Scientific and Technological Experiments on Automatic Space Vehicles and Small Satellites

Estimation of the Spectral Composition of the Signal by the Antenna Composed of Multiple Satellites

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Abstract

The problem of calculation of spectral density of wave process are represented in this work on base of data from antenna array with variable configuration. This antenna array is an aggregate sensing elements moving in space with constant velocities related to laboratory frame system and related one another. The new way of transformation an initial data set obviated difficulties with inhomogeneous Doppler’s shifts in antenna nodes are suggested.

Keywords: multidimensional spectral analysis of signals, spatial-temporary spectra, antenna array with variable configuration, the method of normal principal components, an estimation of spectral densities

1. Introduction

One of the most effective methods of analyzing the characteristics of dynamics of wave processes on the basis of experimental data in a variety of physical systems are methods of spectral analysis of time series and related methods for estimating the space-time spectra using discrete antenna arrays. Such an analysis is used in problems of radio and acoustic location of objects, in the study of wave processes by the remote methods in the atmosphere and ocean, as well as in various problems of astronomy, astrophysics and space physics. One of the main elements of this approach is a stationary antenna array, consisting of a small number of point nodes, which contain sensors that measure the time-varying physical parameter, which is the wave process indicator. Any physical parameter can be an indicator, such as strength of magnetic and electric fields, a temperature and pressure in the environment, etc.

Usually assumes that a discrete antenna array has a set of some properties with high precision. Distances between

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lattice sites are known with highest accuracy. Also assumes that the signals at nodes of the antenna array are measured synchronously with the highest possible precision. Accuracy of finding the angular position signals sources define the error in synchronization measurements. The maximum distance between the lattice sites is called the aperture, and determines the angular resolution of the antenna array. Evaluation wavelength of a harmonic signal at determined frequency and its direction of arrival [1–3] formally based on simple calculating shifts of the phase of the signal between nodes $a, b$ of the antenna array by the system of equations:

$$\left(k, (x_a - x_b)\right) = \Delta\varphi_{ab}(\omega),$$

(1)

where $k = k(\omega)$ – wave vector at a frequency of $\omega$; $x_a, x_b$ – radius-vector of node of antenna array with index $a$; $\Delta\varphi_{ab}(\omega)$ - phase shift between nodes $a$ and $b$.

However, in reality, due to the presence of noise in the environment and various random errors which are perceived as noise, the problem of estimating the wavelengths and their directions at a fixed frequency is complicated and solved by using the procedures of spectral estimation [1, 2].

Problem occurs in many modern tasks when it is necessary to process data sets from sensors that are not constitute components of the antenna array in the original sense and continuously moving in space relative to each other. For example, satellite systems of remote measurement, which compiled of individual satellites, equipped with the same devices. Because of different parameters of individual orbits of each satellite, the distance between them is constantly changing, which leads to changing Doppler shifts, which vary for different pairs of lattice sites. Nowadays there are a lot of satellites at the Earth orbit with similar observational programs and the type of used sensors, for example, meteorological satellites of NOAA and Meteor type geostationary satellites GOES, METEOSAT, etc. However, there is no method to combine data from various satellites in an interferometric data set, which would make it possible to investigate not only the frequency spectra of the waves in the surrounding space but the spatial characteristics of the waves in the form of space-time spectra.

In this paper we propose the way of representing the dynamic antenna array which nodes represent different satellites, as well as a method of estimation of space-time spectra, that allows using data from this lattice. Special transformation of source data can eliminate inhomogeneous Doppler shift in the individual nodes of the dynamic lattice, which moving relative to each other with constant velocities. It is also offered an implementation of computational procedure for the case of discrete time series and the ability to process data in real time.

