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Procedia Engineering 15 (2011) 2495 – 2499

Procedia Engineering

www.elsevier.com/locate/procedia

Advanced in Control Engineering and Information Science

Self-tuning fusion Wiener filter for multisensor multi-channel AR signals with common disturbance noise

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Abstract

For the multisensor multi-channel autoregressive (AR) signals with common disturbance noise, when model parameters and noise variances are unknown, the estimates of model parameters and noise variances can be obtained based on the multi-dimension recursive extended least squares (RELS) algorithm and the correlation method. Further, a self-tuning fusion Wiener filter is presented based on the modern time series analysis method by substituting the estimates for the true values. A simulation example shows the consistence of the estimates of the model parameters and noise variances, and the tracking characteristics of the self-tuning fusion Wiener filter.

Selection and/or peer-review under responsibility of [CEIS 2011]

Keywords: Self-tuning; multisensor; multi-channel AR signals; common disturbance.

1. Introduction

The term "filtering" originates from radio field, and its meaning is to filter out the noise and restore the original state or signal. The optimal filters mentioned above are effective only when the model parameters and noise variances are known. In many practical applications, the model parameters and noise variances are often completely or partly unknown, so many results about the self-tuning signal filter based on mlutisensor have been obtained, but most of them are just for the single-channel signals [1,2] or for the multi-channel signals with independent noises [3,4]. In this paper, for solving the problem of the more complex multi-channel signal filtering, the multi-dimension recursive extended least squares (RELS) [5] algorithm and the correlation method [6] are applied to obtain the unknown model parameters and noise variances. Further, a self-tuning fusion Wiener filter is presented based on the modern time series analysis method by substituting the estimates for the true values. It is obvious that the estimates of the model

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doi:10.1016/j.proeng.2011.08.469

parameters and noise variances converge to their true values, and the self-tuning information fusion Wiener filter can track the signal well.

2. Problem Formulation

Consider the multisensor multi-channel autoregressive (AR) signals with common disturbance noise

$$A(q^{-1})s(t) = w(t-1) \tag{1}$$

$$y_i(t) = s(t) + \xi(t) + e_i(t), i = 1,\cdots,L \tag{2}$$

where $s(t)\in R^m$ is the signal to be estimated, $y_i(t)\in R^m$ is the measurement of the ith sensor, $w(t)\in R^m, \xi(t)\in R^m, e_i(t)\in R^m$ are uncorrelated white noises with zero mean and variances Q_w, Q_ξ, Q_{ei}, respectively. q^{-1} is the backward shift operator, $A(q^{-1})$ is a stable polynomial with form $A(q^{-1}) = I_m + A_1 q^{-1} + \cdots + A_n q^{-n}$, I_m denotes the $m\times m$ unite matrix.

The problem is to find the self-tuning fusion Wiener filter $\hat{s}_0^s(t\,|\,t)$ of $s(t)$ weighted by scalar when Q_w, Q_ξ, Q_{ei} and $A(q^{-1})$ are unknown.

3. Estimates of Model Parameters and Noise Variances

From (1) and (2), an ARMA innovation model can be obtained:

$$A(q^{-1})y_i(t) = D_i(q^{-1})\varepsilon_i(t) \tag{3}$$

where $D_i(q^{-1}) = D_{i0} + D_{i1}q^{-1} + \cdots + D_{in}q^{-n}$ is stable (i.e. all zeros of $D_i(x)$ lie outside the unit circle), $D_{i0} = I_m$, innovation process $\varepsilon_i(t)\in R^m$ is white noise with zero mean and variance $Q_{\varepsilon i}$, and

$$D_i(q^{-1})\varepsilon_i(t) = w(t-1) + A(q^{-1})e_i(t) + A(q^{-1})\xi(t) \tag{4}$$

$D_i(q^{-1})$ and $Q_{\varepsilon i}$ can be obtained by Gevers-Wouters iterative algorithm [7]. For the ARMA innovation model (3), applying the multi-dimension RELS algorithm, the local estimates $\hat{A}_{ij}, \hat{D}_{ij}$ and $\hat{Q}_{\varepsilon i}$ at time t can be obtained, $i = 1\cdots L, j = 1,\cdots,n$. Then the fused estimate $\hat{A}_j$ of A_j is defined as

$$\hat{A}_j = \frac{1}{L}\sum_{i=1}^{L}\hat{A}_{ij}, j = 1,\cdots,L \tag{5}$$

And it has been proved [5] that the RELS estimator of the ARMA innovation model parameters is strongly consistent, i.e. $\hat{\theta}\to\theta, \hat{D}_{ij}\to D_{ij}, \hat{Q}_{\varepsilon i}\to Q_{\varepsilon i}$, as $t\to\infty$, w.p.1, where $\theta = [A_1 \ \cdots \ A_n]^{\mathrm{T}}$.
Introduce a new measurement $r_i(t)$ as $r_i(t) = A(q^{-1})y_i(t)$, thus from (3) and (4), we have

