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A Morita type equivalence for dual operator algebras

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Abstract

We generalize the main theorem of Rieffel for Morita equivalence of W^*-algebras to the case of unital dual operator algebras: two unital dual operator algebras $\mathcal{A}, \mathcal{B}$ have completely isometric normal representations α, β such that $\alpha(\mathcal{A}) = [\mathcal{M}^* \beta(\mathcal{B}) \mathcal{M}]^{-w^*}$ and $\beta(\mathcal{B}) = [\mathcal{M} \alpha(\mathcal{A}) \mathcal{M}^*]^{-w^*}$ for a ternary ring of operators $\mathcal{M}$ (i.e. a linear space $\mathcal{M}$ such that $\mathcal{M}\mathcal{M}^*\mathcal{M} \subset \mathcal{M}$) if and only if there exists an equivalence functor $\mathcal{F} : {}_{\mathcal{A}}\mathfrak{M} \to {}_{\mathcal{B}}\mathfrak{M}$ which "extends" to a $*$-functor implementing an equivalence between the categories ${}_{\mathcal{A}}\mathfrak{D}\mathfrak{M}$ and ${}_{\mathcal{B}}\mathfrak{D}\mathfrak{M}$. By ${}_{\mathcal{A}}\mathfrak{M}$ we denote the category of normal representations of $\mathcal{A}$ and by ${}_{\mathcal{A}}\mathfrak{D}\mathfrak{M}$ the category with the same objects as ${}_{\mathcal{A}}\mathfrak{M}$ and $\Delta(\mathcal{A})$-module maps as morphisms ($\Delta(\mathcal{A}) = \mathcal{A} \cap \mathcal{A}^*$). We prove that this functor is equivalent to a functor "generated" by a $\mathcal{B}, \mathcal{A}$ bimodule, and that it is normal and completely isometric.

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1. Introduction

At the beginning of the 70's, Rieffel [9] (see also [10]) introduced to operator theory the notion of Morita equivalence. Rieffel's work was concerned with the equivalence of representations of C^* and W^* algebras. With the development of the theory of operator spaces, it was natural to seek extensions of this theory to the class of (abstract) operator algebras.

The papers [4,1] deal with Morita equivalence of not necessarily self-adjoint (norm closed) operator algebras. To this day however, as far as we know, there is no complete theory of Morita equivalence for dual operator algebras. A natural requirement for such a theory would be to respect the additional topological structure that dual operator algebras possess as dual operator spaces. A step in this direction is taken in [2], where Rieffel's theory of Hilbert modules is extended to (dual) modules over dual (non-self-adjoint) operator algebras. In this paper we are able to generalize Rieffel's theory in a different direction. We study a new notion of equivalence for representations of dual operator algebras on Hilbert spaces. This equivalence coincides in the W^*-algebra case with the one studied by M. Rieffel; in the non-self-adjoint case there are differences in that two distinct categories have to be simultaneously equivalent. We will say that two unital dual operator algebras are *Δ-equivalent* when there is an equivalence functor between their normal representations which not only preserves intertwiners of representations of the algebras, but also

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preserves intertwiners of restrictions to the diagonals (see Definition 1.4). In [6] a new notion of equivalence between concrete w^* closed operator algebras was developed:

Definition 1.1 (*[6]*). Let $\mathcal{A}, \mathcal{B}$ be w^* closed algebras acting on Hilbert spaces H_1 and H_2 respectively. If there is a TRO $\mathcal{M} \subset B(H_1, H_2)$ (i.e. a subspace of $B(H_1, H_2)$ satisfying $\mathcal{M}\mathcal{M}^*\mathcal{M} \subset \mathcal{M}$)) such that $\mathcal{A} = [\mathcal{M}^*\mathcal{B}\mathcal{M}]^{-w^*}$ and $\mathcal{B} = [\mathcal{M}\mathcal{A}\mathcal{M}^*]^{-w^*}$ we write $\mathcal{A} \overset{\mathcal{M}}{\sim} \mathcal{B}$. The algebras $\mathcal{A}, \mathcal{B}$ are called **TRO equivalent** if there is a TRO $\mathcal{M}$ such that $\mathcal{A} \overset{\mathcal{M}}{\sim} \mathcal{B}$.

Our first main theorem (Theorem 1.3) which generalizes the main result of [9] is that two (abstract) unital dual operator algebras $\mathcal{A}, \mathcal{B}$ are Δ-equivalent if and only if they have completely isometric normal representations α, β such that the algebras $\alpha(\mathcal{A}), \beta(\mathcal{B})$ are TRO equivalent. The second main theorem (Theorem 3.3) states that every Δ-equivalent functor is (unitarily) equivalent to a functor "generated" by an algebra bimodule. The bimodule is generated by "saturating" the TRO which implements the equivalence.

We present some symbols used below. If $\mathcal{A}$ is an operator algebra we denote its diagonal $\mathcal{A} \cap \mathcal{A}^*$ by $\Delta(\mathcal{A})$. The symbol $[\mathcal{S}]$ denotes the linear span of $\mathcal{S}$. The commutant of a set $\mathcal{L}$ of bounded operators on a Hilbert space H is denoted as $\mathcal{L}'$. If $\mathcal{U}$ is a linear space and $n, m \in \mathbb{N}$ we denote by $M_{n,m}(\mathcal{U})$ the space of $n \times m$ matrices with entries from $\mathcal{U}$ and by $M_n(\mathcal{U})$ the space $M_{n,n}(\mathcal{U})$. If $\mathcal{U}, \mathcal{V}$ are linear spaces, α is a linear map from $\mathcal{U}$ to $\mathcal{V}$ and $n, m \in \mathbb{N}$ we denote the linear map

$$M_{n,m}(\mathcal{U}) \to M_{n,m}(\mathcal{V}) : (A_{ij})_{i,j} \to (\alpha(A_{ij}))_{i,j}$$

again by α. If $\mathcal{U}$ is a subspace of $B(H, K)$ for H, K Hilbert spaces we equip $M_{n,m}(\mathcal{U})$, $n, m \in \mathbb{N}$ with the norm inherited from the embedding $M_{n,m}(\mathcal{U}) \subset B(H^n, K^m)$. If $(\mathcal{X}, \|\cdot\|)$ is a normed space we denote by $\mathrm{Ball}(\mathcal{X})$ the unit ball of $\mathcal{X} : \{X \in \mathcal{X} : \|X\| \leq 1\}$. If $x_1, \ldots, x_n$ are in a vector space $\mathcal{V}$, we write $(x_1, \ldots, x_n)^t$ for the column vector in $M_{n,1}(\mathcal{V})$.

We present some definitions and concepts used in this work. A C^* algebra which is a dual Banach space is called a W^* **algebra**. A **dual operator algebra** is an operator algebra which is the dual of an operator space. Every W^* algebra is a dual operator algebra. For every dual operator algebra $\mathcal{A}$ there exists a Hilbert space H_0 and an algebraic homomorphism $\alpha_0 : \mathcal{A} \to B(H_0)$ which is a complete isometry and a w^*-continuous map [3].

Lemma 1.1 (*[3, 8.5.32]*). *Let $\mathcal{C}, \mathcal{E}$ be von Neumann algebras acting on Hilbert spaces H_1 and H_2 respectively, $\theta : \mathcal{C} \to \mathcal{E}$ be a $*$-isomorphism and*

$$\mathcal{M} = \{T \in B(H_1, H_2) : TA = \theta(A)T \text{ for all } A \in \mathcal{C}\}.$$

Then the space $\mathcal{M}$ is an essential TRO, i.e. the algebras $[\mathcal{M}^*\mathcal{M}]^{-w^*}$, $[\mathcal{M}\mathcal{M}^*]^{-w^*}$ contain the identity operators.

We now define the category ${}_{\mathcal{A}}\mathfrak{M}$ for a unital dual operator algebra $\mathcal{A}$ [3]. The objects of ${}_{\mathcal{A}}\mathfrak{M}$ are pairs (H, α) where H is a Hilbert space and $\alpha : \mathcal{A} \to B(H)$ is a **normal representation** of $\mathcal{A}$, i.e. a unital completely contractive w^*-continuous homomorphism. If (H_i, α_i), $i = 1, 2$, are objects of the category ${}_{\mathcal{A}}\mathfrak{M}$ the space of homomorphisms $\mathrm{Hom}_{\mathcal{A}}(H_1, H_2)$ is the following:

$$\mathrm{Hom}_{\mathcal{A}}(H_1, H_2) = \{T \in B(H_1, H_2) : T\alpha_1(A) = \alpha_2(A)T \text{ for all } A \in \mathcal{A}\}.$$

Observe that the map $\alpha_i|_{\Delta(\mathcal{A})}$ is a $*$-homomorphism since α_i is a contraction [3]. We also define the category ${}_{\mathcal{A}}\mathfrak{D}\mathfrak{M}$ which has the same objects as ${}_{\mathcal{A}}\mathfrak{M}$ but for every pair of objects (H_i, α_i), $i = 1, 2$, the space of homomorphisms $\mathrm{Hom}_{\mathcal{A}}^{\mathfrak{D}}(H_1, H_2)$ is given by

$$\mathrm{Hom}_{\mathcal{A}}^{\mathfrak{D}}(H_1, H_2) = \{T \in B(H_1, H_2) : T\alpha_1(A) = \alpha_2(A)T \text{ for all } A \in \Delta(\mathcal{A})\}.$$

If $\mathcal{A}$ is a W^*-algebra the categories ${}_{\mathcal{A}}\mathfrak{M}$ and ${}_{\mathcal{A}}\mathfrak{D}\mathfrak{M}$ are the same. Also observe that $\mathrm{Hom}_{\mathcal{A}}(H_1, H_2) \subset \mathrm{Hom}_{\mathcal{A}}^{\mathfrak{D}}(H_1, H_2)$.

