapped to the anomalous dimension of the Yang–Mills operator and more precise dictionaries for the correspondence have been developed [5–11].

The previously mentioned development was made in type IIB side. In type IIA side, the matrix theory on the plane wave background has been important focus in relation to the better understanding of the M-theory on the plane wave background [2,12]. Recently, simple type IIA string theory on the plane wave background was proposed by the Kaluza–Klein compactification of the M-theory [13,14]. The resulting string theory has many nice features. It admits light cone gauge where the string theory spectrum is that of the free massive theory as happens in type IIB [1]. Furthermore the worldsheet enjoys $(4,4)$ supersymmetry [13]. The structure of the supersymmetry is simpler than that of type IIB in the sense that the supersymmetry commutes with the Hamiltonian so that all

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members of the same supermultiplet has the same mass. In the extended version, more detailed exposition was given and the various $1/2$ BPS D-branes states were analyzed in the light cone gauge [15], which are compatible with the BPS branes in matrix model [16,17].

The purpose of this Letter is to carry out the analysis about the D-branes states in the covariant setting. We follow the logic of Lambert and West [18] and consider the kappa symmetric string action in the plane wave background and figure out how the supersymmetry is reduced when we impose suitable boundary conditions on the boundary of the worldsheet. Similar work [19] has been done in type IIB side and given results in agreement with the previous results [20–22]. See also [23,24] for recent alternative study. Various aspects of kappa symmetry and worldsheet supersymmetry in the plane wave background is discussed in [25]. In our study, as expected, for the D-brane located at the origin of the plane wave background, the analysis in the covariant setting coincides with that in the lightcone gauge [15]. The merit of the covariant analysis is that we are able to work out other D-brane states, which are difficult to analyze in the lightcone gauge. For example, we work out the supersymmetry of D-particle (in fact, non-supersymmetry) and analyze the supersymmetry of D-branes located away from the origin. In the investigation we found out some potential subtleties arising in the D-brane analysis in the lightcone gauge. We will comment on this in the later section. We think that the analysis in the covariant setting is a good starting point to sort out various, sometimes conflicting, claims [26] on the number of supersymmetry of various D-branes in the pp-wave.

2. Covariant Wess–Zumino action of type IIA string

The covariant description of D-branes via open string may be given by investigating the boundary contributions in the kappa symmetry variation of the Wess–Zumino part of the superstring action [19]. In this section, starting from the superspace geometry of $AdS_4 \times S^7$ [27] whose Penrose limit leads to the eleven-dimensional pp-wave background, we derive the covariant Wess–Zumino action of type IIA superstring in the IIA pp-wave background of Refs. [13,14] up to quartic order in the fermionic coordinate θ.

The eleven-dimensional superspace geometry of $AdS_4 \times S^7$ [27] is encoded in the super elfbein $\widehat{E}^{\hat{A}} = (\widehat{E}^{\hat{r}}, \widehat{E})$ and the three form superfield $\widehat{B}$.[1] The super elfbein is

$$\begin{aligned} \widehat{E}^{\hat{r}} &= dx^{\hat{\mu}}\, \hat{e}_{\hat{\mu}}^{\hat{r}} + 2\sum_{n=0}^{15} \frac{1}{(2n+2)!} \bar{\theta}\Gamma^{\hat{r}} \mathcal{M}^{2n} \widehat{D}\theta, \\ \widehat{E} &= \sum_{n=0}^{16} \frac{1}{(2n+1)!} \mathcal{M}^{2n} \widehat{D}\theta, \end{aligned} \tag{2.1}$$

where $\hat{e}_{\hat{\mu}}^{\hat{r}}$ is the elfbein and $\widehat{D}\theta$, the covariant derivative of θ, is given by

$$\widehat{D}\theta = d\theta + \frac{1}{4}\hat{\omega}^{\hat{r}\hat{s}} \Gamma_{\hat{r}\hat{s}} + \hat{e}^{\hat{r}} T_{\hat{r}}^{\hat{s}\hat{t}\hat{u}\hat{v}} \widehat{F}_{\hat{s}\hat{t}\hat{u}\hat{v}}\theta \tag{2.2}$$

