1st International Conference on Applied Economics and Business, ICAEB 2015

Developing a New Algorithm for Finding the Local Minimums of the Multi-Echelon Inventory Control Systems with Random Parameters

Sayyed Mohammad Reza Davoodi \(^a\), Fariborz Jolai\(^b\)
Ali Mohaghar\(^c\), Mohammad Reza Mehregan\(^c\)

\(^a\) Ph.D. Student, Department of Industrial Management, Kish International Campus, Tehran University, Kish Island, Iran
\(^b\) Department of Industrial Engineering, University of Tehran
\(^c\) Department of Industrial Management, University of Tehran

Abstract

The present study aimed to develop a new simulation-based algorithm for finding the local minimums of multi-level inventory control systems with random parameters. The optimization refers to minimization of cost function along with maximization of customer service level of the units. In developing the algorithm, the authors were determined to achieve a local optimum point through linear localization of limitations and Genetic Algorithm. Since point estimations of goal function and repletion rates have been done through Monte Carlo Simulation Technique, the statistical test have been employed for examining possibility and improvability of solutions. Finally, the proposed algorithm has been used in an example with three levels.

© 2016 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
Peer-review under responsibility of SCIJOUR-Scientific Journals Publisher

Keywords: Supply Chain Management; Simulation-based Optimization; Test of Statistical Hypotheses; Local Optimization

1. Introduction

Optimization of the inventory control systems is considered as one of the most important issues in the supply chain management. The main purpose of classic models of inventory control systems was optimization of system gives certain conditions and presumptions. The multi-level inventory system is a generalized version of classic models in

* Corresponding author. Tel.: +989132290367
E-mail address: smrdavoodi@ut.ac.ir

2212-5671 © 2016 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
Peer-review under responsibility of SCIJOUR-Scientific Journals Publisher
doi:10.1016/S2212-5671(16)30036-3
which different units such as retailer or wholesaler cooperating in a given chain. The main challenge of inventory management system is optimization of order time and quantity given the system costs. The studies of multi-level inventory control systems were started from 1950s. Clark and Scarf (1960) are one of the most famous authors in this area. Deuermeyer and Schwarz (1981), Graves (1985), Axsater (1990), Axsater (2002), Axsate (2006), Cachon (2001), Amiri and Seifbarghy, Olfat and Razavi Hajiagha (2012), Gao and Wang (2008), Kochel and Nielander (2005), Sherbrook (1968), and Ivanov, Dolgui, and Sokolov (2012) are the main authors that have entered the random parameters such as customer to the model.

Supply chain activities are so complex that different mathematical methods are needed for model simplification. This is why supply chain management models have some adaptability with reality and such methods have not efficiency and favorability. In addition, such models require utilization of simulation methods. As a result, the combinative methods (both simulation and optimization) have been developed rapidly. In this regard, Schwartz, Wang, and Rivera (2006) combine simulation methods and random control systems. Also Chu, You, and Wassick (2014), Chu, You, Wass1ck, and Agarwal (2014), Jung, Blau, Pekny, Reklaitis, and Eversdyk (2008), Jung, Blau, Pekny, Reklaitis, and Eversdyk (2004), Almeder, Preusser, and Hartl (2009), and Nikolopoul and Ierapetritou (2012) combine simulation and programming methods. Mele, Guillen, Espuna, and Puigjaner (2006) and Silva, Sousa, Runkler, and Dacosta (2006) combine simulation methods and hyper-creative algorithms such Colony and Genetic algorithms. Chu, You, Wasslck, and Agarwal (2014) attempt to localize goal function and limitations of multi-level inventory control systems problem through simulation methods. The above-mentioned algorithm is employed in the present study.

2. Simulation of multi-level inventory control system

The distribution network, which has been studied in the present study, is a divergent network in which there is a factory with extreme inventory. The final layer of network faces with customer demand that its quantity follows an especial distribution function. If a customer order cannot be satisfied, it will be satisfied as soon as possible. The final units not only respond the customer order, but also they satisfy demand of the lower units. In the simulation model of present study, the units respond the lower unit needs and then satisfy the customer needs. Time gap between satisfaction of customer order by the unit and satisfaction of customer order by the higher unit is a random variable. It is known as processing time. If the higher unit cannot satisfy the customer needs, time gap will be increased. In the studied network, all of the units follow the policy of continuous inventory control. The network is shown in Fig 1.