Definition 1.2. Let $\mathcal{A}, \mathcal{B}$ be unital dual operator algebras and $\mathcal{F} : {}_{\mathcal{A}}\mathfrak{M} \to {}_{\mathcal{B}}\mathfrak{M}$ be a functor. We say that the functor $\mathcal{F}$ has a Δ-extension if there is a functor $\mathcal{G} : {}_{\mathcal{A}}\mathfrak{D}\mathfrak{M} \to {}_{\mathcal{B}}\mathfrak{D}\mathfrak{M}$ such that the following diagram is commutative:

$$\begin{array}{ccc} {}_{\mathcal{A}}\mathfrak{M} & \hookrightarrow & {}_{\mathcal{A}}\mathfrak{D}\mathfrak{M} \\ \mathcal{F}\downarrow & & \mathcal{G}\downarrow \\ {}_{\mathcal{B}}\mathfrak{M} & \hookrightarrow & {}_{\mathcal{B}}\mathfrak{D}\mathfrak{M}. \end{array}$$

The following extends Rieffel's definition [9].

Definition 1.3. Let $\mathcal{A}, \mathcal{B}$ be unital dual operator algebras and $\mathcal{F} : {}_{\mathcal{A}}\mathfrak{D}\mathfrak{M} \to {}_{\mathcal{B}}\mathfrak{D}\mathfrak{M}$ be a functor. We say that $\mathcal{F}$ is a $*$-**functor** if for every pair of objects H_1, H_2 of ${}_{\mathcal{A}}\mathfrak{D}\mathfrak{M}$ every operator $F \in \mathrm{Hom}^{\mathfrak{D}}_{\mathcal{A}}(H_1, H_2)$ satisfies $\mathcal{F}(F^*) = \mathcal{F}(F)^*$.

Definition 1.4. Let $\mathcal{A}, \mathcal{B}$ be unital dual operator algebras. If there exists an equivalence functor $\mathcal{F} : {}_{\mathcal{A}}\mathfrak{M} \to {}_{\mathcal{B}}\mathfrak{M}$ which has a Δ-extension as a $*$-functor implementing an equivalence between the categories ${}_{\mathcal{A}}\mathfrak{D}\mathfrak{M}, {}_{\mathcal{B}}\mathfrak{D}\mathfrak{M}$, we say that $\mathcal{A}$ and $\mathcal{B}$ are Δ-**equivalent** algebras.

In [9] two W^* algebras $\mathcal{A}, \mathcal{B}$ are called Morita equivalent if there exists an equivalence of ${}_{\mathcal{A}}\mathfrak{M}$ with ${}_{\mathcal{B}}\mathfrak{M}$ implemented by $*$-functors. The main theorem of Rieffel for Morita equivalence of W^*-algebras can be formulated as follows [3, 8.5.38]:

Theorem 1.2. *Two W^* algebras $\mathcal{A}, \mathcal{B}$ are Morita equivalent if and only if they have faithful normal representations α, β on Hilbert spaces such that the algebras $\alpha(\mathcal{A}), \beta(\mathcal{B})$ are TRO equivalent.*

We will generalize this to dual operator algebras:

Theorem 1.3. *Two unital dual operator algebras $\mathcal{A}, \mathcal{B}$ are Δ-equivalent if and only if they have completely isometric normal representations α, β on Hilbert spaces such that the algebras $\alpha(\mathcal{A}), \beta(\mathcal{B})$ are TRO equivalent.*

For the proof, we use a recent result obtained jointly with Paulsen [8] (see the Concluding remarks): If the unital dual operator algebras $\mathcal{A}, \mathcal{B}$ have completely isometric normal representations with TRO equivalent images then they are stably isomorphic, i.e. there exists a Hilbert space H such that the algebras $\mathcal{A}\bar{\otimes}B(H)$ and $\mathcal{B}\bar{\otimes}B(H)$ (where $\bar{\otimes}$ denotes the normal spatial tensor product [3]) are isomorphic as dual operator algebras. One easily checks that the algebras $\mathcal{A}$ and $\mathcal{A}\bar{\otimes}B(H)$ (resp. $\mathcal{B}$ and $\mathcal{B}\bar{\otimes}B(H)$) are Δ-equivalent.

For the converse direction of the proof we need some definitions and facts from [5]. Let $\mathcal{A}$ be a unital dual operator algebra. If $K \subset H$ are objects of ${}_{\mathcal{A}}\mathfrak{M}$, we say that K is $\mathcal{A}$-complemented in H if the projection of H onto K belongs to the space $\mathrm{Hom}_{\mathcal{A}}(H, H)$. We say that the object H is $\mathcal{A}$-*universal* if every object K of ${}_{\mathcal{A}}\mathfrak{M}$ is ${}_{\mathcal{A}}\mathfrak{M}$-isomorphic to an $\mathcal{A}$-complemented object in a direct sum of copies of H. In [5] it is proved that there exist $\mathcal{A}$-universal objects and that if (H, α) is an $\mathcal{A}$-universal object then α is a complete isometry and $\alpha(\mathcal{A}) = \alpha(\mathcal{A})''$. Also it is proved that there exists a W^* algebra $W^*(\mathcal{A})$ and a w^*-continuous completely isometric homomorphism $j : \mathcal{A} \to W^*(\mathcal{A})$ whose range generates $W^*(\mathcal{A})$ as a W^* algebra and which possesses the following universal property: given any normal representation $\alpha : \mathcal{A} \to B(H)$, there exists a unique normal $*$-representation $\tilde{\alpha} : W^*(\mathcal{A}) \to B(H)$ extending α. An object H is $\mathcal{A}$-universal if and only if it is $W^*(\mathcal{A})$-universal.

We now fix unital dual operator algebras $\mathcal{A}, \mathcal{B}$ and an equivalence functor $\mathcal{F} : {}_{\mathcal{A}}\mathfrak{M} \to {}_{\mathcal{B}}\mathfrak{M}$ which has a Δ-extension as a $*$-functor implementing an equivalence between the categories ${}_{\mathcal{A}}\mathfrak{D}\mathfrak{M}$ and ${}_{\mathcal{B}}\mathfrak{D}\mathfrak{M}$. We still denote the Δ-extension of this functor by $\mathcal{F}$. We need the following lemma.

Lemma 1.4. *The functor $\mathcal{F}$ restricts to an equivalence $*$-functor between the categories ${}_{W^*(\mathcal{A})}\mathfrak{M}$ and ${}_{W^*(\mathcal{B})}\mathfrak{M}$.*

Proof. If $T \in \mathrm{Hom}_{W^*(\mathcal{A})}(H_1, H_2)$, using the fact that $W^*(\mathcal{A})$ (resp. $W^*(\mathcal{B})$) is a W^*-algebra generated by a copy of $\mathcal{A}$ (resp. $\mathcal{B}$) and $\mathcal{F}$ is a $*$-functor we can check that $\mathcal{F}(T) \in \mathrm{Hom}_{W^*(\mathcal{B})}(\mathcal{F}(H_1), \mathcal{F}(H_2))$. Since the objects of ${}_{W^*(\mathcal{A})}\mathfrak{M}$ and ${}_{\mathcal{A}}\mathfrak{M}$ coincide, as do the objects of ${}_{W^*(\mathcal{B})}\mathfrak{M}$ and ${}_{\mathcal{B}}\mathfrak{M}$, we can define a functor $\mathcal{G} : {}_{W^*(\mathcal{A})}\mathfrak{M} \to {}_{W^*(\mathcal{B})}\mathfrak{M}$ by sending every object K to the object $\mathcal{F}(K)$ and every homomorphism T to the homomorphism $\mathcal{F}(T)$. Clearly $\mathcal{G}$ is a $*$-functor. For every $H_1, H_2 \in {}_{W^*(\mathcal{A})}\mathfrak{M}$ the map $\mathcal{G} : \mathrm{Hom}_{W^*(\mathcal{A})}(H_1, H_2) \to \mathrm{Hom}_{W^*(\mathcal{B})}(\mathcal{F}(H_1), \mathcal{F}(H_2))$ is faithful, being a restriction of $\mathcal{F}$. Also it is onto because for every $S \in \mathrm{Hom}_{W^*(\mathcal{B})}(\mathcal{F}(H_1), \mathcal{F}(H_2))$ we can check that $\mathcal{F}^{-1}(S) \in \mathrm{Hom}_{W^*(\mathcal{A})}(H_1, H_2)$. If $K \in {}_{W^*(\mathcal{B})}\mathfrak{M}$, since $\mathcal{F} : {}_{\mathcal{A}}\mathfrak{M} \to {}_{\mathcal{B}}\mathfrak{M}$ is an equivalence functor, there exists an object $H \in {}_{\mathcal{A}}\mathfrak{M}$ and a unitary $U \in \mathrm{Hom}_{\mathcal{B}}(\mathcal{F}(H), K)$. We can easily check that U belongs to $\mathrm{Hom}_{W^*(\mathcal{B})}(\mathcal{F}(H), K)$. It follows that $\mathcal{G}$ is an equivalence $*$-functor. See for example [11, Theorem 1, Section IV-4]. $\square$