with the eleven-dimensional spin connection $\hat{\omega}^{\hat{r}\hat{s}}$. The matrix $\mathcal{M}^2$ is

$$\mathcal{M}^2_{ab} = 2\big(T_{\hat{r}}^{\hat{s}\hat{t}\hat{u}\hat{v}} \widehat{F}_{\hat{s}\hat{t}\hat{u}\hat{v}}\theta\big)_a \big(\bar{\theta}\Gamma^{\hat{r}}\big)_b - \frac{1}{4}\big(\Gamma^{\hat{r}\hat{s}}\theta\big)_a \big(\bar{\theta} S_{\hat{r}\hat{s}}^{\hat{t}\hat{u}\hat{v}\hat{w}} \widehat{F}_{\hat{t}\hat{u}\hat{v}\hat{w}}\big)_b. \tag{2.3}$$

[1] We note that $\widehat{E}$ means $\widehat{E}^{\hat{a}}$. The index notations adopted here are as follows: $M, N, \ldots$ are used for the target superspace indices while $A, B, \ldots$ for tangent superspace. As usual, a superspace index is the composition of two types of indices such as $M = (\mu, \alpha)$ and $A = (r, a)$. $\mu, \nu, \ldots$ $(r, s, \ldots)$ are the ten-dimensional target (tangent) space–time indices. $\alpha, \beta, \ldots$ $(a, b, \ldots)$ are the ten-dimensional (tangent) spinor indices. For the eleven-dimensional case, we denote quantities and indices with hat to distinguish from those of ten dimensions. $m, n, \ldots$ are the worldsheet vector indices with values τ and σ. The convention for the worldsheet antisymmetric tensor is taken to be $\epsilon^{\tau\sigma} = 1$.

The definitions for the tensor structures are as follows:

$$T_{\hat{r}}^{\hat{s}\hat{t}\hat{u}\hat{v}} \equiv \frac{1}{288}\big(\Gamma_{\hat{r}}^{\hat{s}\hat{t}\hat{u}\hat{v}} - 8\delta_{\hat{r}}^{[\hat{s}}\Gamma^{\hat{t}\hat{u}\hat{v}]}\big), \qquad S_{\hat{r}\hat{s}}^{\hat{t}\hat{u}\hat{v}\hat{w}} \equiv \frac{1}{72}\big(\Gamma_{\hat{r}\hat{s}}^{\hat{t}\hat{u}\hat{v}\hat{w}} + 24\delta_{\hat{r}}^{[\hat{t}}\delta_{\hat{s}}^{\hat{u}}\Gamma^{\hat{v}\hat{w}]}\big). \tag{2.4}$$

The three form superfield is given by

$$\widehat{B} = \frac{1}{6}\hat{e}^{\hat{r}} \wedge \hat{e}^{\hat{s}} \wedge \hat{e}^{\hat{t}}\widehat{C}_{\hat{r}\hat{s}\hat{t}} - \int_0^1 dt\, \bar{\theta}\Gamma_{\hat{r}\hat{s}}\widehat{E} \wedge \widehat{E}^{\hat{r}} \wedge \widehat{E}^{\hat{s}}, \tag{2.5}$$

where $\widehat{C}_{\hat{r}\hat{s}\hat{t}}$ is three form gauge field whose field strength is $\widehat{F}_{\hat{r}\hat{s}\hat{t}\hat{u}} = 4\partial_{[\hat{r}}\widehat{C}_{\hat{s}\hat{t}\hat{u}]}$. We note that the super elfbeins in the second term on the right-hand side have t dependence in a way that θ's in (2.1) are replaced as $\theta \to t\theta$.