2. The case of harmonic signals.

First, consider the situation where the harmonic signal is detected by the sensors at the nodes of the antenna array with the frequency $\omega$ and wave vector $k$ in the laboratory reference system. Suppose that the nodes of the antenna array are moving with constant velocity $V_a$, $a = 1, ..., M$ relative to the laboratory reference frame, where the index $a$ corresponds to the node number of the antenna array, the number of which is equal to $M$. Then the frequencies of the measured signal have a Doppler shift at the nodes of the antenna array and respectively equal to:

$$\omega_a = \omega - (V_a, k)$$

(2)

This ratio in some senses is similar to (1). Using set of these relations we can establish some characteristics of a harmonic signal - its frequency, relative to the laboratory frame of reference, and with a sufficient number of nodes of the antenna array, its wave vector. In the case of three-dimensional space we could be completely restore the signal with only four nodes array. The system of equations (2) can be rewritten in the next form:

$$\left(V_a - V_b, k\right) = \omega_a - \omega_b, a < b = 1, ..., M.$$

(3)

For $M = 4$ there are six pairs of equations, and only any three of these equations of the system are linearly independent. These three equations are representing a system of equations for the calculation of components of the wave vector $k = \left(k_x, k_y, k_z\right)$. The rest of equations are will be done automatically. Matrix of the system (3) is non-degenerate only if all vectors $V_a$ are non-collinear in pairs. These properties are similar to conventional phase antenna array relatively on their radius-vectors but not the velocity of the nodes. Therefore, such antenna arrays could be called a Doppler's antenna arrays.
In the presence of noise in the signal and in receiving equipment, the condition of consistency of measurements in the individual nodes may be disturbed, similar to phase antenna arrays. In this case, every three equations (3) will be giving its assessment of the wave vector, i.e. estimation problem becomes incorrect. Due to this problem it needs to construct a correct algorithm that takes into account the noise in the data.

3. The covariance matrix for antenna array with variable configuration

The Doppler shift at each element of the array antenna will have a different meaning. In addition, the phase shifts between the values of the harmonic signal propagating in a fixed environment, relative to the laboratory reference system, in different antenna array elements will change over time.

Let the process under study in the laboratory reference system could be represented in the form of wide-sense stationary process \( u(x,t) \), where \( x \) – Cartesian coordinates at the laboratory reference system, \( t \) – time. The condition of stationarity in the wide sense means that

\[
\langle u(x,t) \rangle = a = \text{const},
\]

\[
\langle u(x,t)u(x',t') \rangle = R(x-x', t-t').
\]

Without loss of generality, we assume that \( a = 0 \). Then this process can be represented in the form of expansion in a Fourier integral:

\[
u(x,t) = \int a(k,\omega) e^{i(k,x) - i\omega t} dk^3 d\omega,
\]

where Fourier components of the process satisfy the conditions:

\[
\langle a(k,\omega) \rangle = 0, \quad S(k,\omega)\delta(k-k')\delta(\omega-\omega') = \langle a(k,\omega) a^*(k',\omega') \rangle.
\]

The function \( S(k,\omega) \) is called the spectral density of the process.

Process values are measured by the sensor at each node of the antenna array with index \( a(1,\ldots,M) \), which moves with the speed \( V_a \), will be given by the following Galilean transformation:

\[
v_a(t) = u(x_a - V_a t, t), \quad a = 1,\ldots,M.
\]

Here we find that \( v_a(t) \) – processes, which can also be represented in the form of Fourier integrals:

\[
v_a(t) = \int a(k,\omega) e^{i(k(x_a - V_a t) - i\omega t)} dk^3 d\omega.
\]

The function of the cross-correlation matrix of these processes in this case will have the form:

\[
R_{ab}(t,t') = \langle v_a(t)v_b(t') \rangle = \int S(k,\omega) e^{i(k(x_a - x_b) - (x_a - x_b)(t - t') - i\omega(t - t'))} dk^3 d\omega.
\]

These relations imply that the processes \( v_a(t) \) are not mutually stationary in a wide sense, because their cross-correlation matrix does not depend only on \( t - t' \), but also on both points of time \( t \) and \( t' \). However, we can use this dependence to build a new estimate of the spectral density \( S(k,\omega) \). For example, the variance and averages of all the processes are remaining independent of time.

