Extreme Mechanics Letters 9 (2016) 91–96

Contents lists available at ScienceDirect

Extreme Mechanics Letters

journal homepage: www.elsevier.com/locate/eml

Harnessing structural hierarchy to design stiff and lightweight phononic crystals

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A R T I C L E   I N F O

Article history:
Received 4 April 2016
Received in revised form 18 May 2016
Accepted 18 May 2016
Available online 24 May 2016

Keywords:
Phononic crystals
Hierarchical
Wave propagation
Honeycombs
Band gaps

A B S T R A C T

In this letter we report a class of hierarchically architected honeycombs in which structural hierarchy can be exploited to achieve prominent wave attenuation and load-carrying capabilities. The hierarchically architected honeycombs can exhibit broad and multiple phononic band gaps. The mechanisms responsible for these band gaps depend on the geometric features of the hierarchical honeycombs rather than their composition. Furthermore, the introduction of structural hierarchy also endows the hierarchical honeycombs with enhanced stiffness. We predict that the proposed hierarchical honeycombs can realize a unique combination of wave attenuation and load-carrying capabilities, thereby providing opportunities to design lightweight and stiff phononic crystals for various engineering applications.

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Phononic crystals have gained increasing research interests because of their rationally designed periodic architectures and compositions enabling to modify phononic dispersion relations, thereby providing avenues to tailor group velocities and hence the flow of vibrational energy [1]. When the structural periodicity of phononic crystals is comparable to the wavelength of propagating waves, Bragg interference of elastic waves scattered by the compositions arises. This mechanism gives rise to complete wave band gaps: frequency ranges where incident elastic waves are not allowed to propagate. This fundamental property offers a variety of promising applications, including wave filtering [2,3], waveguiding [4–6], and energy harvesting [7–9]. However, the inherent architectures and compositions, if not designed properly, could not generate desired wave band gaps and even lead to mechanical instability that is inapplicable for load-carrying conditions.

Structural hierarchy has been employed as an important strategy to explore improved mechanical properties and other unusual physical properties. Typical examples range from Eiffel tower to hierarchically architected nanotrusses with multiple length scales [10–12]. Recent studies show that it is possible to manipulate wave propagation by harnessing multiscale characteristic of hierarchical architectures [13–15]. These rationally designed hierarchical architectures can give rise to multiple and broadband phononic band gaps as well as low frequency band gaps. In addition, these hierarchical architectures also enable the coexisting of multiband wave filtering and waveguiding in an ultrawide frequency range [16].

Despite these considerable efforts, challenges still remain. For example, to achieve the desired wave attenuation, soft materials using thermally coupled dissipation mechanism are often employed in engineering practice. As a result, the wave attenuation capability strongly depends on the thickness of the materials, thus posing a great challenge to design lightweight and stiff materials with strong wave attenuation ability. Furthermore, conventional phononic crystals with periodic architectures can only provide limited frequency band gaps since the Bragg interference requires that the wavelength must be comparable to the given structural periodicity. To overcome

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http://dx.doi.org/10.1016/j.eml.2016.05.009
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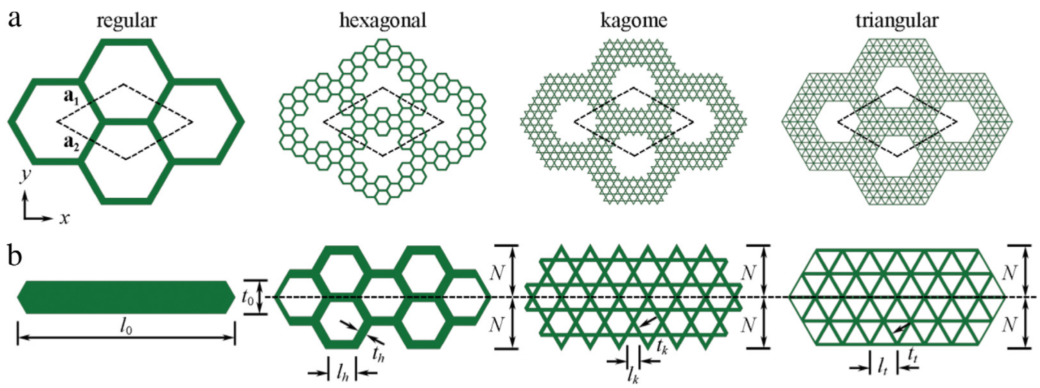

Fig. 1. (a) Schematics of regular honeycomb and hierarchical honeycombs. Here $\mathbf{a_1} = (3l_0/2, \sqrt{3}l_0/2)$, $\mathbf{a_2} = (3l_0/2, -\sqrt{3}l_0/2)$ are lattice constants; l_0 and t_0 are the length and thickness of cell walls. The dash lines indicate the supercells; (b) schematics of cell walls of regular honeycombs hexagonal, kagome, and triangular hierarchical honeycombs, respectively.

the bandwidth limitation, numerical approaches such as topology optimization have been developed to maximize the band gap size [17,18]. It is worth noting that the objective function of this approach is to maximize a single band gap size, and the resultant architectures are still spatially periodic. Lightweight and stiff phononic crystals with broadband and multiband wave attenuation ability remain unrealized.