The component fields in Eqs. (2.1) and (2.5) are for the $AdS_4 \times S^7$. As shown in [4], by taking the Penrose limit [28], they become the fields describing the eleven-dimensional pp-wave background. After some rotation in a certain plane, say 49-plane, for our convenience in ten dimensions, the eleven-dimensional pp-wave background becomes as follows:[2]

$$\begin{aligned} &\hat{e}^+ = dx^+, \qquad \hat{e}^- = dx^- + \frac{1}{2}A\big(x^I\big)\,dx^+, \\ &\hat{e}^I = dx^I, \qquad \hat{e}^9 = dx^9 + \frac{\mu}{3}x^4\,dx^+, \qquad \widehat{F}_{+123} = \mu, \end{aligned} \tag{2.6}$$

where μ is constant characterizing the pp-wave, $I = 1, \ldots, 8$ and

$$A\big(x^I\big) = \left(\frac{\mu}{3}\right)^2 \sum_{i=1}^{4}\big(x^i\big)^2 + \left(\frac{\mu}{6}\right)^2 \sum_{i'=5}^{8}\big(x^{i'}\big)^2. \tag{2.7}$$

We now turn to the ten-dimensional background, which is the type IIA pp-wave background obtained from the circle compactification of the eleven-dimensional pp-wave (2.6). If we take x^9 as the direction of compactification, then the usual Kaluza–Klein dimensional reduction leads us to have the following ten-dimensional background:

$$\begin{aligned} &e^+ = dx^+, \qquad e^- = dx^- + \frac{1}{2}A\big(x^I\big)\,dx^+, \\ &e^I = dx^I, \qquad F_{+123} = \mu, \qquad F_{+4} = -\frac{\mu}{3}. \end{aligned} \tag{2.8}$$

In terms of these ten-dimensional fields and by using the logic of the Kaluza–Klein reduction, one can express the elementary pieces of the eleven-dimensional superfields (2.1) and (2.5), which are $\widehat{D}\theta$ and the matrix $\mathcal{M}^2$. Firstly, the eleven-dimensional supercovariant derivative becomes

$$\widehat{D}\theta = D\theta + \frac{\mu}{6}\big(\Gamma^{+4}h_-\theta\big)\hat{e}^9, \tag{2.9}$$

where $D\theta$ is the ten-dimensional supercovariant derivative of θ and $h_\pm$ is the operator projecting spinor states onto the states with eigenvalue ± 1 of Γ^{12349}:

$$h_\pm = \frac{1}{2}\big(1 \pm \Gamma^{12349}\big). \tag{2.10}$$

The covariant derivative $D\theta$ is given by

$$D\theta = d\theta + \frac{1}{4}\omega^{rs}\Gamma_{rs}\theta + \Omega\theta, \tag{2.11}$$

[2] For detailed derivation of this background and its ten-dimensional reduction, see [13].

where the non-vanishing ten-dimensional spin connection is

$$\omega^{-I} = \frac{1}{2}\partial_I A\, dx^+ \tag{2.12}$$

and the definition for Ω is

$$\Omega = \frac{\mu}{12}\big[-e^+\big(\Gamma^-\Gamma^{+123} + 2\Gamma^{49}(2h_- - h_+)\big) + 2e^i\Gamma^{+i}\Gamma^{123} + 2e^4\Gamma^{+9}h_+ - e^{i'}\Gamma^{+i'}\Gamma^{123}\big], \tag{2.13}$$

where $i = 1,2,3$ and $i' = 5,6,7,8$. The explicit expression of matrix $\mathcal{M}^2$ in term of ten-dimensional quantities is obtained as