4. Formation of spatio-temporal spectrum

We assume that the function of matrix covariance with components \( R_{ab}(t,t') \) can be measured with sufficient accuracy. Then the problem of construction \( S(k,\omega) \) will consist, firstly in searching a suitable inverse to (5) of the
formula expressing \( S(k) \) through \( R_{ab}(t,t') \), and secondly in allowing of keeping the noise in the data, which is typical for various adaptive methods for constructing estimates. Unfortunately in this case, there is no possibility to use the standard spectral analysis methods [4-6]. It’s related to the fact that between each pair of nodes of the array has its own Doppler shift. Therefore, the cross spectrum will contain it, and autospectrum - no. In this case, it needs to use the method of the component, which is a kind of general method of principal normal modes [7].

For this we consider the problem of the eigenvectors and eigenvalues of function of the covariance matrix. Eigenvectors \( \xi^{(A)}_{a}(t) \) must satisfy condition.

\[
\sum_{b=1}^{M} \int R_{ab}(t,t') \xi^{(A)}_{b}(t',\lambda) dt' = \lambda \xi^{(A)}_{a}(t,\lambda). \tag{6}
\]

Superscript \( A(A=1,..,M) \) in \( \xi^{(A)}_{a} \) numbers eigenvectors for a fixed value of the eigenvalue \( \lambda \). Matrix kernel of this integral equation, according to the general theory of Fredholm integral equations can be represented as follows:

\[
R_{ab}(t,t') = \sum_{A=1}^{M} \lambda \xi^{(A)*}_{a}(t,\lambda) \xi^{(A)}_{b}(t',\lambda) d\lambda.
\]

Using (5), we arrive at the following equation:

\[
S(k,\omega)e^{i(k-x_{a})\omega} = \sum_{A=1}^{M} \lambda \xi^{(A)*}_{a}(t,\lambda) \xi^{(A)}_{b}(t',\lambda) d\lambda.
\]

Transforming the left-hand side of equation (6) and multiplying it by \( e^{-i(k-x_{a})\omega} \), in the summation over a and integrating with respect to \( \omega \) we obtain the following integral equation for the function \( \xi^{(A)}_{a}(k,\omega,\lambda) \):

\[
\int S(k',\omega')\Delta(k'-k,\omega'-\omega) \xi^{(A)}(k',\omega',\lambda) dk'^{2}d\omega' = \xi^{(A)}(k,\omega,\lambda), \tag{7}
\]

where

\[
\sum_{a} e^{i(k-x_{a})\omega} \xi^{(A)}(k,\omega,\lambda) = \sum_{b=1}^{M} e^{-i(k-x_{b})\omega} g^{(A)}(\omega+(k,V_{b}),\lambda)
\]

\[
g^{(A)}(\omega,\lambda) = \int e^{i\omega \xi^{(A)}_{b}(t',\lambda)} dt'.
\]

Equation (7) is analogous to (4), which determines the properties of the Fourier components of the wide-sense stationary process. In a sense, the summation over the eigenvalues \( \lambda \) of covariance matrix is equivalent to the expectation.

Equation (7) is an integral equation for the eigenfunctions and eigenvalues. The functions \( \xi(k,\omega,\lambda) \) are eigenfunctions for Hermitian series \( S(k,\omega)\Delta(k'-k,\omega'-\omega) \). Consequently, the kernel can be written as:

\[
S(k',\omega')\Delta(k'-k,\omega'-\omega) = \sum_{A=1}^{M} \lambda \xi^{(A)*}_{a}(k,\omega,\lambda) \xi^{(A)}(k',\omega',\lambda) dV(\lambda). \tag{8}
\]

This ratio serves as a basis for constructing estimates of the spectral density \( S(k,\omega) \).