Corollary 1.5. *If H is an $\mathcal{A}$-universal object then $\mathcal{F}(H)$ is a $\mathcal{B}$-universal object.*

Proof. Let $\mathcal{G} : {}_{W^*(\mathcal{A})}\mathfrak{M} \to {}_{W^*(\mathcal{B})}\mathfrak{M}$ be the restriction of $\mathcal{F}$ as in Lemma 1.4. Every $\mathcal{A}$-universal object H is $W^*(\mathcal{A})$-universal. Since $\mathcal{G}$ is an equivalence, $\mathcal{F}(H)$ is a $W^*(\mathcal{B})$-universal object [9], and hence $\mathcal{B}$-universal. □

We now return to the proof of Theorem 1.3. Choose an $\mathcal{A}$-universal object (H, α) and denote by $(\mathcal{F}(H), \beta)$ the corresponding object. By the previous corollary this object is $\mathcal{B}$-universal. As we remarked in the discussion before Lemma 1.4 the normal representations α, β are complete isometries and the algebras $\alpha(\mathcal{A}), \beta(\mathcal{B})$ have the double commutant property: $\alpha(\mathcal{A}) = \alpha(\mathcal{A})'', \beta(\mathcal{B}) = \beta(\mathcal{B})''$. We denote by σ the map

$$\mathcal{F} : \mathrm{Hom}_{\mathcal{A}}^{\mathfrak{D}}(H, H) = \alpha(\Delta(\mathcal{A}))' \to \beta(\Delta(\mathcal{B}))' = \mathrm{Hom}_{\mathcal{B}}^{\mathfrak{D}}(\mathcal{F}(H), \mathcal{F}(H)),$$

that is $\sigma(T) = \mathcal{F}(T),\ T \in \alpha(\Delta(\mathcal{A}))'$. Since $\mathcal{F} : {}_{\mathcal{A}}\mathfrak{D}\mathfrak{M} \to {}_{\mathcal{B}}\mathfrak{D}\mathfrak{M}$ is an equivalence $*$-functor this map is a $*$-isomorphism. By the Δ-extension property σ maps the space $\mathrm{Hom}_{\mathcal{A}}(H, H) = \alpha(\mathcal{A})'$ into $\mathrm{Hom}_{\mathcal{B}}(\mathcal{F}(H), \mathcal{F}(H)) = \beta(\mathcal{B})'$. Since $\mathcal{F} : {}_{\mathcal{A}}\mathfrak{M} \to {}_{\mathcal{B}}\mathfrak{M}$ is an equivalence functor we have $\sigma(\alpha(\mathcal{A})') = \beta(\mathcal{B})'$. We define the space

$$\mathcal{M} = \{M : MA = \sigma(A)M \text{ for all } A \in \alpha(\Delta(\mathcal{A}))'\}.$$

By Lemma 1.1 this space is an essential TRO. Choose $M, N \in \mathcal{M}, B \in \mathcal{B}$. For all $A \in \alpha(\mathcal{A})'$ we have $M^*\beta(B)NA = M^*\beta(B)\sigma(A)N$. Since $\sigma(A) \in \beta(\mathcal{B})'$ the last operator equals $M^*\sigma(A)\beta(B)N = AM^*\beta(B)N$. We proved that $\mathcal{M}^*\beta(\mathcal{B})\mathcal{M} \subset \alpha(\mathcal{A})$. Symmetrically we can prove $\mathcal{M}\alpha(\mathcal{A})\mathcal{M}^* \subset \beta(\mathcal{B})$. It follows from [6, 2.1] that $\alpha(\mathcal{A}) \overset{\mathcal{M}}{\sim} \beta(\mathcal{B})$.

2. The generated functor

In this section we fix unital dual operator algebras $\mathcal{A}, \mathcal{B}$ acting on Hilbert spaces H_0, K_0 respectively which are TRO equivalent. We are going to construct a functor $\mathcal{F}_{\mathcal{U}}$ generated by a $\mathcal{B}, \mathcal{A}$ bimodule $\mathcal{U}$. In Section 3 we shall prove that every functor implementing the equivalence of Theorem 1.3 is unitarily equivalent to such a functor $\mathcal{F}_{\mathcal{U}}$.

In [6, 2.8] it is shown that the TRO $\mathcal{M} \subset B(H_0, K_0)$ implementing the equivalence can be chosen so that $[\mathcal{M}^*\mathcal{M}]^{-w^*} = \Delta(\mathcal{A}), \quad [\mathcal{M}\mathcal{M}^*]^{-w^*} = \Delta(\mathcal{B})$. Define $\mathcal{U} = [\mathcal{B}\mathcal{M}]^{-w^*}, \mathcal{V} = [\mathcal{M}^*\mathcal{B}]^{-w^*}$. One can now check that $\mathcal{U} = [\mathcal{M}\mathcal{A}]^{-w^*}, \mathcal{V} = [\mathcal{A}\mathcal{M}^*]^{-w^*}$ and

$$\mathcal{B}\mathcal{U}\mathcal{A} \subset \mathcal{U}, \qquad \mathcal{A}\mathcal{V}\mathcal{B} \subset \mathcal{V}, \qquad [\mathcal{V}\mathcal{U}]^{-w^*} = \mathcal{A}, \qquad [\mathcal{U}\mathcal{V}]^{-w^*} = \mathcal{B}.$$

If $n \in \mathbb{N}$ and $S \in \mathrm{Ball}(M_{n,1}(\mathcal{V}))$ we define on the algebraic tensor product $\mathcal{U} \otimes H$ a sesquilinear form by the formula

$$\langle T_1 \otimes x_1, T_2 \otimes x_2 \rangle_S = \langle \alpha(ST_1)x_1, \alpha(ST_2)x_2 \rangle_{H^n}.$$

We write $\|\cdot\|_S$ for the associated seminorm and $\mathcal{L}_S$ for its kernel. The completion of $((\mathcal{U} \otimes H)/\mathcal{L}_S, \|\cdot\|_S)$ will be denoted by H_S and the symbol $\|\cdot\|_S$ will be used for the norm of H_S as well. Let $\pi_S : \mathcal{U} \otimes H \to H_S$ be the quotient map. Again on the algebraic tensor product $\mathcal{U} \otimes H$ we define the following seminorm:

$$\left\| \sum_{j=1}^m T_j \otimes x_j \right\|_{\mathcal{F}_{\mathcal{U}}(H)} = \sup_{S \in \mathrm{Ball}(M_{n,1}(\mathcal{V})), n \in \mathbb{N}} \left\| \sum_{j=1}^m T_j \otimes x_j \right\|_S.$$

Since the seminorm $\|\cdot\|_S$ satisfies the parallelogram identity for all S, the previous seminorm satisfies the parallelogram identity too. If $\mathcal{L} = \{z \in \mathcal{U} \otimes H : \|z\|_{\mathcal{F}_{\mathcal{U}}(H)} = 0\}$ the space $(\mathcal{U} \otimes H)/\mathcal{L}$ is a pre-Hilbert space. We denote its completion by $\mathcal{F}_{\mathcal{U}}(H)$ and we use the same symbol $\|\cdot\|_{\mathcal{F}_{\mathcal{U}}(H)}$ for the corresponding norm. We write $\pi : \mathcal{U} \otimes H \to \mathcal{F}_{\mathcal{U}}(H)$ for the quotient map. The following lemma is essentially due to Paschke; see for example [3, 8.5.23].

Lemma 2.1. *There exist partial isometries $\{W_k, k \in J\} \subset \mathcal{M}(\{V_k, k \in I\} \subset \mathcal{M})$ such that $W_k^*W_k \perp W_m^*W_m (V_kV_k^* \perp V_mV_m^*)$ for $k \neq m$ and $I_{H_0} = \sum_k \oplus W_k^*W_k (I_{K_0} = \sum_k \oplus V_kV_k^*)$.*

The following proposition says that we can calculate the norm $\|\cdot\|_{\mathcal{F}_{\mathcal{U}}(H)}$ using only the operators $\{S : S \in \mathrm{Ball}(M_{n,1}(\mathcal{M}^*)), n \in \mathbb{N}\}$.