$$\begin{aligned}\mathcal{M}^2_{ab} = -i\frac{\mu}{6}\Big[&\big(\Gamma^-\Gamma^{+123}\theta + 3\Gamma^{123}\theta\big)_a\big(\bar\theta\Gamma^+\big)_b + \big(\Gamma^{+i'}\Gamma^{123}\theta\big)_a\big(\bar\theta\Gamma^{i'}\big)_b + \big(\Gamma^{+4}\Gamma^{123}\theta\big)_a\big(\bar\theta\Gamma^4\big)_b\\ &+ \big(\Gamma^{+9}\Gamma^{123}\theta\big)_a\big(\bar\theta\Gamma^9\big)_b + \big(\Gamma^{+i'}\theta\big)_a\big(\bar\theta\Gamma^{-+i'}\Gamma^{123}\big)_b + \big(\Gamma^{+4}\theta\big)_a\big(\bar\theta\Gamma^{-+4}\Gamma^{123}\big)_b\\ &+ \big(\Gamma^{+9}\theta\big)_a\big(\bar\theta\Gamma^{-+9}\Gamma^{123}\big)_b + \frac{1}{2}\big(\Gamma^{i'j'}\theta\big)_a\big(\bar\theta\Gamma^{+i'j'}\Gamma^{123}\big)_b + \big(\Gamma^{i'4}\theta\big)_a\big(\bar\theta\Gamma^{+i'4}\Gamma^{123}\big)_b\\ &+ \big(\Gamma^{i'9}\theta\big)_a\big(\bar\theta\Gamma^{+i'9}\Gamma^{123}\big)_b + \big(\Gamma^{+ij}\theta\big)_a\big(\bar\theta\Gamma^{ij}\Gamma^{123}\big)_b - \big(\Gamma^{+ij}\Gamma^{123}\theta\big)_a\big(\bar\theta\Gamma^{ij}\big)_b\\ &- \big(\Gamma^{ij}\theta\big)_a\big(\bar\theta\Gamma^{+ij}\Gamma^{123}\big)_b\Big].\end{aligned} \tag{2.14}$$

Here we note that, going from eleven to ten dimensions, the fermionic coordinate θ splits into two Majorana–Weyl spinors with opposite $SO(1,9)$ chiralities measured by Γ^9:

$$\theta = \theta^1 + \theta^2, \tag{2.15}$$

where $\Gamma^9\theta^1 = \theta^1$ and $\Gamma^9\theta^2 = -\theta^2$.

We now have all the ingredients for writing down the covariant Wess–Zumino action of type IIA string in the pp-wave background, (2.8). In the superfield formalism, the Wess–Zumino action is given by

$$S_{\mathrm{WZ}} = \frac{1}{2\pi\alpha'}\int_\Sigma d^2\sigma\,\frac{1}{2!}\epsilon^{mn}\Pi^A_m\Pi^B_n B_{BA}, \tag{2.16}$$

where $\Pi^A_m = \partial_m Z^M E^A_M$ with supercoordinate $Z^M = (X^\mu, \theta^\alpha)$ and B_{BA} is the two form superfield. Σ represents the worldsheet of open string. In the context of this Letter, it is useful to remind the well known fact that S_{WZ} can be viewed as the action obtained from the Wess–Zumino action for the eleven-dimensional super membrane through the double dimensional reduction [29]. Since we compactify the super membrane along the x^9 direction, the two form superfield is identified with the eleven-dimensional three form superfield with the index 9, that is, $\widehat{B}_{9NM}$. Then the above Wess–Zumino action is rewritten as

$$S_{\mathrm{WZ}} = \frac{1}{2\pi\alpha'}\int_\Sigma d^2\sigma\,\frac{1}{2!}\epsilon^{mn}\partial_m Z^M\partial_n Z^N\widehat{B}_{9NM}. \tag{2.17}$$