![Fig. 1. two-Echelon distribution network](image)

In the above model, all units have both maintenance and order costs. Therefore, cost function can be shown as equation 1.
\[
INV\text{COST} = \sum_{i=1}^{N_F} \sum_{t=1}^{N_T} C_i HH_{i\text{it}} + \sum_{i=1}^{N_F} \sum_{t=1}^{N_T} C_i R_{i\text{it}}
\]

In the above model, “NT” is the number of simulation days and “NF” is the number of network units (except factory). Also “CHi” is maintenance cost per every product. In the model, “CRI” is order cost of every product and “IOHit” is inventory level of end of day. In this regard, SOR (0, 1) is satisfaction or dissatisfaction of the order. In the present study, the customer service is defined as equation 2.

\[
\forall i : FR_i = \frac{SFD_i}{SRD_i}
\]

In the above formula, “FRi” is repletion rate of unit “i”; “SFD"i is total order that is satisfied in the unit “i” in the simulation period. In the model, “SRDi" is total order that is satisfied by unit “i” in the simulation period.

Suppose in a given network with “NF” units, “RI” is order point of unit “i” and “Qi” is quantity of order of unit “i”. Now, the purpose is to find the following vector(equation 3)

\[
X = [R_1, R_2, ..., R_N, F, Q_1, Q_2, ..., Q_N, F]
\]

The above vector attempts to minimize the cost function and maximize the repletion rate of units. Equations 4,5 define this concept.

\[
\forall i : FR_i \geq f_i^{\min}
\]

In which:

\[
\theta = \{od_1, ..., od_{N_F}, N_T, ot_1, ..., ot_{N_T, N_T}\}
\]

In this regard, “otit” is the number of daily orders of customers in the unit “i” in day “t”. Accordingly, when functions of \(g_1, g_2, ..., g_N\) is available that(equation 6)

\[
INV\text{COST} = f(X, \theta)
\]

\[
\forall i : FR_i = g_i(X, \theta)
\]

After introduction of equation 7 and 8 Mathematical form of the optimization model will be as equation 9.

\[
\Phi(x) = E_{\theta}(f(x, \theta))
\]

\[
\forall i : \psi_i(X) = E_{\theta}(SFD_i(X, \theta)) / E_{\theta}(SRD_i(X, \theta))
\]

\[
\min \Phi(X) \text{ INVOPT}
\]

\[
\text{S.t}
\]

\[
\forall i : \psi_i(X) \geq f_i^{\min}
\]

In which “x” is a positive vector.
In order to estimate $\psi_i(x)$ and $\Phi(x)$, Monte Carlo Simulation Technique is used. In this technique, we need a random sample like to the equation 10.

$$\theta^r = \{\theta^{(r,1)}, \theta^{(r,2)}, \ldots, \theta^{(r,MC)}\}$$

$$r = 1, 2, \ldots, N^R$$

(10)

In the above formula, “Nmc” is sample size and $\theta^{(r,i)}$ is a matrix with “NT” columns. The simulation program examines $NT$ networks in “Nmc” times for conducting the estimations (equation 11, 12, 13, 14, 15)

$$f^{(r,s)}(x) = f(x, \theta^{(r,s)})$$

(11)

$$SFD_i^{(r,s)} = SFD_i(x, \theta^{(r,s)})$$

(12)

$$SRD_i^{(r,s)} = SRD_i(x, \theta^{(r,s)})$$

(13)

$$\hat{\Phi}(x) = \frac{1}{N_{MC}} \sum_{s=1}^{N_{MC}} f^{(r,s)}(x)$$

(14)

$$\forall i: \hat{\psi}^f_i(x) = \frac{\sum_{s=1}^{N_{MC}} SFD_i^{(r,s)}(x)}{\sum_{s=1}^{N_{MC}} SFR_i^{(r,s)}(x)}$$

(15)

Suppose that “$X^P$” is a possible point in the INVOPT model in which cost and rate of repletion is satisfied through sample “r”. In the next section, an algorithm will be introduced that its goal is to find possible point of “$X^{P+1}$” of “$X^P$” for decreasing cost. Now, suppose that the algorithm is solved and repletion rate is calculated through sample r+1 (Equation 16).