$$r_i(t) = w(t-1) + A(q^{-1})e_i(t) + A(q^{-1})\xi(t) \tag{6}$$

So it is evident that $r_i(t)$ is a stationary stochastic process with correlation function $R_{rij}(k) = \mathrm{E}[r_i(t)r_j^{\mathrm{T}}(t-k)]$, $i, j = 1,\cdots,L$, where E denotes the mathematical expectation. Computing the correlation function of the stochastic processes of two sides for (6), we obtain that

$$R_{rij}(0) = Q_w + \delta_{ij}\sum_{\alpha=0}^{n}A_\alpha Q_{ei}A_{\alpha-k}^{\mathrm{T}} + \sum_{\alpha=0}^{n}A_\alpha Q_\xi A_{\alpha-k}^{\mathrm{T}};\ R_{rii}(n) = A_n Q_{vi} + A_n Q_\xi,\ \ R_{rij}(n) = A_n Q_\xi,\ i\neq j \tag{7}$$

where $\delta_{ii} = 1, \delta_{ij} = 0 (i\neq j)$. At time t, based on the measurement processes $r_i(t), r_i(t-1),\cdots$, the sampled correlation function $R_{rij}^t(k)$ has the recursive formula $R_{rij}^t(k) = R_{rij}^{t-1}(k) + \frac{1}{t}(r_i(t)r_j^{\mathrm{T}}(t-k) - R_{rij}^{t-1}(k))$.

Defining the estimator of $r_i(t)$ as $\hat{r}_i(t) = \hat{A}(q^{-1})y_i(t)$, then the estimate of the sampled correlation function $R_{rij}^t(k)$ can be derived. Substituting the estimates $\hat{R}_{rij}^t(k)$ and $\hat{A}(q^{-1})$ into (7) yields the estimators of the noise variances at time t as

$$\hat{Q}_\xi = \frac{2}{L(L-1)}\sum_{i,j=1}^{L}\hat{A}_n^{-1}\hat{R}_{rij}^t(n),\quad \hat{Q}_{ei} = \hat{A}_n^{-1}(\hat{R}_{rii}^t(n) - \hat{A}_n\hat{Q}_\xi),\quad i = 1,\ldots,L, i \neq j \tag{8}$$

$$\hat{Q}_w = \frac{1}{L\times L}\sum_{i,j=1}^{L}(\hat{R}_{rij}^t(0) - \delta_{ij}\sum_{\alpha=0}^{n}\hat{A}_\alpha\hat{Q}_{ei}\hat{A}_{\alpha-k}^{\mathrm{T}} - \sum_{\alpha=0}^{n}\hat{A}_\alpha\hat{Q}_\xi\hat{A}_{\alpha-k}^{\mathrm{T}}) \tag{9}$$

4. Self-tuning Fusion Wiener Filter

The AR signal system (1) and (2) can be transformed into state space model as

$$x(t+1) = \Phi(\theta)x(t) + \Gamma w(t) \tag{10}$$

$$y_i(t) = Hx(t) + v_i(t), v_i(t) = \xi(t) + e_i(t) \tag{11}$$

$$s(t) = Hx(t) \tag{12}$$

with the following block companion form $\Phi(\theta) = \begin{bmatrix} -A_1 & & \\ \vdots & & I_{(n-1)m} \\ -A_n & 0 & \cdots & 0 \end{bmatrix}$, $\Gamma = \begin{bmatrix} I_m \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, $H = [I_m \quad 0 \quad \cdots \quad 0]$. $w(t)$ and $v_i(t)$ are white noises with zero means and variances with the relationship

$$\mathrm{E}\left\{\begin{bmatrix} w(t) \\ v_i(t) \end{bmatrix}[w^{\mathrm{T}}(k) \quad v_j^{\mathrm{T}}(k)]\right\} = \begin{bmatrix} Q_w & 0 \\ 0 & R_{ij} \end{bmatrix}\delta_{tk}, \text{ where } R_{ii} = Q_\xi + Q_{vi}, R_{ij} = Q_\xi, i \neq j \tag{13}$$