Proposition 2.2. *If* $\sum_{j=1}^{m} T_j \otimes x_j \in \mathcal{U} \otimes H$ *then*

$$\left\| \sum_{j=1}^{m} T_j \otimes x_j \right\|_{\mathcal{F}_{\mathcal{U}}(H)} = \sup_{S \in \mathrm{Ball}(M_{n,1}(\mathcal{M}^*)), n \in \mathbb{N}} \left\| \sum_{j=1}^{m} T_j \otimes x_j \right\|_{S}.$$

Proof. For $\epsilon > 0$ there exist $n \in \mathbb{N}$ and $S \in \mathrm{Ball}(M_{n,1}(\mathcal{V}))$ such that

$$\left\| \sum_{j=1}^{m} T_j \otimes x_j \right\|_{\mathcal{F}_{\mathcal{U}}(H)} - \epsilon < \left\| \sum_{j=1}^{m} \alpha(ST_j)x_j \right\|_{H^n} - \frac{\epsilon}{2}.$$

Using Lemma 2.1 and the fact that α is w^*-continuous we can find partial isometries $\{V_1, \dots, V_N\} \subset \mathcal{M}$ such that the operator $\sum_{i=1}^{N} V_i V_i^*$ is a projection and

$$\begin{aligned}\left\| \sum_{j=1}^{m} \alpha(ST_j)x_j \right\|_{H^n} - \frac{\epsilon}{2} &\le \left\| \sum_{j=1}^{m} \alpha\left(S \sum_{k=1}^{N} V_k V_k^* T_j \right) x_j \right\|_{H^n} \\ &= \left\| \alpha\left(S(V_1, \dots, V_N)\right) \sum_{j=1}^{m} \alpha((V_1^*, \dots, V_N^*)^t T_j) x_j \right\|_{H^n}.\end{aligned}$$

Observe that $(V_1^*, \dots, V_N^*)^t$ is in $\mathrm{Ball}(M_{N,1}(\mathcal{M}^*))$. So since α is a complete contraction we have

$$\left\| \sum_{j=1}^{m} T_j \otimes x_j \right\|_{\mathcal{F}_{\mathcal{U}}(H)} - \epsilon \le \sup_{S \in \mathrm{Ball}(M_{r,1}(\mathcal{M}^*)), r \in \mathbb{N}} \left\| \sum_{j=1}^{m} T_j \otimes x_j \right\|_{S}.$$

Since ϵ is arbitrary the proof is complete. □

For all $n \in \mathbb{N}$ and $S \in \mathrm{Ball}(M_{n,1}(\mathcal{V}))$ we have $\|\pi_s(\xi)\|_S \le \|\pi(\xi)\|_{\mathcal{F}_{\mathcal{U}}(H)}$ for every $\xi \in \mathcal{U} \otimes H$. This shows that the map $\pi(\xi) \to \pi_S(\xi)$ is well defined and extends to a contraction $\theta_S : \mathcal{F}_{\mathcal{U}}(H) \to H_S$ between the associated completions.

Lemma 2.3. *If* $\theta_S : \mathcal{F}_{\mathcal{U}}(H) \to H_S$ *is given by* $\theta_S(\pi(\xi)) = \pi_S(\xi)$,

$$\mathcal{F}_{\mathcal{U}}(H) = [\theta_S^*(\pi_S(T \otimes x)) : S \in \mathrm{Ball}(M_{m,1}(\mathcal{V})), m \in \mathbb{N}, T \in \mathcal{U}, x \in H]^-.$$

Proof. Let $z \in \mathcal{F}_{\mathcal{U}}(H)$ be such that $\langle \theta_S^*(\pi_S(T \otimes x)), z \rangle_{\mathcal{F}_{\mathcal{U}}(H)} = 0$ for all $m \in \mathbb{N}$, $S \in \mathrm{Ball}(M_{m,1}(\mathcal{V}))$, $T \in \mathcal{U}$ and $x \in H$. Then $\langle \pi_S(T \otimes x), \theta_S(z) \rangle_{H_S} = 0$ for all $m \in \mathbb{N}$, $S \in \mathrm{Ball}(M_{m,1}(\mathcal{V}))$, $T \in \mathcal{U}$ and $x \in H$. It follows that $\theta_S(z) = 0$ for all $m \in \mathbb{N}$, $S \in \mathrm{Ball}(M_{m,1}(\mathcal{V}))$. But

$$\|z\|_{\mathcal{F}_{\mathcal{U}}(H)} = \sup_{S \in \mathrm{Ball}(M_{n,1}(\mathcal{V})), n \in \mathbb{N}} \|\theta_S(z)\|_S.$$

Indeed, this holds when $z \in \pi(\mathcal{U} \otimes H)$ and it is a standard fact that the equality extends to all $z \in \mathcal{F}_{\mathcal{U}}(H)$. It follows that $z = 0$. □

We will show below that the space $\pi(\mathcal{M} \otimes H)$ is dense in $\mathcal{F}_{\mathcal{U}}(H)$. In fact we shall prove the following stronger result:

Lemma 2.4. *Let* L *be an invariant projection for* $\alpha(\mathcal{A})$. *If* $T \in \mathcal{U}$ *and* $x \in H$ *then*

$$\pi(T \otimes L(x)) \in [\pi(N \otimes L(y)) : N \in \mathcal{M}, y \in H]^{-\mathcal{F}_{\mathcal{U}}(H)}.$$

Proof. On the algebraic tensor product $\mathcal{M}^* \otimes \mathcal{U} \otimes L(H)$ we define the following sesquilinear form:

$$\langle M_1^* \otimes T_1 \otimes L(x_1), M_2^* \otimes T_2 \otimes L(x_2) \rangle = \langle \alpha(M_1^* T_1) L(x_1), \alpha(M_2^* T_2) L(x_2) \rangle_H.$$

If $\mathcal{K}$ is the kernel of $\langle \cdot, \cdot \rangle$ we denote by K the completion of $(\mathcal{M}^* \otimes \mathcal{U} \otimes L(H))/\mathcal{K}$ under the corresponding norm and by π_K the quotient map $\mathcal{M}^* \otimes \mathcal{U} \otimes L(H) \to K$. Since the identity operator belongs to $[\mathcal{M}^* \mathcal{M}]^{-w^*}$ and α

is w^*-continuous we can check that the space K_1 generated by vectors of the form $\pi_K(M^* \otimes N \otimes L(y))$ where $M, N \in \mathcal{M}$, $y \in H$ is dense in K.

Claim. For every $N, M_0 \in \mathcal{M}$, $T \in \mathcal{U}$ and $x \in H$,

$$\pi(NM_0^*T \otimes L(x)) \in [\pi(M \otimes L(y)) : M \in \mathcal{M}, y \in H]^{-\mathcal{F}_{\mathcal{U}}(H)}.$$

Proof. For every $n \in \mathbb{N}$, $S \in \mathrm{Ball}(M_{n,1}(\mathcal{V}))$, $M_i \in \mathcal{M}$, $T_i \in \mathcal{U}$, $x_i \in H$, $i = 1, \ldots, m$, and $N \in \mathrm{Ball}(\mathcal{M})$ we have

$$\begin{aligned}\left\|\alpha\left(SN\sum_{i=1}^m M_i^*T_i\right)L(x_i)\right\|_{H^n} &= \left\|\alpha(SN)\sum_{i=1}^m \alpha(M_i^*T_i)L(x_i)\right\|_{H^n} \\ &\le \left\|\sum_{i=1}^m \alpha(M_i^*T_i)L(x_i)\right\|_H = \left\|\pi_K\left(\sum_{i=1}^m M_i^* \otimes T_i \otimes L(x_i)\right)\right\|_K .\end{aligned}$$

It follows from the definition of $\|\cdot\|_{\mathcal{F}_{\mathcal{U}}(H)}$ that

$$\left\|\sum_{i=1}^m NM_i^*T_i \otimes L(x_i)\right\|_{\mathcal{F}_{\mathcal{U}}(H)} \le \left\|\pi_K\left(\sum_{i=1}^m M_i^* \otimes T_i \otimes L(x_i)\right)\right\|_K . \tag{2.1}$$

Now fix $N \in \mathrm{Ball}(\mathcal{M})$, $M_0 \in \mathcal{M}$, $T \in \mathcal{U}$, $x \in H$ and $\epsilon > 0$. By the density of K_1 in K there exist $N_i, M_i \in \mathcal{M}$, $x_i \in H$, $i = 1, \ldots m$, such that

$$\left\|\pi_K(M_0^* \otimes T \otimes L(x)) - \pi_K\left(\sum_{i=1}^m M_i^* \otimes N_i \otimes L(x_i)\right)\right\|_K < \epsilon.$$

It follows from (2.1) that

$$\left\|NM_0^*T \otimes L(x) - \sum_{i=1}^m NM_i^*N_i \otimes L(x_i)\right\|_{\mathcal{F}_{\mathcal{U}}(H)} < \epsilon.$$

This proves the Claim. Let $T \in \mathcal{U}$ and $x \in H$. It now suffices to show that

$$\pi(T \otimes L(x)) \in [\pi(NM^*U \otimes L(y)) : N, M \in \mathcal{M}, U \in \mathcal{U}, y \in H]^{-\mathcal{F}_{\mathcal{U}}(H)}.$$

Recall the partial isometries $\{V_k, k \in I\} \subset \mathcal{M}$ from Lemma 2.1. We have

$$\begin{aligned}&\lim_{E\subset I,\text{finite}}\left\langle \pi(T \otimes L(x)) - \sum_{k\in E}\pi(V_kV_k^*T \otimes L(x)), \theta_S^*(\pi_S(U \otimes y))\right\rangle_{\mathcal{F}_{\mathcal{U}}(H)} \\ &= \lim_E\left\langle \theta_S\left(\pi\left(T \otimes L(x) - \sum_{k\in E} V_kV_k^*T \otimes L(x)\right)\right), \pi_S(U \otimes y)\right\rangle_S \\ &= \lim_E\left\langle \pi_S\left(T \otimes L(x) - \sum_{k\in E} V_kV_k^*T \otimes L(x)\right), \pi_S(U \otimes y)\right\rangle_S \\ &= \lim_E\left\langle \alpha(ST)L(x) - \sum_{k\in E}\alpha(SV_kV_k^*T)L(x), \alpha(SU)(y)\right\rangle_{H^n} \\ &= \lim_E\left\langle \alpha\left(S\left(I - \sum_{k\in E}V_kV_k^*\right)T\right)L(x), \alpha(SU)(y)\right\rangle_{H^n} = 0.\end{aligned}$$

Since this net is uniformly bounded from Lemma 2.3 the equality $\pi(T \otimes L(x)) = \sum_{k\in I}\pi(V_kV_k^*T \otimes L(x))$ follows and the proof is complete. □

Corollary 2.5. *The subspace $\pi(\mathcal{M} \otimes H)$ of $\mathcal{F}_{\mathcal{U}}(H)$ is dense.*

We define a map $\beta : \mathcal{B} \to B(\mathcal{F}_{\mathcal{U}}(H))$ given by

$$\beta(B)(\pi(T \otimes x)) = \pi(BT \otimes x), \quad B \in \mathcal{B}, T \in \mathcal{U}, x \in H.$$

This is a well-defined unital algebraic homomorphism and a contraction. We shall prove the following stronger result.