If $\hat{\Phi}^{r+1}(X^{P+1}) < \hat{\Phi}(x^P)$, $\forall i: \hat{\psi}^f_i(x^{P+1}) \geq f_{i}^{\min}$

(16)

We can conclude that $X^{P+1}$ are better than $X^P$. According to hypothesis 17 we have:

$$\Phi(x^{P+1}) < \Phi(x^P), \forall i: \psi_i(x^{P+1}) \geq f_{i}^{\min}$$

(17)

Since we deal with samples, difference may not be significant. Indeed, difference may be random. As a result, we face hypotheses 18, 19, 20, 21.

$$H_0: \forall i \psi_i(X^{P+1}) \geq f_{i}^{\min}$$

(18)

$$H_1: \exists i \psi_i(X^{P+1}) < f_{i}^{\min}$$

(19)
And

\[ H_0 : \Phi(X^{P+1}) \leq \Phi(X^P) \]

\[ H_1 : \Phi(X^{P+1}) > \Phi(X^P) \] (20)

According to Chu, You, Wassle, and Agarwal (2014), in the first hypothesis testing, when \( H_0 \) will be accepted that \( \forall i : \hat{\psi}_i(x) \geq f_{r_i}^{\text{min}} + d_p \) is significant in \( dp_i = sp_i : t_{1-\alpha(N_{\text{MC}}-1)} \) and significance level can be calculated through formula 22.

\[
\forall i \quad sp_i = \frac{1}{N_{R} - 1} \sum_{r=1}^{N_{R}} \frac{\sum_{i=1}^{N_{R}} \hat{\psi}_i^r(x)}{(\sum_{i=1}^{N_{R}} \hat{\psi}_i^r(x))^2} \] (22)

According to Chu, You, Wassle, and Agarwal (2014), the value of the second hypothesis is calculated as equation 23.

\[
t_f = \frac{\Phi_r^{r+1}(x^{P+1}) - \Phi_r(x^P)}{(zf_{r+1})^2 + (zf_r)^2} \sqrt{N_{\text{MC}}} \] (23)

In which, \( zf_{r+1}, zf_r \) are standard deviations of \( f^{(r+1)}(x^{P+1}) \) and \( f^{(r)}(x^P) \). Therefore, when \( H_0 \) will be rejected that \( t_f > t_{1-\alpha(N_{\text{MC}}-1)} \).

3. Algorithm of finding local minimum

In this section of article, an algorithm is presented that its purpose is finding possible point of “\( X^{P+1} \)” that can be calculated through following method. (Equation 24)

\[
Q_1 \ldots x_{2N_F} \ldots Q_{N_F} \quad x^P = [x_1^P, \ldots, x_{N_F}^P = R_{N_F}^P, x_{N_F}^P + 1 \] (24)

This localization can be shown as followed.

\[
= f_c^P + \sum_i (x_i - x_i^P) f_{a_{i,i}}^P \quad \text{and} \quad \hat{\psi}_{i,p}(x) = (x - x_i^P)^T h_{a_{i,i}} + h_{b_{i,j}} \] (25)
In order to find unknown coefficients of \( p_a, p_{a,1}, p_{b,1}, p_b \), the later formulas can be calculated as following. (equation 27)

\[
D = \{ x^P_1, (x^P_1 - \delta), x^P_2, \ldots, x^P_{2N_F} \}
\]

In which, \( \delta \) is a fixed value. Now, \( X^{P+1} \) is solution of the following problem (Hypotheses 28,29).

\[
\begin{align*}
\min & \Phi^P (X) \\
\text{S.T} & \forall i : \hat{\psi}^r_{i,p}(X) \geq fr^i_{\text{min}} + dp_i \\
& \forall i : x^p_i - \delta \leq x^p_i \leq x^p_i + \delta
\end{align*}
\]

(28)

In order to solve INVOPT 1, Genetic Algorithm has been used. In this regard, the following points should be noted.