When θ is known, we denote $\Phi(\theta) = \Phi$, $x(t) = [x_1(t),\ldots,x_n(t)]^{\mathrm{T}}$, $x_i(t) \in R^m$. Substituting the estimates for the true values, the ith sensor subsystem has the local self-tuning Wiener filter of the state $x(t)$ as [8]

$$\hat{\psi}_i(q^{-1})\hat{x}_i^s(t\,|\,t) = \hat{K}_i(q^{-1})y_i(t), i = 1,\ldots,L \tag{14}$$

$$\hat{\psi}_i(q^{-1}) = \det(I_{mn} - q^{-1}\hat{\Psi}_{fi}),\ \hat{K}_i(q^{-1}) = adj(I_{mn} - q^{-1}\hat{\Psi}_{fi})\hat{K}_{fi} \tag{15}$$

$$\hat{\Psi}_{fi} = [I_{mn} - \hat{K}_{fi}H]\Phi(\hat{\theta}),\ \hat{K}_{fi} = \begin{bmatrix} H \\ H\Phi(\hat{\theta}) \\ \vdots \\ H\Phi^{\beta_i-1}(\hat{\theta}) \end{bmatrix}^{+}\begin{bmatrix} I_m - \hat{R}_{ii}/\hat{Q}_{\varepsilon i} \\ \hat{M}_{i1} \\ \vdots \\ \hat{M}_{i,\beta_i-1} \end{bmatrix} \tag{16}$$

where $\hat{\Psi}_{fi}$ is a stable matrix, $\hat{K}_{fi}$ is the filter gain, the pseudo-inverse of matrix X is defined as $X^+ = (X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}$, and $\hat{M}_{ij}$ can recursively be computed as $\hat{M}_{ij} = -\hat{A}_1\hat{M}_{i,j-1} - \cdots - \hat{A}_n\hat{M}_{i,j-n} + \hat{D}_{ij}$ with $\hat{M}_{ij} = 0 (j<0), \hat{M}_{i0} = I_m$. The local self-tuning filtering error cross-covariance $\hat{P}_{ij}(t) = \hat{P}_{ij}$ at time t satisfy the Lyapunov equations $\hat{P}_{ij}(t) = \hat{\Psi}_{fi}\hat{P}_{ij}(t-1)\hat{\Psi}_{fj}^{\mathrm{T}} + [I_{mn} - \hat{K}_{fi}H]\Gamma\hat{Q}\Gamma^{\mathrm{T}}[I_{mn} - \hat{K}_{fj}H]^{\mathrm{T}} + \hat{K}_{fi}\hat{R}_{ij}\hat{K}_{fj}^{\mathrm{T}}$, $i, j = 1,\ldots,L$.
From (12) and (14), applying the projective theorem, the ith sensor subsystem has the local self-tuning Wiener signal filter

$$\hat{\psi}_i(q^{-1})\hat{s}_i^s(t\,|\,t) = H\hat{K}_i(q^{-1})y_i(t), i = 1,\ldots,L \tag{17}$$

And the self-tuning filtering error cross-covariance of $s(t)$ can be given by $\hat{P}_{sij} = H\hat{P}_{ij}H^{\mathrm{T}}$. Then the self-tuning fused Wiener filter of $s(t)$ weighted by scalar is obtained by

$$\hat{s}_0^s(t\,|\,t)=\sum_{i=1}^{L}\hat{\omega}_i\hat{s}_i^s(t\,|\,t) \tag{18}$$

where the self-tuning weighting vector $\hat{\omega}=[\hat{\omega}_1,\cdots,\hat{\omega}_L]$ is given by $\hat{\omega}=[e^{\mathrm{T}}\hat{P}_{str}^{-1}e]^{-1}e^{\mathrm{T}}\hat{P}_{str}^{-1}$, $e^{\mathrm{T}}=[1,\cdots,1]$, $\hat{P}_{str}$ is a $L\times L$ matrix whose the (i,j)th element is $tr\hat{P}_{sij}$. The fused error variance is given by $\hat{P}_{s0}=\sum_{i=1}^{L}\sum_{j=1}^{L}\hat{\omega}_i\hat{\omega}_j\hat{P}_{sij}$, and $tr\hat{P}_{s0}\le tr\hat{P}_{sii}$. When the measurement data $y_i(t)$ is bounded for each sensor i , it can be easily proved [1] that the self-tuning information fusion Wiener filter $\hat{s}_0^s(t\,|\,t)$ has asymptotic optimality.

5. Simulation Example

Consider the multisensor multi-channel AR signals with common disturbance noise as (1) and (2)

$$(I_2+A_1q^{-1})s(t)=w(t-1) \tag{19}$$

$$y_i(t)=s(t)+\xi(t)+e_i(t),i=1,2,3 \tag{20}$$

In this simulation, we take the unknown model parameters and noise variances as

$A_1=\begin{bmatrix}0.9 & 0.5\\ -0.8 & 0.3\end{bmatrix}$, $Q_w=\begin{bmatrix}2 & 0\\ 0 & 4\end{bmatrix}$, $Q_\xi=\begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix}$, $Q_{e1}=\begin{bmatrix}0.6 & 0\\ 0 & 0.3\end{bmatrix}$, $Q_{e2}=\begin{bmatrix}0.3 & 0\\ 0 & 0.5\end{bmatrix}$, $Q_{e3}=\begin{bmatrix}0.4 & 0\\ 0 & 0.1\end{bmatrix}$.