Now by using Eqs. (2.1), (2.5), (2.8), (2.9) and (2.14), the Wess–Zumino action is given in component form and expanded in terms of θ. Although it has expansion up to 32th order in θ, we will give the expansion up to quartic order since, as we shall see in Section 3, the non-trivial information for the description of D-branes is obtained already at the quartic order. The resulting Wess–Zumino action of type IIA string in the IIA pp-wave background is then

$$S_{\mathrm{WZ}} = \frac{i}{2\pi\alpha'} \int\limits_{\Sigma} d^2\sigma\, \epsilon^{mn} \biggl[-\bar\theta \Gamma_{r9} D_m\theta e^r_n - \frac{i}{2}\bigl(\bar\theta\Gamma_{r9}D_m\theta\bigr)\bigl(\bar\theta\Gamma^r D_n\theta\bigr) - \frac{\mu}{12}\bigl(\bar\theta\Gamma_{rs}\Gamma^{+4}h_-\theta\bigr)e^r_m e^s_n$$
$$+ i\frac{\mu}{12}\bigl(\bar\theta\Gamma^r\Gamma^{+4}h_-\theta\bigr)\bigl(\bar\theta\Gamma_{rs}D_m\theta\bigr)e^s_n + i\frac{\mu}{12}\bigl(\bar\theta\Gamma^9\Gamma^{+4}h_-\theta\bigr)\bigl(\bar\theta\Gamma_{9r}D_m\theta\bigr)e^r_n$$
$$- i\frac{\mu}{12}\bigl(\bar\theta\Gamma_{rs}\Gamma^{+4}h_-\theta\bigr)\bigl(\bar\theta\Gamma^r D_m\theta\bigr)e^s_n - i\frac{\mu}{12}\bigl(\bar\theta\Gamma_{9r}\Gamma^{+4}h_-\theta\bigr)\bigl(\bar\theta\Gamma^9 D_m\theta\bigr)e^r_n$$
$$- \frac{1}{12}\bigl(\bar\theta\Gamma_{r9}\mathcal{M}^2 D_m\theta\bigr)e^r_n - \frac{\mu}{144}\bigl(\bar\theta\Gamma_{rs}\mathcal{M}^2\Gamma^{+4}h_-\theta\bigr)e^r_m e^s_n + \mathcal{O}\bigl(\theta^6\bigr)\biggr], \tag{2.18}$$

where $e^r_m = \partial_m X^\mu e^r_\mu$ and $D_m\theta = \partial_m X^\mu (D\theta)_\mu$.

3. Boundary conditions from kappa symmetry

The κ symmetry transformation rules are read off from

$$\delta Z^M E^r_M = 0, \qquad \delta Z^M E^a_M = \bigl(1 - \Gamma\Gamma^9\bigr)^a{}_b\kappa^b, \tag{3.1}$$

where $\Gamma = \frac{1}{2!}\epsilon^{mn}\Pi^r_m\Pi^s_n\Gamma_{rs}$. From (3.1), one can see that the variation of X^μ is given by

$$\delta X^\mu = -i\bar\theta^1\Gamma^\mu\delta\theta^1 - i\bar\theta^2\Gamma^\mu\delta\theta^2 + \mathcal{O}\bigl(\theta^3\bigr). \tag{3.2}$$

Here we retain the variations up to the quadratic in θ since we are interested in the kappa variation up to the quartic in θ. As shown in [19], the kinematic parts of the kappa symmetric action does not produce the boundary terms, so we just consider the variations of the Wess–Zumino terms. We divide the resulting kappa variations as three parts, i.e., μ independent part, μ dependent part with no position dependence and finally the part with both μ and position dependence.

μ independent part gives the same result as in the flat case, which gives the well-known result [18]. The relevant variation is given by

$$\delta S_{\mathrm{WZ}} \to \int\limits_{\partial\Sigma} \Bigl[i\bigl(\bar\theta^1\Gamma_r\delta\theta^1 - \bar\theta^2\Gamma_r\delta\theta^2\bigr)\, dX^\mu e^r_\mu - \bigl(\bar\theta^1\Gamma_r\, d\theta^1\, \bar\theta^1\Gamma^r\delta\theta^1 - \bar\theta^2\Gamma_r\, d\theta^2\, \bar\theta^2\Gamma^r\delta\theta^2\bigr)\Bigr], \tag{3.3}$$

where $\partial\Sigma$ represents the boundary of Σ, that is, the boundary of open string worldsheet. Here the arrow means that we are ignoring overall coefficients in front of the Wess–Zumino terms. In order to have vanishing variation on the boundary, we impose the usual half-BPS boundary conditions,