5. Simple regularization

Equation (8) contains as a factor to the function \( S(k,\omega) \) being evaluated a singular function \( \Delta(k,\omega) \). This requires additional regularization of constructions for finding complete algorithm for constructing the assessment. It can be seen that at point \( k = 0 \) the function \( \Delta(k,\omega) \) has the form:

\[
\Delta(0,\omega) = M \delta(\omega),
\]
where $M$ - the number of antenna array elements.

Thus, instead of (8), we obtain

$$ S(k, \omega) \delta(\omega - \omega') = \frac{1}{M} \sum_{A=1}^{M} \int \lambda \gamma^{(A)r}(k, \omega, \lambda) \gamma^{(A)}(k, \omega', \lambda) d\nu(\lambda). \quad (9) $$

The left-hand side of this equation has exactly the same with the left side (4). But the essential difference (9) of (4) is that the operation of averaging over the ensemble is hidden here in the eigenvectors $\gamma^{(A)}(k, \omega', \lambda)$.

In the expression (9) $\delta$-function on the left is because of the orthogonality of the eigenfunctions $\gamma^{(A)}(k, \omega', \lambda)$ corresponding to different eigenvalues, while in (4), this function arises as a consequence of the properties of Fourier transforms of the wide-sense stationary process. However, the formal similarity between the left sides of (9) and (4) can be used to regularize the estimates based on the formula. Assuming that the complex processes from

$$ \delta(k, x_0 - V \nu) \zeta^{(A)}(t, \lambda) \quad (10) $$

are themselves random stationary processes in a wide sense for each $k, A, \lambda$, then in this case there is a standard Wiener-Khintchine:

$$ \langle \gamma^{(A)r}(k, \omega, \lambda) \gamma^{(A)}(k, \omega', \lambda) \rangle = S^{(A)}(k, \omega, \lambda) \delta(\omega - \omega'), \quad (11) $$

where $S^{(A)}(k, \omega, \lambda)$ - spectral densities of processes $\chi^{(A)}(k, t, \lambda)$.

Substituting (11) into (9) and comparing the right and left sides, we obtain

$$ S(k, \omega) = \frac{1}{M} \sum_{A=1}^{M} \int \lambda S^{(A)}(k, \omega, \lambda) d\nu(\lambda). \quad (12) $$

This means that for the construction of the spectral density of the process under investigation it is necessary to calculate the spectral density of the individual one-dimensional random processes (10), for each $A, \lambda$ and $k$, which are linear combinations of vector-functions of the covariance matrix function. The sum of such spectral densities of all the eigenvalues of the covariance matrix at a given wave number $k$ and it would be the final formula for the spectral density of the original process. Since relations are exact, provided stationary in the wide sense of the original field $u(x, t)$ and consistency $V_a$, the formal averaging in (11) is unnecessarily. However, when using this approach for the construction of spectral density estimates for the real data, this averaging is necessary because of the appearance of an additional stochastic eigenvectors of matrix associated with noise in the receivers, the limited sample, the fluctuations of the velocity sensors and rounding errors in calculations.

6. Transition to the discrete series

Now we consider the discrete analogue of the obtained estimates. In the case of a discrete series time steps $\Delta t$ the covariance matrix can be written as:

$$ R_{nm} = R_{ab}(n\Delta t, m\Delta t), \quad n, \quad m = 0, \ldots , L, $$

where $L$ - the number of time steps for which covariance matrix estimated.

As a result, the relation (5) can now be rewritten as:

$$ R_{nm} = \int_{-1/2}^{1/2} \int S(k, f) e^{i(k [x_n - x_m] - \Delta t (V_n - V_m)) - i2\pi f (n - m)/L} \, dk \, df, \quad (13) $$

where $f = \omega \Delta t / \left(4\pi (L + 1)\right)$ - normalized dimensionless frequency; values $f_n = \pm 1/2$ correspond to the Nyquist frequency of the discrete process.