Proposition 2.6. *The map β is a complete contraction.*

Proof. Let $n \in \mathbb{N}$ and $(B_{ij}) \in M_n(\mathcal{B})$. Fix vectors $z_j = \sum_{i=1}^{k_j} \pi(T_i^j \otimes x_i^j)$, $j = 1, \ldots, n$, of the space $\mathcal{F}_{\mathcal{U}}(H)$ and denote by z the vector $(z_1, \ldots, z_n)^t$. Also write $y = \beta((B_{ij}))(z)$. Then

$$\|y\|^2 = \sum_{k=1}^{n} \left\| \sum_{j=1}^{n} \sum_{i=1}^{k_j} B_{kj} T_i^j \otimes x_i^j \right\|_{\mathcal{F}_{\mathcal{U}}(H)}^2 .$$

By the definition of the norm of the space $\mathcal{F}_{\mathcal{U}}(H)$, given $\epsilon > 0$ there exist $r \in \mathbb{N}$, $S_k \in \mathrm{Ball}(M_{r,1}(\mathcal{V}))$, $k = 1, \ldots, n$, such that

$$\|y\|^2 - \epsilon \leq \sum_{k=1}^{n} \left\| \sum_{j=1}^{n} \sum_{i=1}^{k_j} \alpha(S_k B_{kj} T_i^j)(x_i^j) \right\|_{H^r}^2 - \frac{\epsilon}{2}.$$

Since α is w^*-continuous from Lemma 2.1 we can find partial isometries $V_1, \ldots, V_N \in \mathcal{M}$ such that

$$\sum_{k=1}^{n} \left\| \sum_{j=1}^{n} \sum_{i=1}^{k_j} \alpha(S_k B_{kj} T_i^j)(x_i^j) \right\|_{H^r}^2 - \frac{\epsilon}{2} \leq \sum_{k=1}^{n} \left\| \sum_{j=1}^{n} \sum_{i=1}^{k_j} \alpha\left(S_k B_{kj} \sum_{l=1}^{N} V_l V_l^* T_i^j \right)(x_i^j) \right\|^2 .$$

Let $V = (V_1, \ldots, V_N)$. Now α is an algebraic homomorphism, and hence

$$\|y\|^2 - \epsilon \leq \left\| \alpha((S_i B_{ij} V)_{1 \leq i, j \leq n}) \begin{bmatrix} \sum_{i=1}^{k_1} \alpha(V^* T_i^1)(x_i^1) \\ \vdots \\ \sum_{i=1}^{k_n} \alpha(V^* T_i^n)(x_i^n) \end{bmatrix} \right\|^2 .$$

Since $(S_i B_{ij} V)_{1 \leq i, j \leq n} = (S_1 \oplus \cdots \oplus S_n)(B_{ij})(V \oplus \cdots \oplus V)$ and $\|(S_1 \oplus \cdots \oplus S_n)\| \leq 1$, $\|(V \oplus \cdots \oplus V)\| \leq 1$ it follows that $\|(\alpha(S_i B_{ij} V))\| \leq \|(B_{ij})\|$ and hence

$$\begin{aligned} \|y\|^2 - \epsilon &\leq \|(B_{ij})\|^2 \sum_{j=1}^{n} \left\| \sum_{i=1}^{k_j} \alpha(V^* T_i^j)(x_i^j) \right\|^2 \\ &\leq \sum_{j=1}^{n} \left\| \sum_{i=1}^{k_j} T_i^j \otimes x_i^j \right\|_{\mathcal{F}_{\mathcal{U}}(H)}^2 \|(B_{ij})\|^2 = \|z\|^2 \|(B_{ij})\|^2. \end{aligned}$$

But ϵ is arbitrary and so $\left\|\beta((B_{ij}))(z)\right\| = \|y\| \leq \left\|(B_{ij})\right\| \|z\|$; hence $\left\|\beta((B_{ij}))\right\| \leq \left\|(B_{ij})\right\|$. Since n is arbitrary, this shows that β is a complete contraction. □

Proposition 2.7. *The map β is w^*-continuous.*

Proof. Since β is a bounded map it suffices to show that given a net $(B_i) \subset \mathrm{Ball}(\mathcal{B})$ which converges to 0 in the weak operator topology, the net $(\beta(B_i))$ also converges to 0 in the weak operator topology. Indeed, for all $T_1, T_2 \in \mathcal{U}$, $x_1, x_2 \in H$, $n \in \mathbb{N}$ and $S \in \mathrm{Ball}(M_{n,1}(\mathcal{V}))$,

$$\begin{aligned} \left\langle \beta(B_i)(\pi(T_1 \otimes x_1)), \theta_S^*(\pi_S(T_2 \otimes x_2)) \right\rangle_{\mathcal{F}_{\mathcal{U}}(H)} &= \langle \theta_S(\pi((B_i T_1 \otimes x_1))), \pi_S(T_2 \otimes x_2) \rangle_{H_S} \\ &= \langle \alpha(S B_i T_1)(x_1), \alpha(S T_2)(x_2) \rangle \to 0. \end{aligned}$$

The conclusion follows from Lemma 2.3. □

In the rest of this section if $H \in {}_{\mathcal{A}}\mathfrak{M}$ we identify $\mathcal{U} \otimes H$ with its image in $\mathcal{F}_{\mathcal{U}}(H)$. From the above discussion we have a correspondence $H \in {}_{\mathcal{A}}\mathfrak{M} \to \mathcal{F}_{\mathcal{U}}(H) \in {}_{\mathcal{B}}\mathfrak{M}$. If $(H_i, \alpha_i) \in {}_{\mathcal{A}}\mathfrak{M}, i = 1, 2$, we define a map $\mathcal{F}_{\mathcal{U}}(F)$ from the space $\mathcal{F}_{\mathcal{U}}(H_1)$ into the space $\mathcal{F}_{\mathcal{U}}(H_2)$ by the formula

$$\mathcal{F}_{\mathcal{U}}(F)(T \otimes x) = T \otimes F(x) \quad \text{for all } T \in \mathcal{U}, x \in H_1.$$

We can easily check that this map is bounded with norm at most $\|F\|$ and $\mathcal{F}_{\mathcal{U}}(F) \in \mathrm{Hom}_{\mathcal{B}}(\mathcal{F}_{\mathcal{U}}(H_1), \mathcal{F}_{\mathcal{U}}(H_2))$. This definition completes the definition of the functor $\mathcal{F}_{\mathcal{U}} : {}_{\mathcal{A}}\mathfrak{M} \to {}_{\mathcal{B}}\mathfrak{M}$.

Theorem 2.8. *The functor $\mathcal{F}_{\mathcal{U}}$ has a Δ-extension.*

Proof. Let $F \in \mathrm{Hom}_{\mathcal{A}}{}^{\mathfrak{D}}(H_1, H_2)$. Suppose that $M_1, \ldots, M_m \in \mathcal{M}$ and $x_1, \ldots, x_m \in H$. If $n \in \mathbb{N}$ and $S \in \mathrm{Ball}(M_{n,1}(\mathcal{M}^*))$ we have

$$\begin{aligned}\left\|\sum_{i=1}^{m} \alpha_2(SM_i)F(x_i)\right\| &= \left\|F^{(n)}\sum_{i=1}^{m} \alpha_1(SM_i)(x_i)\right\| \qquad (F^{(n)} = (F \oplus F \oplus \cdots \oplus F))\\ &\leq \|F\| \left\|\sum_{i=1}^{m} \alpha_1(SM_i)(x_i)\right\| \leq \|F\| \left\|\sum_{i=1}^{m} M_i \otimes x_i\right\|_{\mathcal{F}_{\mathcal{U}}(H_1)}.\end{aligned}$$

From Proposition 2.2 it follows that

$$\left\|\sum_{i=1}^{m} M_i \otimes F(x_i)\right\|_{\mathcal{F}_{\mathcal{U}}(H_2)} \leq \|F\| \left\|\sum_{i=1}^{m} M_i \otimes x_i\right\|_{\mathcal{F}_{\mathcal{U}}(H_1)}.$$

So we can define a map $\delta(F)$ from the subspace $\mathcal{M} \otimes H_1$ of $\mathcal{F}_{\mathcal{U}}(H_1)$ into the space $\mathcal{F}_{\mathcal{U}}(H_2)$ by the formula

$$\delta(F)(M \otimes x) = M \otimes F(x) \quad \text{for all } M \in \mathcal{M}, x \in H_1. \tag{2.2}$$

The map $\delta(F)$ is bounded with norm at most $\|F\|$. By Corollary 2.5 the space $\mathcal{M} \otimes H_1$ is dense in $\mathcal{F}_{\mathcal{U}}(H_1)$, so this map extends to $\mathcal{F}_{\mathcal{U}}(H_1)$. Since $\Delta(\mathcal{B})\mathcal{M} \subset \mathcal{M}$, equality (2.2) shows that $\delta(F) \in \mathrm{Hom}_{\mathcal{B}}^{\mathfrak{D}}(\mathcal{F}_{\mathcal{U}}(H_1), \mathcal{F}_{\mathcal{U}}(H_2))$. Observe that if $F \in \mathrm{Hom}_{\mathcal{A}}(H_1, H_2)$ then $\mathcal{F}_{\mathcal{U}}(F) = \delta(F)$, because both operators are bounded and coincide in the dense subspace $\mathcal{M} \otimes H_1$ of $\mathcal{F}_{\mathcal{U}}(H_1)$. Therefore we may define a functor ${}_{\mathcal{A}}\mathfrak{D}\mathfrak{M} \to {}_{\mathcal{B}}\mathfrak{D}\mathfrak{M}$ by sending every object H to $\mathcal{F}_{\mathcal{U}}(H)$ and every homomorphism F to $\delta(F)$. Clearly this functor is a Δ-extension of the functor $\mathcal{F}_{\mathcal{U}}$. □

Definition 2.1. In the sequel the Δ-extension of the functor $\mathcal{F}_{\mathcal{U}}$ will be denoted again by $\mathcal{F}_{\mathcal{U}}$ and every homomorphism $\delta(F)$ defined by Eq. (2.2) by $\mathcal{F}_{\mathcal{U}}(F)$.