$$\theta^2 = P\theta^1 \tag{3.4}$$

with

$$P = \Gamma^{+-i_1 i_2\cdots i_{(p-1)}}, \tag{3.5}$$

where $+-i_1 i_2\cdots i_{(p-1)}$ denotes the Neumann directions of the D-brane considered. And p should be even since θ^1 and θ^2 have opposite chiralities. Then[3]

$$\bar\theta^2\Gamma_r\delta\theta^2 = \begin{cases} +\bar\theta^1\Gamma_r\delta\theta^1: & r \in N, \\ -\bar\theta^1\Gamma_r\delta\theta^1: & r \in D. \end{cases} \tag{3.6}$$

[3] $r \in N(D)$ means that r is the direction of Neumann (Dirichlet) boundary condition.

It is clear that the boundary conditions eliminate the boundary terms.

The kappa variation of the Wess–Zumino terms which have dependence on μ with no position dependence is given by

$$\begin{aligned}\delta S_{\mathrm{WZ}} \to \int_{\partial\Sigma} dX^\mu\, e^r_\mu \Big[&\big(\bar\theta^1\Gamma^s\delta\theta^1 + \bar\theta^2\Gamma^s\delta\theta^2\big) \\ &\times \left(\bar\theta^1\left(2\Gamma_{[r}\Omega_{s]} - \frac{\mu}{12}\Gamma_{rs}\Gamma^{+4}h_-\right)\theta^2 - \bar\theta^2\left(2\Gamma_{[r}\Omega_{s]} + \frac{\mu}{12}\Gamma_{rs}\Gamma^{+4}h_-\right)\theta^1\right) \\ &+ \big(\bar\theta^2\Gamma^s\delta\theta^2\big)\big(\bar\theta^1\Gamma_s\Omega_r\theta^2\big) - \big(\bar\theta^1\Gamma^s\delta\theta^1\big)\big(\bar\theta^2\Gamma_s\Omega_r\theta^1\big) \\ &- \frac{\mu}{12}\big(\bar\theta^1\Gamma_{rs}\delta\theta^2 + \bar\theta^2\Gamma_{rs}\delta\theta^1\big)\big(\bar\theta^1\Gamma^s\Gamma^{+4}h_-\theta^1 + \bar\theta^2\Gamma^s\Gamma^{+4}h_-\theta^2\big) \\ &- \frac{\mu}{12}\big(\bar\theta^1\delta\theta^2 - \bar\theta^2\delta\theta^1\big)\big(\bar\theta^1\Gamma_r\Gamma^{+4}h_-\theta^1 - \bar\theta^2\Gamma_r\Gamma^{+4}h_-\theta^2\big) + \frac{i}{12}\big(\bar\theta\Gamma_{r9}\mathcal{M}^2\delta\theta\big)\Big]. \end{aligned} \tag{3.7}$$

First consider terms of the structure $\bar\theta\Gamma_{rs}\delta\theta$

$$\bar\theta^1\Gamma_{rs}\delta\theta^2 + \bar\theta^2\Gamma_{rs}\delta\theta^1 = \begin{cases} 0: & r\in N,\quad s\in D(N),\quad \text{for } p=2,6\ (4,8), \\ 2\bar\theta^1\Gamma_{rs}P\delta\theta^1: & r\in N,\quad s\in N(D),\quad \text{for } p=2,6\ (4,8). \end{cases} \tag{3.8}$$

We see that for $p=4,8$ (2, 6) with $s\in D$ (N), $\bar\theta^1\Gamma^s\Gamma^{+4}h_-\theta^1 + \bar\theta^2\Gamma^s\Gamma^{+4}h_-\theta^2$ should vanish. However, it does not vanish only with the boundary condition $\theta^2 = P\theta^1$. Some constraints should be imposed on the structure of P and thus picks up the branes with particular orientations. Let us label the matrix P by three non-negative integers n, n_4 and n' with $n+n_4+n' = p-1$:

$$P^{(n,n_4,n')}.$$

$n(n')$ denotes the number of gamma matrices with indices in 123 (5678) directions and n_4 the presence or the absence of Γ^4 thus taking value of 0 or 1, respectively, in Eq. (3.5).