The convergences of the model parameters and noise variance estimates are shown in Fig 1 and Fig 2, where the straight lines denote the true values, the curves denote the fusion estimates. It is obvious that these estimates have the consistency. In Fig 3, the solid lines denote the signal $s(t)$, the dotted lines denote the self-tuning fusion Wiener filter $\hat{s}_0^s(t\,|\,t)$, it yields that the self-tuning fusion Wiener filter can track the signal well.

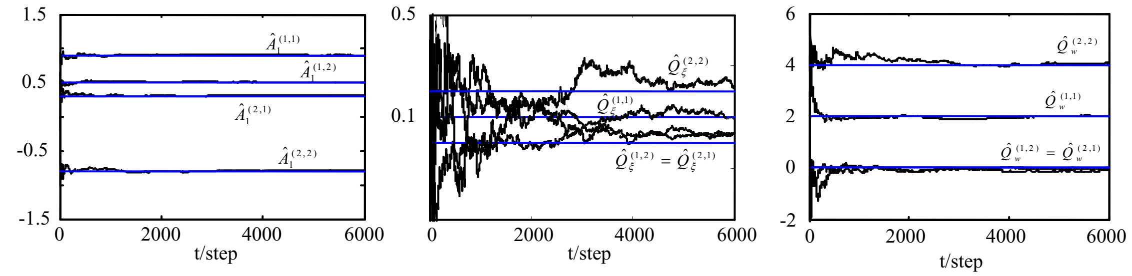

Fig. 1. (a) the estimate of A_1 ; (b) the estimate of Q_ξ ; (c) the estimate of Q_w

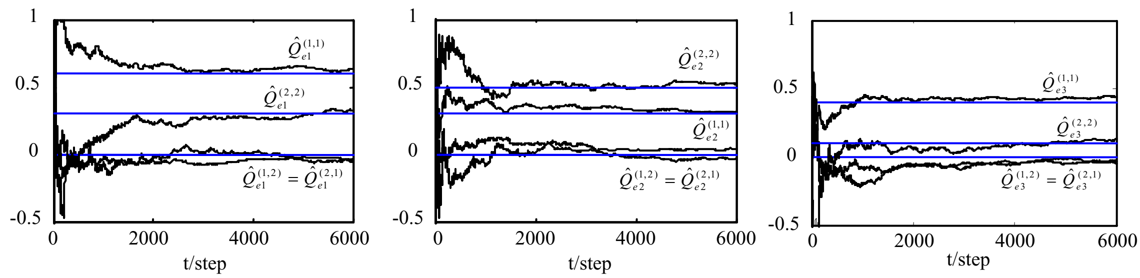

Fig. 2. (a) the estimate of Q_{e1} ; (b) the estimate of Q_{e2} ; (c) the estimate of Q_{e3}

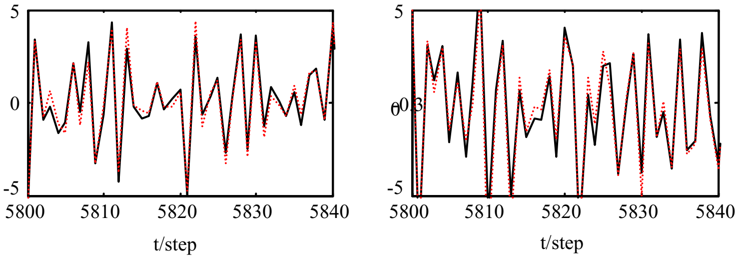

Fig. 3. (a) the first component of signal $s(t)$ and its self-tuning fusion Wiener filter; (b) the second component of signal $s(t)$ and its self-tuning fusion Wiener filter

6. Conclusion

For the multisensor multi-channel AR signals with common disturbance noise, when the model parameters and noise variances are unknown, the information fusion estimates of the model parameters and noise variances are obtained by the multi-dimension RELS algorithm and the correlation method. The so called information fusion estimates mean taking the average of all local estimates. Then a self-tuning fusion Wiener filter with asymptotical optimality is presented based on the modern time series analysis method, which can track the signal well.

Acknowledgements

The authors thank to the Support Program for Young Professionals in Regular Higher Education Institutions of Heilongjiang Province (1155G13).

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