Now we will prove that the Δ-extension of $\mathcal{F}_{\mathcal{U}}$ is a $*$-functor.

Lemma 2.9. *If $U \in \mathrm{Hom}_{\mathcal{A}}{}^{\mathfrak{D}}(H_1, H_2)$ is a partial isometry then*

$$\mathcal{F}_{\mathcal{U}}(U^*) = \mathcal{F}_{\mathcal{U}}(U)^*.$$

Proof. Let $M_j \in \mathcal{M}, x_j \in H_1, 1 \leq j \leq m,\ S = (N_1^*, \ldots, N_n^*)^t \in \mathrm{Ball}(M_{n,1}(\mathcal{M}^*))$. We have

$$\begin{aligned}\left\|\sum_{j=1}^{m} \alpha_1(SM_j)U^*U(x_j)\right\|^2 &= \sum_{i=1}^{n}\left\|\sum_{j=1}^{m} \alpha_1(N_i^*M_j)U^*U(x_j)\right\|^2\\ &= \sum_{i=1}^{n}\left\|U^*\left(U\sum_{j=1}^{m} \alpha_1(N_i^*M_j)(x_j)\right)\right\|^2 = \sum_{i=1}^{n}\left\|\sum_{j=1}^{m} U\alpha_1(N_i^*M_j)(x_j)\right\|^2\\ &= \sum_{i=1}^{n}\left\|\sum_{j=1}^{m} \alpha_2(N_i^*M_j)U(x_j)\right\|^2 = \left\|\sum_{j=1}^{m} \alpha_2(SM_j)U(x_j)\right\|^2.\end{aligned}$$

Since S was arbitrary in $\mathrm{Ball}(M_{n,1}(\mathcal{M}^*))$ we have

$$\left\|\sum_{j=1}^{m} M_j \otimes U^*U(x_j)\right\|_{\mathcal{F}_{\mathcal{U}}(H_1)} = \left\|\sum_{j=1}^{m} M_j \otimes U(x_j)\right\|_{\mathcal{F}_{\mathcal{U}}(H_2)}$$

or equivalently

$$\left\|\mathcal{F}_{\mathcal{U}}(U^*U)\left(\sum_{j=1}^{m} M_j \otimes x_j\right)\right\|_{\mathcal{F}_{\mathcal{U}}(H_1)} = \left\|\mathcal{F}_{\mathcal{U}}(U)\left(\sum_{j=1}^{m} M_j \otimes x_j\right)\right\|_{\mathcal{F}_{\mathcal{U}}(H_2)}.$$

By Corollary 2.5 we have that

$$\|\mathcal{F}_{\mathcal{U}}(U^*U)(z)\|_{\mathcal{F}_{\mathcal{U}}(H_1)} = \|\mathcal{F}_{\mathcal{U}}(U)(z)\|_{\mathcal{F}_{\mathcal{U}}(H_2)} \quad \text{for all } z \in \mathcal{F}_{\mathcal{U}}(H_1). \tag{2.3}$$

We proved in Theorem 2.8 that the map $\mathcal{F}_{\mathcal{U}}$ between the spaces of homomorphisms is a contraction; therefore $\mathcal{F}_{\mathcal{U}}(U^*U)$ is an orthogonal projection. It follows now by (2.3) that

$$\langle\mathcal{F}_{\mathcal{U}}(U^*U)(z), z\rangle_{\mathcal{F}_{\mathcal{U}}(H_1)} = \langle\mathcal{F}_{\mathcal{U}}(U)^*\mathcal{F}_{\mathcal{U}}(U)(z), z\rangle_{\mathcal{F}_{\mathcal{U}}(H_1)}$$

for all $z \in \mathcal{F}_{\mathcal{U}}(H_1)$ and so $\mathcal{F}_{\mathcal{U}}(U^*)\mathcal{F}_{\mathcal{U}}(U) = \mathcal{F}_{\mathcal{U}}(U^*U) = \mathcal{F}_{\mathcal{U}}(U)^*\mathcal{F}_{\mathcal{U}}(U)$. Let $W = \mathcal{F}_{\mathcal{U}}(U)$, $V = \mathcal{F}_{\mathcal{U}}(U^*)$. We have proved that $VW = W^*W$. Similarly working with the partial isometry U^* we obtain $WV = V^*V$. Now we have $V = \mathcal{F}_{\mathcal{U}}(U^*) = \mathcal{F}_{\mathcal{U}}(U^*UU^*) = VWV$. It follows that $V = W^*WV \Rightarrow V^* = V^*W^*W = V^*VW = WVW = \mathcal{F}_{\mathcal{U}}(UU^*U) = \mathcal{F}_{\mathcal{U}}(U) = W$ or equivalently $\mathcal{F}_{\mathcal{U}}(U^*) = \mathcal{F}_{\mathcal{U}}(U)^*$. □

Theorem 2.10. *The functor $\mathcal{F}_{\mathcal{U}} : {}_{\mathcal{A}}\mathfrak{DM} \to {}_{\mathcal{B}}\mathfrak{DM}$ is a $*$-functor.*

Proof. Let $T \in \mathrm{Hom}_{\mathcal{A}}{}^{\mathfrak{D}}(H_1, H_2)$ with polar decomposition $T = U|T|$. Observe that $(|T| + \epsilon I)^{-1} \in \alpha_1(\Delta(\mathcal{A}))'$ for every $\epsilon > 0$. Since $U = w^* - \lim_{\epsilon\to 0} T(|T| + \epsilon I)^{-1}$ it follows that $U \in \mathrm{Hom}_{\mathcal{A}}{}^{\mathfrak{D}}(H_1, H_2)$. The map

$$\mathcal{F}_{\mathcal{U}} : \mathrm{Hom}_{\mathcal{A}}{}^{\mathfrak{D}}(H_1, H_1) = \alpha_1(\Delta(\mathcal{A}))' \to \beta_1(\Delta(\mathcal{B}))' = \mathrm{Hom}_{\mathcal{B}}^{\mathfrak{D}}(\mathcal{F}_{\mathcal{U}}(H_1), \mathcal{F}_{\mathcal{U}}(H_1))$$

is an algebraic homomorphism between von Neumann algebras. We also proved in Theorem 2.8 that it is a contraction. It follows that it is a $*$-homomorphism. Therefore $\mathcal{F}_{\mathcal{U}}(|T|) \geq 0$. Using the previous lemma we obtain

$$\begin{aligned}\mathcal{F}_{\mathcal{U}}(T^*) &= \mathcal{F}_{\mathcal{U}}(|T|U^*) = \mathcal{F}_{\mathcal{U}}(|T|)\mathcal{F}_{\mathcal{U}}(U^*)\\ &= \mathcal{F}_{\mathcal{U}}(|T|)\mathcal{F}_{\mathcal{U}}(U)^* = (\mathcal{F}_{\mathcal{U}}(U)\mathcal{F}_{\mathcal{U}}(|T|))^* = \mathcal{F}_{\mathcal{U}}(T)^*. \quad \square\end{aligned}$$

3. Equivalence functors

In this section we prove that every functor $\mathcal{F}$ implementing the equivalence of Theorem 1.3 is equivalent to a functor of the form $\mathcal{F}_{\mathcal{U}}$ for some $\mathcal{B}$, $\mathcal{A}$ bimodule $\mathcal{U}$ and we also prove that $\mathcal{F}$ is normal and completely isometric.