Careful analysis shows that

$$\bar\theta^1\Gamma^s\Gamma^{+4}h_-\theta^1 + \bar\theta^2\Gamma^s\Gamma^{+4}h_-\theta^2 = 0 \tag{3.9}$$

if we bear in mind that $s\in N(D)$ for $p=2,6$ (4, 8) and impose the following constraints,

$$\begin{aligned} &p=2,6:\quad n=\text{odd},\quad n_4=0, \\ &p=4,8:\quad n=\text{even},\quad n_4=1. \end{aligned} \tag{3.10}$$

Interestingly, one can check, with lengthy calculation, that all other remaining terms in (3.7) vanishes if we impose the constraints (3.10).

Possible D-brane configurations making the above boundary contributions vanish are given by the following choices of (n, n_4, n') for Dp-brane.

$$\begin{aligned} &p=2:\quad (1,0,0), \\ &p=4:\quad (0,1,2),\quad (2,1,0), \\ &p=6:\quad (1,0,4),\quad (3,0,2), \\ &p=8:\quad (2,1,4). \end{aligned} \tag{3.11}$$

This exactly coincides with the previous result obtained in the light-cone gauge formulation [15].

Finally the kappa variations which have both dependence on μ and the position are given by

$$\delta S_{\mathrm{WZ}} \to \int\limits_{\partial\Sigma} dX^\mu\, e^r_\mu \Big[\big(\bar\theta^1 \Gamma^s \delta\theta^1 + \bar\theta^2 \Gamma^s \delta\theta^2\big)\big(\bar\theta^1 \omega_{[r}{}^{-I} \Gamma_{s]} \Gamma^{+I} \theta^1 - \bar\theta^2 \omega_{[r}{}^{-I} \Gamma_{s]} \Gamma^{+I} \theta^2\big) + \frac{1}{2}\big(\bar\theta^1 \Gamma^s \delta\theta^1\big)\big(\bar\theta^2 \omega_r^{-I} \Gamma_s \Gamma^{+I} \theta^2\big) - \frac{1}{2}\big(\bar\theta^2 \Gamma^s \delta\theta^2\big)\big(\bar\theta^1 \omega_r^{-I} \Gamma_s \Gamma^{+I} \theta^1\big)\Big]. \tag{3.12}$$

Note that the position dependence comes from the spin connection ω_r^{-I}. From this action, we need to consider only for $r \in N$ since $dX^\mu e^r_\mu = 0$ for $r \in D$. First term vanishes for $s \in D$ or $I \in N$. However, for $s \in N$ and $I \in D$, it does not vanish and becomes

$$4\big(\bar\theta^1 \Gamma^s \delta\theta^1\big)\big(\bar\theta^1 \omega_{[r}{}^{-I} \Gamma_{s]} \Gamma^{+I} \theta^1\big). \tag{3.13}$$

The remaining terms combine to vanish for $I \in N$ but for $I \in D$ we have

$$\pm\big(\bar\theta^1 \Gamma^s \delta\theta^1\big)\big(\bar\theta^1 \omega_r^{-I} \Gamma_s \Gamma^{+I} \theta^1\big), \tag{3.14}$$

where $+(-)$ sign corresponds to $s \in D(N)$. At this point, we have to impose additional boundary condition

$$\Gamma^+ \theta^1\big|_{\partial\Sigma} = 0. \tag{3.15}$$

This leads to 1/4-BPS.