- In calculating localization coefficients, we have:

\[
\forall i : h^P_{a,i} = \hat{\psi}^r_{i,p}(X^P), f^P_c = \hat{\Phi}^r (X^P)
\]

(30)

- The second order localization has more accuracy than linear localization (\( f^P_b = 0 \)).
- The main purpose of utilization of localization in the present study is in equation 31,32.

\[
\forall i : \hat{\psi}^r_{i,p}(X^P) = \hat{\psi}^r_{i}(X^P), \hat{\Phi}^r_p(X^P) = \hat{\Phi}^r (X^P)
\]

(31)

Since \( \forall i : x^p_i - \delta \leq x^p_i \leq x^p_i + \delta, \hat{\Phi}^r_p(X^{P+1}) \leq \hat{\Phi}^r (X^P) \)

(32)

will be more corrected.

- \( dp_i \) is added to the second limitation of INVOPT 1 for increasing confidence of \( X^{P+1} \).

After calculating \( X^{P+1} \) and investigating the results of statistical tests and ensuring that \( X^{P+1} \) is more possible and better than \( X^P \), we should conduct localization based on the \( X^{P+2} \).

Based on the conditions of KKT, we know that minimizing \( X^P \) requires the following conditions. (hypotheses 33,34,35,36.)

\[
\nabla \Phi(X^P) = \sum_i \lambda_i \nabla \psi_i(X^P)
\]

(33)

\[
\forall i : \lambda_i (\psi_i(X^P) - f^i_{\text{min}} - dp_i) = 0 \quad (2)
\]

(34)

\[
\forall i : \psi_i(X^P) - f^i_{\text{min}} \geq 0 \quad (3)
\]

(35)
In the \( I = \{ i : \psi_i(x^P) = f_i^\min + d \psi_i \} \), \( X^P \) is active. Since we used statistical estimations, the following statistical indexes should be used. Hypothesis 37.

\[
H_0 : \forall i : \psi_i(x) = f_i^\min + d \psi_i \\
H_1 : \exists i : \psi_i(x) > f_i^\min + d \psi_i
\]

(37)

According to Chu, You, Wasslck, and Agarwal (2014), according to hypothesis 38 when \( H_0 \) will be acceptable that

\[
\psi_i^\ast(x) < f_i^\min + 2d \psi_i.
\]

(38)

The accuracy of regression model will be calculated through following formula(equation 39)

\[
\tilde{r} = \sum_{i \in I} \lambda_i \left( \psi_i^\ast(x^P) \right)
\]

(39)

The above formula is a basis for formula in the conditions of KKT. According to equation 40 The findings showed that

\[
\nabla \tilde{r}^i = h_{a,i} \nabla \tilde{r}^i = f_d^P.
\]

(40)

The following formula can be used for calculating accuracy of above formula.(equation 41)

\[
R^2 = 1 - \frac{\varepsilon}{\nabla \tilde{r}^T \nabla \tilde{r}}
\]

In the formula, \( \varepsilon \) is solution of minimization problem. According to hypotheses 41,42,43 This means that:

\[
\varepsilon = \min(\nabla \tilde{r}^T - \sum_{i \in I} \lambda_i \nabla \tilde{r}^i(X^P))
\]

(41)

\[
(\nabla \tilde{r}^T - \sum_{i \in I} \lambda_i \nabla \tilde{r}^i(X^P))
\]

(42)

\[
\forall i \notin I : \lambda_i \geq 0
\]

(43)

As much as \( R^2 \) is closer to 1, the correctness of regression model will be increased.

4. A numerical example

The model of our example is shown in Fig 2.