Equation for the eigenvectors of this matrix can be written in the following form:
\[
\sum_{b=1}^{M} \sum_{m=0}^{L} R_{ab}^{mn} \xi_{m,b}^{(A)}(q) = \lambda_{A,q}^{(A)}(q).
\]

(14)

For computational convenience \( R_{ab}^{mn} \) can be represented as a square matrix of dimensions \( M \times (L + 1) \), consisting of blocks. Blocks are numbered by indexes \( a, b = 1, \ldots, M \), and the numbering of elements inside the blocks - indexes \( n, m \). In this case, (10) does not differ from the task on the eigenvectors obtained symmetric positive definite square matrix \( R \). According to the general properties of this matrix, we have

\[
R_{ab}^{mn} = \sum_{A,q} \lambda_{A,q}^{(A)}(q) \xi_{m,b}^{(A)}(q).
\]

Substituting (13) into (14), and then multiplying the resulting value by the \( e^{-i(k,v)n/L} \) summing by \( a \) and \( l \), we obtain an analogue of (7):

\[
\int_{-1/2}^{1/2} \int_{-1/2}^{1/2} S(k', f') \Delta(k' - k, \omega' - \omega) \xi^{(A,q)}(k', f') dk' df' = \lambda_{A,q}^{(A,q)}(k, \omega),
\]

where

\[
\xi^{(A,q)}(k, f) = \sum_{n=0}^{L} \sum_{a=1}^{M} \xi_{n,b}^{(A)}(q) e^{-i(k,v)n/L} e^{2i\pi fn/L}.
\]

\[
\Delta(k, f) = \sum_{n=0}^{L} \sum_{a=1}^{M} e^{i(k,v)n/L} e^{2i\pi fn/L}.
\]

Because the kernel of this integral equation is Hermitian, we have the relation

\[
S(k', f') \Delta(k' - k, \omega' - \omega) = \sum_{A,q} \lambda_{A,q}^{(A,q)}(k', f') \xi^{(A,q)}(k, f).
\]

When \( k = k' \) and \( f = f' \)

\[
\Delta(0, 0) = ML.
\]

As a result, we finally find:

\[
S(k, f) = \frac{1}{ML} \sum_{A,q} \lambda_{A,q}^{(A,q)}(k, f) \xi^{(A,q)}(k, f).
\]

(15)

Using the idea of an additional averaging this formula by analogy with (12) it can be written as

\[
S(k, f) = \frac{1}{ML} \sum_{A,q} \lambda_{A,q}^{(A,q)}(k, f) \xi^{(A,q)}(k, f).
\]

(16)

7. Algorithm for constructing estimates

The main difficulty in the implementation of such approach to the construction \( \hat{S}(k, f) \) of the spectral density estimate based on the finite series of measurements consists in finding a sufficiently reliable estimate covariance matrix \( \hat{R}_{ab}^{mn} \) of a sufficiently large number of time shifts. For this could be use the natural method of estimating the covariance averaging time on the basis of a sequence of segments of fixed length starting with the beginning of the series shifted by the same number of time samples.

Algorithm for computing the estimates \( \hat{S}(k, f) \) on the basis of (16) should consist of the following steps:

1) Construction estimates for the covariance matrix \( \hat{R}_{ab}^{mn} \) by a discrete set of synchronous data for nodes of the array;

2) Calculation of the eigenvectors and eigenvalues of the covariance matrix;
3) Calculating estimates of the Fourier transform $\tilde{\zeta}^{i,j}(k, f)$ of the eigenvectors covariance matrix;

4) Construction their own assessment $\hat{S}(k, f)$ on the basis of the ratio (16).

The construction of the covariance matrix estimation $\hat{R}_{ab}^{m,n}$ can be carried out with the replacement of averaging over the ensemble averaging over time, namely, let $x_i^{(a)}$, $i = 1, ..., N$, $a = 1, ..., M$ - data set from M nodes of the antenna array at equidistant synchronous timing $t_n = n\Delta t$. For each node of the antenna array of a series of $X_i^{(a)}$ we create a set of $\left( L + 1 \right)$ series long $N_i = N - L$ by the rule [8]:

$$u_{i,n}^{(a)} = x_i^{(a)} - X_n^{(a)}, n = 0, ..., L; i = 0, ..., N - L,$$

where

$$X_n^{(a)} = \frac{1}{N - L + 1} \sum_{i=0}^{N-L} x_i^{(a)}$$

represent the average values of the individual segments of the original series.