What would be the physical consequence of the results obtained above? It is well known that the appropriate sigma model of the open string coupled to open string background is given by

$$S = \frac{1}{4\pi\alpha'} \int\limits_{\Sigma} d^2\sigma \sqrt{-h}\, h^{ab} \partial_a X^\mu \partial_b X^\nu - \int\limits_{\partial\Sigma} ds \left(A_\mu \frac{\partial}{\partial s} X^\mu + \phi^i \frac{\partial}{\partial\sigma} X^i \right), \tag{3.16}$$

for bosonic case where X^i s denote the Dirichlet directions. If we consider supersymmetric case, we should consider a suitable supersymmetric generalization of (3.16). For the flat background, such model were considered in [30] in the RNS formalism. Even though the detailed form of the action is not known for the plane wave background, we expect that the one-dimensional boundary theory defined above should have different form for the boundary located away from the origin from that at the origin, since the number of supersymmetries are different. Part of such differences can be captured by the Dirac–Born–Infeld action, which can be derived by the condition of the vanishing beta function of (3.16) or its suitable supersymmetric generalization. Currently this issue is on the investigation [31].

So far we work out the kappa variation up to the quartic terms in θ coordinates. Thus it is interesting to see if the results obtained above are persistent at the higher orders, which we suspect so. Especially for the half BPS branes where the analysis in the light cone gauge is available, the higher terms should not modify the analysis at the quartic order. For the type IIB case, there are some hand waving argument that the quartic results will go through the higher orders [19]. It will be interesting to see if we can find similar argument in the type IIA theory.

4. D-particle

Now we consider the possible constraints on the supersymmetry of the open string where the D-particle boundary condition is given. This case cannot be covered by the lightcone analysis. As a first attempt, we take $P = \Gamma^+$ which means the D-particle whose worldline lies along x^+. Same thing happens for $P = \Gamma^-$. Then even the boundary contribution (3.3) does not vanish. So we consider the boundary condition $\theta^2 = P\theta^1$ with

$$P = \frac{1}{\gamma}\big(\Gamma^+ + \gamma\Gamma^-\big), \tag{4.1}$$

where γ is a real constant. With this boundary condition, one can easily see that Eq. (3.3) vanishes.

Let us turn to the boundary contribution, Eq. (3.7). First look at the term with the structure of $\bar{\theta}\Gamma_{rs}\delta\theta$, the third line in (3.7). It does not vanish when $r \in N$ and $s \in D$, and is proportional to

$$\left(\bar{\theta}^1 \Gamma_{rs} P \delta\theta^1\right)\left[\bar{\theta}^1 \Gamma^s \Gamma^4 \left(\Gamma^+ \theta^1_- + \gamma \Gamma^- \theta^1_+\right)\right], \tag{4.2}$$

where

$$\theta^1_\pm = h_\pm \theta^1. \tag{4.3}$$

We should impose additional boundary conditions as

$$\Gamma^\pm \theta^1_\mp \big|_{\partial\Sigma} = 0. \tag{4.4}$$

The terms in the first line of (3.7) vanishes basically because of the antisymmetric property between the indices r and s, and $r, s \in N$. (If $r \in D$ or $s \in D$, it automatically vanishes.)

The terms in the second line of (3.7) do not vanish. For example, for $s \in N$, that is, $\Gamma^s = P$, they are proportional to, with (4.4),

$$\left(\bar{\theta}^1_+ \Gamma^+ \delta\theta^1_+ + \gamma \bar{\theta}^1_- \Gamma^- \delta\theta^1_-\right)\left(\theta^1_+ \Gamma^4 \Gamma^- \theta^1_- - \theta^1_- \Gamma^4 \Gamma^+ \theta^1_+\right). \tag{4.5}$$

To eliminate this contribution, we should further require boundary condition as

$$\Gamma^\pm \theta^1_\pm \big|_{\partial\Sigma} = 0. \tag{4.6}$$

All other terms with each boundary conditions for indices vanish. For example, the first term vanishes basically because of the antisymmetric property between the indices r and s, and $r, s \in N$. (If $r \in D$ or $s \in D$, it automatically vanishes.)

We see that D-particle is not supersymmetric.

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