The order processing times follow the uniform distribution with [lower bound, upper bound]. In the Table1, “U” refers to uniform distribution and “N” refers to normal distribution.
Fig. 2: a multi-Echelon inventory system

Table 1. The daily customer demands follow the Gaussian distribution with (mean, standard deviation)

<table>
<thead>
<tr>
<th>Unit</th>
<th>( \text{od}_{1t} \sim N(12.4) )</th>
<th>( \text{ot}_{1t} \sim U(1.3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \text{od}_{2t} \sim N(18.4) )</td>
<td>( \text{ot}_{2t} \sim U(1.3) )</td>
</tr>
<tr>
<td>( \forall t: )</td>
<td>( \text{od}_{3t} \sim N(10.2) )</td>
<td>( \text{ot}_{3t} \sim U(1.3) )</td>
</tr>
<tr>
<td></td>
<td>( \text{od}_{4t} \sim N(10.2) )</td>
<td>( \text{ot}_{4t} \sim U(1.3) )</td>
</tr>
<tr>
<td></td>
<td>( \text{od}_{5t} \sim N(8.2) )</td>
<td>( \text{ot}_{5t} \sim U(1.3) )</td>
</tr>
</tbody>
</table>

In the model, “U” refers to steady distribution and “N” refers to normal distribution. The primary inventory of network is \( \text{Inv}(0) = [200 \ 200 \ 500 \ 600 \ 1000] \). Also maintenance and order costs of all units are \( c_i^M = 1 \) and \( c_i^R = 50 \). \( N_t = 100 \) refer to the number of simulation days. And sample size is 100 (MC= 100). It should be noted that the minimum repletion rate of all units is 0.6 (\( r_i^{\text{min}} = 0.6 \)).

The algorithm is conducted from possible pint of X1 as following equation (equations 44, 45, 46, 47, 48, 49, 50)

\[
X^1 = [100 \ 100 \ 400 \ 500 \ 800 \ 120 \ 300 \ 1000 \ 1200 \ 4000]
\]  

\[
FR^1 = [0/9765 \ 0/9431 \ 0/9479 \ 0/9807 \ 0/9899]
\]  

\[
\text{INVCOST}^1 = 418360
\]

\[
\]

\[
f_b^1 = [36/9005 -18/026 -16/4685 -3/6355 34/7100 14/2037 -1/9964 0/7575 13/7486 6/8346]
\]

\[
h_{a,1}^1 = [0/0037 0/0058 0/0037 0/0019 -0/0004 0/1900 0/0002 -0/0015 -0/0007 -0/0089]
\]

\[
h_{a,2}^1 = [0/0009 0/0013 0/0009 0/0011 -0/0008 0/4122 0/3465 0/7097 0/1101 0/9053]
\]
\[ h_{a,3}^1 = [0/5314 \ 0/4604 \ 2/2405 \ -0/3643 \ 0/7271 \ 0/5674 \ 0/5761 \ 0/5592 \ 0/6354 \ 0/3531] \]

\[ h_{a,4}^1 = [0/3687 \ 1/6925 \ 0/4372 \ 2/1775 \ 0/6935 \ 0/9974 \ 0/1278 \ 0/4988 \ 0/1030 \ 0/3557] \]

\[ h_{a,5}^1 = [0/3856 \ 0/4761 \ 0/3739 \ 0/9371 \ 0/2425 \ 0/2514 \ 0/6675 \ 0/5733 \ 0/0704 \ 0/5270] \]

We gain possible point of \( X^2 \) after conduction of INVOPT 1 which ensures statistical tests of possibility and improvability. (Equations 53, 54, 55)

\[ X^2 = [319 \ 1020 \ 1219 \ 3980] \]

\[ Cost \ X^2 = 32420 \]

\[ FR^2 = [0/6580 \ 0/9161 \ 0/9625 \ 0/9443 \ 0/9898] \].

is the best point in the conditions of KKT for \( X^2 \). (equation 56)

\[ \lambda^* = [50/2226 \ 22/1171 \ 0/0053 \ 0/0095 \ 0/0231 \ 3/3962 \ 7/6187 \ 10/0216 \ 7/7655 \ 6/7558] \]

It can be said that \( X^2 \) is local minimum point.

5. Conclusion

The proposed algorithm in this study is a second order localization model that aimed to find a local minimum point. Since linear localization is an especial form of second order localization, difference between goal and estimated functions is minimum. So it is expected that the point is better than linear localization. Finally, it is suggested that other localization methods are used in the future studies.

References