Throughout this section we fix unital dual operator algebras $\mathcal{A}$, $\mathcal{B}$ and a functor $\mathcal{F}$ implementing the equivalence of Theorem 1.3. We choose an $\mathcal{A}$-universal object (H_0, α_0). Suppose that $(\mathcal{F}(H_0), \beta_0)$ is the corresponding object which is $\mathcal{B}$-universal (Corollary 1.5.) By the proof of Theorem 1.3 (Section 1) the map

$$\mathcal{F} : \mathrm{Hom}_{\mathcal{A}}{}^{\mathfrak{D}}(H_0, H_0) = \alpha_0(\Delta(\mathcal{A}))' \to \beta_0(\Delta(\mathcal{B}))' = \mathrm{Hom}_{\mathcal{B}}^{\mathfrak{D}}(\mathcal{F}(H_0), \mathcal{F}(H_0))$$

is a $*$-isomorphism with the property $\mathcal{F}(\alpha_0(\mathcal{A})') = \beta_0(\mathcal{B})'$, the space

$$\mathcal{M} = \{M \in B(H_0, \mathcal{F}(H_0)) : MF = \mathcal{F}(F)M \text{ for all } F \in \alpha_0(\Delta(\mathcal{A}))'\}$$

is an essential TRO and the algebras $\alpha_0(\mathcal{A})$, $\beta_0(\mathcal{B})$ are TRO equivalent via the space $\mathcal{M}$. We denote by $\mathcal{U}$ and $\mathcal{V}$ the spaces

$$\mathcal{U} = [\mathcal{M}\alpha_0(\mathcal{A})]^{-w^*}, \qquad \mathcal{V} = [\alpha_0(\mathcal{A})\mathcal{M}^*]^{-w^*}$$

which satisfy the following relations:

$$\beta_0(\mathcal{B})\mathcal{U}\alpha_0(\mathcal{A}) \subset \mathcal{U}, \qquad \alpha_0(\mathcal{A})\mathcal{V}\beta_0(\mathcal{B}) \subset \mathcal{V}, \quad [\mathcal{V}\mathcal{U}]^{-w^*} = \alpha_0(\mathcal{A}), \quad [\mathcal{U}\mathcal{V}]^{-w^*} = \beta_0(\mathcal{B}).$$

As in Section 2 we define a functor $\mathcal{F}_{\mathcal{U}} : {}_{\mathcal{A}}\mathfrak{M} \to {}_{\mathcal{B}}\mathfrak{M}$ which has a Δ-extension. In the rest of this section for every $(H, \alpha) \in {}_{\mathcal{A}}\mathfrak{M}$ we identify the element $T \otimes x$ with its image in $\mathcal{F}_{\mathcal{U}}(H)$ (see Section 2). Also we identify the algebra $\alpha_0(\mathcal{A})$ with $\mathcal{A}$ and the algebra $\beta_0(\mathcal{B})$ with $\mathcal{B}$.

Lemma 3.1. (i) *The map* $T \otimes x \to T(x) \quad T \in \mathcal{U},\ x \in H_0$ *extends to a unitary* $U : \mathcal{F}_{\mathcal{U}}(H_0) \to \mathcal{F}(H_0)$ *which belongs to the space* $\mathrm{Hom}_{\mathcal{B}}(\mathcal{F}_{\mathcal{U}}(H_0), \mathcal{F}(H_0))$.

(ii) *For all* $F \in \mathrm{Hom}_{\mathcal{A}}{}^{\mathfrak{D}}(H_0, H_0)$ *the equality* $U\mathcal{F}_{\mathcal{U}}(F) = \mathcal{F}(F)U$ *holds.*

Proof. (i) For all $T_1, \ldots, T_m \in \mathcal{U}, x_1, \ldots, x_m \in H_0$ we have

$$\left\| \sum_{j=1}^m T_j \otimes x_j \right\|_{\mathcal{F}_{\mathcal{U}}(H_0)} = \sup_{S \in \mathrm{Ball}(M_{n,1}(\mathcal{V})), n \in \mathbb{N}} \left\| \sum_{j=1}^m S T_j(x_j) \right\|_{H_0^{(n)}} \le \left\| \sum_{j=1}^m T_j(x_j) \right\|_{\mathcal{F}(H_0)}.$$

For arbitrary $\epsilon > 0$ there exist (Lemma 2.1) partial isometries $V_1, \ldots, V_n \in \mathcal{M}$ such that the operator $\sum_{i=1}^n V_i V_i^*$ is a projection and

$$\begin{aligned} \left\| \sum_{j=1}^m T_j(x_j) \right\|_{\mathcal{F}(H_0)} - \epsilon &\le \left\| \sum_{l=1}^n V_l V_l^* \sum_{j=1}^m T_j(x_j) \right\|_{\mathcal{F}(H_0)} \\ &\le \left\| (V_1^*, \ldots, V_n^*)^t \sum_{j=1}^m T_j(x_j) \right\|_{H_0^{(n)}} \le \left\| \sum_{j=1}^m T_j \otimes x_j \right\|_{\mathcal{F}_{\mathcal{U}}(H_0)}. \end{aligned}$$

It follows that $\left\| \sum_{j=1}^m T_j \otimes x_j \right\|_{\mathcal{F}_{\mathcal{U}}(H_0)} = \left\| \sum_{j=1}^m T_j(x_j) \right\|_{\mathcal{F}(H_0)}$. So the map $T \otimes x \to T(x)$, $T \in \mathcal{U}, x \in H_0$ extends to an isometry $U : \mathcal{F}_{\mathcal{U}}(H_0) \to \mathcal{F}(H_0)$. Since $[\mathcal{U}(H_0)]^- = \mathcal{F}(H_0)$ the image of U is dense in $\mathcal{F}(H_0)$, so U is a unitary. We can easily check that $U \in \mathrm{Hom}_{\mathcal{B}}(\mathcal{F}_{\mathcal{U}}(H_0), \mathcal{F}(H_0))$.

(ii) Let $F \in \mathrm{Hom}_{\mathcal{A}}{}^{\mathfrak{D}}(H_0, H_0)$. For every $M \in \mathcal{M}, x \in H_0$ we have

$$(U\mathcal{F}_{\mathcal{U}}(F))(M \otimes x) = U(M \otimes F(x)) = M(F(x)) = \mathcal{F}(F)M(x) = (\mathcal{F}(F)U)(M \otimes x).$$

By Corollary 2.5 it follows that $U\mathcal{F}_{\mathcal{U}}(F) = \mathcal{F}(F)U$. □

The following lemma is analogous to [9, Proposition 4.9]. The proof is similar, using the Δ-extension of the functors $\mathcal{F}, \mathcal{F}_{\mathcal{U}}$.

Lemma 3.2. *If* $\{H_j : j \in I\}$ *are objects of* ${}_{\mathcal{A}}\mathfrak{M}$, *then there exist unitaries* $W \in \mathrm{Hom}_{\mathcal{B}}(\oplus_j \mathcal{F}(H_j), \mathcal{F}(\oplus_j H_j))$, *and* $V \in \mathrm{Hom}_{\mathcal{B}}(\oplus_j \mathcal{F}_{\mathcal{U}}(H_j), \mathcal{F}_{\mathcal{U}}(\oplus_j H_j))$.

Theorem 3.3. *The functors* $\mathcal{F}, \mathcal{F}_{\mathcal{U}}$ *are equivalent as functors between the categories* ${}_{\mathcal{A}}\mathfrak{M}, {}_{\mathcal{B}}\mathfrak{M}$ *and their* Δ*-extensions are equivalent as* $*$*-functors between the categories* ${}_{\mathcal{A}}\mathfrak{DM}, {}_{\mathcal{B}}\mathfrak{DM}$.

Proof. Since H_0 is an $\mathcal{A}$-universal object, it is also $W^*(\mathcal{A})$-universal (Section 1). Therefore, by [9, Proposition 1.1] for every $K \in {}_{\mathcal{A}}\mathfrak{M}$ there exists a set of indices J_K, projections

$$\{Q_i^K : i \in J_K\} \subset \mathrm{Hom}_{W^*(\mathcal{A})}(H_0, H_0) \subset \mathrm{Hom}_{\mathcal{A}}(H_0, H_0)$$

and a unitary

$$W_K \in \mathrm{Hom}_{W^*(\mathcal{A})}(K, \oplus_i Q_i^K(H_0)) \subset \mathrm{Hom}_{\mathcal{A}}(K, \oplus_i Q_i^K(H_0)).$$

Since the Δ-extensions of $\mathcal{F}, \mathcal{F}_{\mathcal{U}}$ are $*$-functors, the operators $\mathcal{F}(W_K), \mathcal{F}_{\mathcal{U}}(W_K)$ are unitaries. By Lemma 3.2 we can view $\mathcal{F}_{\mathcal{U}}(W_K)$ as an element

$$\mathcal{F}_{\mathcal{U}}(W_K) \in \mathrm{Hom}_{\mathcal{B}}(\mathcal{F}_{\mathcal{U}}(K), \oplus_i \mathcal{F}_{\mathcal{U}}(Q_i^K(H_0)))$$

and

$$\mathcal{F}(W_K) \in \mathrm{Hom}_{\mathcal{B}}(\mathcal{F}(K), \oplus_i \mathcal{F}(Q_i^K(H_0))).$$

Lemma 3.1, ii, shows that $U\mathcal{F}_{\mathcal{U}}(Q_i^K) = \mathcal{F}(Q_i^K)U$. Thus the operator

$$U_i^K = U|_{\mathcal{F}_{\mathcal{U}}(Q_i^K(H_0))} : \mathcal{F}_{\mathcal{U}}(Q_i^K(H_0)) \to \mathcal{F}(Q_i^K(H_0))$$

is a unitary for all $i \in J_K$. So we can define the unitary

$$V_K = \mathcal{F}(W_K^*)(\oplus_i U_i^K)\mathcal{F}_{\mathcal{U}}(W_K) \in \mathrm{Hom}_{\mathcal{B}}(\mathcal{F}_{\mathcal{U}}(K), \mathcal{F}(K)) \subset \mathrm{Hom}_{\mathcal{B}}^{\mathfrak{D}}(\mathcal{F}_{\mathcal{U}}(K), \mathcal{F}(K)).$$

As in the proof of [9, Proposition 5.4] we can prove that the unitaries $\{V_K : K \in {}_{\mathcal{A}}\mathfrak{M}\}$ implement both the required equivalences. □

Definition 3.1. Let $\mathcal{A}_1, \mathcal{B}_1$ be unital dual operator algebras. A functor $\mathcal{G} : {}_{\mathcal{A}_1}\mathfrak{M} \to {}_{\mathcal{B}_1}\mathfrak{M}$ is called **completely isometric (resp. normal)** if for every pair of objects H_1, H_2 the map $\mathcal{G} : \mathrm{Hom}_{\mathcal{A}_1}(H_1, H_2) \to \mathrm{Hom}_{\mathcal{B}_1}(\mathcal{G}(H_1), \mathcal{G}(H_2))$ is a complete isometry (resp. w^*-continuous). And similarly for a functor $\mathcal{G} : {}_{\mathcal{A}_1}\mathfrak{DM} \to {}_{\mathcal{B}_1}\mathfrak{DM}$.