Then the estimate covariance matrix is constructed by

$$\hat{R}_{ab}^{m,n} = \frac{1}{N - L} \sum_{i=0}^{N-L} u_{i,n}^{(a)} u_{i,m}^{(a)}$$

(17)

Estimation of the covariance matrix can be represented as a square symmetric block matrix $R$ of dimensions $K \times K$, where $K = (L \times M)$:

$$R = \begin{pmatrix}
\hat{R}_{11} & \hat{R}_{12} & \ldots & \hat{R}_{1L} \\
\hat{R}_{21} & \hat{R}_{22} & \ldots & \hat{R}_{2L} \\
\vdots & \vdots & \ddots & \vdots \\
\hat{R}_{L1} & \hat{R}_{L2} & \ldots & \hat{R}_{LL}
\end{pmatrix},$$

(18)

here blocks of $\hat{R}_{mm}$ are matrices of dimension $5 \times 5$. $M \times M$.

Solving the problem on eigenvalues and eigenvalues of this matrix

$$R\xi^\mu = \lambda_{(a)}^\mu \xi^\mu, \mu = 1, ..., K,$$

obtain the eigenvectors of this matrix in the form of vectors of dimension $K$ with components:

$$\xi^\mu = \text{column} \begin{bmatrix} \xi_1^\mu, \ldots, \xi_M^\mu, \xi_{M+1}^\mu, \ldots, \xi_{2M}^\mu, \ldots, \xi_{(L-1)M+1}^\mu, \ldots, \xi_{LM}^\mu \end{bmatrix}.$$

Function $\zeta^\mu(k, f)$ is calculated as follows:

$$\zeta^\mu(k, f) = \sum_{j=0}^{L} \left[ \sum_{a=1}^{M} \xi_{aj}^\mu e^{-i(kx_a)L} \Delta(k, V_j)j/L \right] e^{2i\pi fj/L}.$$

Final evaluation of the spectral density is calculated by the formula

$$\hat{S}(k, f) = \frac{1}{ML} \sum_{\mu=1}^{K} \lambda_{(a)}^\mu \left| \langle \zeta^\mu(k, f) \rangle \right|^2.$$

To construct estimates of the spectral densities

$$\hat{S}^{(i)}(k, f) = \left| \langle \zeta^{(i)}(k, f) \rangle \right|^2$$

we can use the well-developed methods of spectral density estimation of time series based on the maximum entropy method or any equivalent method (eg, [3-5, 7]).
8. Conclusion

The proposed method provides a real opportunity for constructing estimates of the spectral density of the initial wave process on the basis of data from the sensors, moving relative to each other with constant speed. As shown, such a procedure relies on three major assumptions. The first assumption concerns the requirements of stationarity in the wide sense of the original process, which is quite natural for the methods of spectral estimation. The second assumption regards the requirement of determinacy, or at least wide-sense stationary auxiliary processes \( \gamma_X(k, t, \lambda) \). This hypothesis requires further study, but we can hope that it will be the case for a broad class of initial processes. The last assumption is that by means of finite series can construct a satisfactory estimate of the covariance matrix \( R_{ab}(m, n) \). This is determined by the properties of the initial series of measurements and requires study in each case. It can be concluded that the method will work in a rather general class of conditions and problems. But due to certain features of the new method, which is not typical for the conventional method of spectral density estimation, associated with the continuous change of the bases node of the antenna array and its aperture, for example, it takes some work to clarify its resolution, which will be clarified in the test problems in the near future.

Also, it is worth noting that the most promising field of application of this approach is the study of wave processes in space on the basis of data from various combined sensors at different spacecrafts. At present there are several satellite clusters, the data from which can be processed by the proposed method.

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