Lemma 3.4. *The functor $\mathcal{F}_{\mathcal{U}} : {}_{\mathcal{A}}\mathfrak{DM} \to {}_{\mathcal{B}}\mathfrak{DM}$ is normal.*

Proof. Let $H_1, H_2 \in {}_{\mathcal{A}}\mathfrak{M}$. We have proved in Theorem 2.8 that $\|\mathcal{F}_{\mathcal{U}}(F)\| \leq \|F\|$ for all $F \in \mathrm{Hom}_{\mathcal{A}}^{\mathfrak{D}}(H_1, H_2)$. So it suffices to show that if (F_i) is a bounded net of the space $\mathrm{Hom}_{\mathcal{A}}(H_1, H_2)$ which converges in the weak operator topology to 0 then the net $(\mathcal{F}_{\mathcal{U}}(F_i))$ converges in the weak operator topology to 0 too. We recall from Section 2 the contractions $\theta_S : \mathcal{F}_{\mathcal{U}}(H_2) \to H_{2,S}$ and the quotient maps π, π_S where $S \in \mathrm{Ball}(M_{n,1}(\mathcal{V})), n \in \mathbb{N}$. If $M \in \mathcal{M}, x \in H_1, T \in \mathcal{U}$ and $y \in H_2$ then

$$\begin{aligned}\left\langle \mathcal{F}_{\mathcal{U}}(F_i)(\pi(M \otimes x)), \theta_S^*(\pi_S(T \otimes y))\right\rangle_{\mathcal{F}_{\mathcal{U}}(H_2)} &= \langle \theta_S(\pi(M \otimes F_i(x))), \pi_S(T \otimes y)\rangle_{H_{2,S}} \\ &= \langle \pi_S(M \otimes F_i(x)), \pi_S(T \otimes y)\rangle_{H_{2,S}} = \langle \alpha_2(SM)F_i(x), \alpha_2(ST)(y)\rangle \to 0\end{aligned}$$

We recall from Lemma 2.3 that

$$\mathcal{F}_{\mathcal{U}}(H_2) = [\theta_S^*(\pi_S(T \otimes y)) : S \in \mathrm{Ball}(M_{m,1}(\mathcal{V})), m \in \mathbb{N}, T \in \mathcal{U}, y \in H_2]^-$$

and from Corollary 2.5 that the space $\pi(\mathcal{M} \otimes H_1)$ is dense in $\mathcal{F}_{\mathcal{U}}(H_1)$. Since the net $(\mathcal{F}_{\mathcal{U}}(F_i))$ is bounded it follows that $\langle \mathcal{F}_{\mathcal{U}}(F_i)(z), \xi\rangle \to 0$ for all $z \in \mathcal{F}_{\mathcal{U}}(H_1), \xi \in \mathcal{F}_{\mathcal{U}}(H_2)$. □

Lemma 3.5. *The functor $\mathcal{F}_{\mathcal{U}} : {}_{\mathcal{A}}\mathfrak{DM} \to {}_{\mathcal{B}}\mathfrak{DM}$ is completely isometric.*

Proof. Let $(H_1, \alpha_1), (H_2, \alpha_2) \in {}_{\mathcal{A}}\mathfrak{M}$ and $(F_{ij}) \in M_n(\mathrm{Hom}_{\mathcal{A}}^{\mathfrak{D}}(H_1, H_2))$ for $n \in \mathbb{N}$. Fix vectors $z_j = \sum_{k=1}^{m_j} M_k^j \otimes x_k^j \in \mathcal{M} \otimes H_1, j = 1, \dots, n$, and denote by z the vector $(z_1, \dots, z_n)^t$. Then

$$\left\|(\mathcal{F}_{\mathcal{U}}(F_{ij}))(z)\right\|^2 = \sum_{i=1}^n \left\|\sum_{j=1}^n \sum_{k=1}^{m_j} M_k^j \otimes F_{ij}(x_k^j)\right\|_{\mathcal{F}_{\mathcal{U}}(H_2)}^2.$$

For $\epsilon > 0$ by Proposition 2.2 there exist $r \in \mathbb{N}$ and $S_i = (S_1^i, \dots, S_r^i)^t \in \mathrm{Ball}(M_{r,1}(\mathcal{M}^*)), i = 1, \dots, n$, such that

$$\begin{aligned}\left\|(\mathcal{F}_{\mathcal{U}}(F_{ij}))(z)\right\|^2 - \epsilon &\leq \sum_{i=1}^n \left\|\sum_{j=1}^n \sum_{k=1}^{m_j} \alpha_2(S_i M_k^j)F_{ij}(x_k^j)\right\|_{H_2^r}^2 = \sum_{i=1}^n \sum_{l=1}^r \left\|\sum_{j=1}^n \sum_{k=1}^{m_j} \alpha_2(S_l^i M_k^j)F_{ij}(x_k^j)\right\|_{H_2}^2 \\ &= \sum_{i=1}^n \sum_{l=1}^r \left\|\sum_{j=1}^n \sum_{k=1}^{m_j} F_{ij}\alpha_1(S_l^i M_k^j)(x_k^j)\right\|_{H_2}^2 = \sum_{l=1}^r \left\|(F_{ij})\begin{bmatrix}\sum_{k=1}^{m_1} \alpha_1(S_l^1 M_k^1)(x_k^1) \\ \vdots \\ \sum_{k=1}^{m_n} \alpha_1(S_l^n M_k^n)(x_k^n)\end{bmatrix}\right\|_{H_2^n}^2\end{aligned}$$

$$\leq \|(F_{ij})\|^2 \sum_{l=1}^{r} \left\| \begin{bmatrix} \sum_{k=1}^{m_1} \alpha_1(S_l^1 M_k^1)(x_k^1) \\ \vdots \\ \sum_{k=1}^{m_n} \alpha_1(S_l^n M_k^n)(x_k^n) \end{bmatrix} \right\|_{H_1^n}^2 = \|(F_{ij})\|^2 \sum_{i=1}^{n} \left\| \sum_{k=1}^{m_i} \alpha_1(S_i M_k^i)(x_k^i) \right\|_{H_1^r}^2$$

$$\leq \left\|(F_{ij})\right\|^2 \sum_{i=1}^{n} \|z_i\|_{\mathcal{F}_{\mathcal{U}}(H_1)}^2 = \left\|(F_{ij})\right\|^2 \|z\|_{\mathcal{F}_{\mathcal{U}}(H_1)^n}^2 .$$

Since ϵ was arbitrary we have $\left\|(\mathcal{F}_{\mathcal{U}}(F_{ij}))(z)\right\| \leq \|(F_{ij})\| \|z\|$ for all $z \in M_{n,1}(\mathcal{M} \otimes H_1)$. From Corollary 2.5 it follows that $\left\|(\mathcal{F}_{\mathcal{U}}(F_{ij}))\right\| \leq \|(F_{ij})\|$. By Theorem 3.3, $\mathcal{F}_{\mathcal{U}}$ is an equivalence functor; hence there is a functor $\mathcal{G}$ such that $\mathcal{G} \circ \mathcal{F}_{\mathcal{U}}$ is equivalent to the identity functor. As above we see that $\mathcal{G}$ can be taken of the form $\mathcal{G}_{\mathcal{W}}$ for a suitable bimodule $\mathcal{W}$. Hence the reverse inequality follows. $\square$

Combining Lemma 3.4, 3.5 and Theorem 3.3 we obtain the next theorem:

Theorem 3.6. *Every functor implementing the equivalence of Theorem 1.3 is a normal and completely isometric functor.*

Concluding remarks

1. In a companion paper [7] we show that every functor implementing the equivalence of Theorem 1.3 maps completely isometric representations to completely isometric representations and reflexive algebras to reflexive algebras. Also we present examples of Δ-equivalent and Δ-inequivalent CSL algebras.
2. The original proof (see ArXiv:math.OA/0607489v.3) of one direction of Theorem 1.3 (if the algebras have completely isometric normal representations with TRO equivalent images then they are Δ-equivalent) was by proving that the functor $\mathcal{F}_{\mathcal{U}}$, constructed in Section 2, is an equivalence functor. After this work was submitted, the present author and V.I. Paulsen proved in [8] that TRO equivalent algebras are stably isomorphic. We thank the referee for suggesting that we use this result to shorten our original proof.

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