Here, we report a class of hierarchically architected honeycombs in which structural hierarchy is exploited to simultaneously improve the wave attenuation and load-carrying capabilities. The proposed hierarchical architectures are constructed by replacing the cell walls of the regular honeycombs with hexagonal, kagome, and triangular lattices, respectively (referred to as hexagonal, kagome, and triangular hierarchical honeycombs for simplicity in the following, Fig. 1(a)–(e)). For the purpose of fair comparison, kagome and triangular hierarchical honeycombs are subsequently obtained by connecting the midpoints and vertices of the hexagonal lattice, respectively. The proposed hierarchical honeycombs are characterized by two geometric parameters, hierarchical length ratio, $\gamma = l_h/l_0$, and the number of hexagonal lattice away from the central axis, N, where l_0 and l_h are the length of cell walls of regular lattice and hexagonal lattice, respectively. The length and thickness of the hexagonal, kagome, and triangular lattices are determined by mass/volume equivalence between regular honeycombs and hierarchical honeycombs (See Supporting Information, Appendix A). The composition of the regular honeycombs and hierarchical honeycombs is a glassy polymer, SU-8, whose properties are characterized by a Young's modulus $E_s = 3.3$ GPa, Poisson's ratio $\nu = 0.33$, a yield stress $\sigma_y = 105$ MPa, and density $\rho_s = 1200$ kg/m^3 [19].

To investigate the wave attenuation capability of the proposed hierarchical honeycombs, phononic dispersion relations are constructed by performing eigenfrequency analysis within the finite element framework using the commercial package COMSOL Multiphysics. Note that we focus on the in-plane wave propagation in the hierarchical honeycombs, thus a plane strain assumption is made without loss of generality. To capture the periodic feature of the hierarchical honeycombs, Bloch's periodic boundary conditions are applied at the boundaries of the supercell. The supercell is discretized using 6-node triangular elements. We then solve the wave equation by scanning the wave vectors in the first irreducible Brillouin zone. More details concerning the modeling of wave propagation are provided in the Supporting Information (see Appendix A).

We start by examining the phononic dispersion relations of hierarchical honeycombs with $\gamma = 1/5$, $N = 1$, and relative density $\rho/\rho_s = 0.06$. For the purpose of comparison, phononic dispersion relation of the associated regular honeycomb is also reported. For the regular honeycomb, we only observe one narrow band gap at $\varpi = 0.059$–0.061 (Fig. 2(a)). By contrast, the introduction of structural hierarchy in the regular honeycombs leads to much broader band gaps (Fig. 2(b)–(d)). Specifically, the maximum band gaps in hexagonal, kagome, and triangular hierarchical honeycombs are $\varpi = 0.047$–0.079, $\varpi = 0.108$–0.133, and $\varpi = 0.064$–0.078, respectively. In addition, the introduction of structural hierarchy also gives rise to multiple band gaps, as shown in the phononic dispersion relations. To gain a deeper understanding, we plot the eigenmodes of the high-symmetry points $\bar{\Gamma}$, $\bar{M}$, and $\bar{K}$ at the lower band edges of the band gaps (Red lines in Fig. 2). For hexagonal and kagome hierarchical honeycombs, the vibrational modes of the high-symmetry points exhibit a global nature, indicating a Bragg-type band gap. Interestingly, localized vibrational modes are observed for the triangular hierarchical honeycombs, suggesting that local resonances are responsible for the broad band gaps [20–23]. This is also supported by the flat band edge of the band gaps. A direct comparison between the geometric features of the regular honeycomb and hierarchical honeycombs leads us to believe that different mechanisms of band gaps formation are intrinsically dictated by the slenderness ratio and coordination number of the lattice. It should be pointed out that damping effect resulting from the viscoelastic feature of the glassy polymer may make some contribution to the wave attenuation [24]. However, recent experimental results indicate that the damping effect will not swamp the band gaps in the phononic dispersion relations [25].

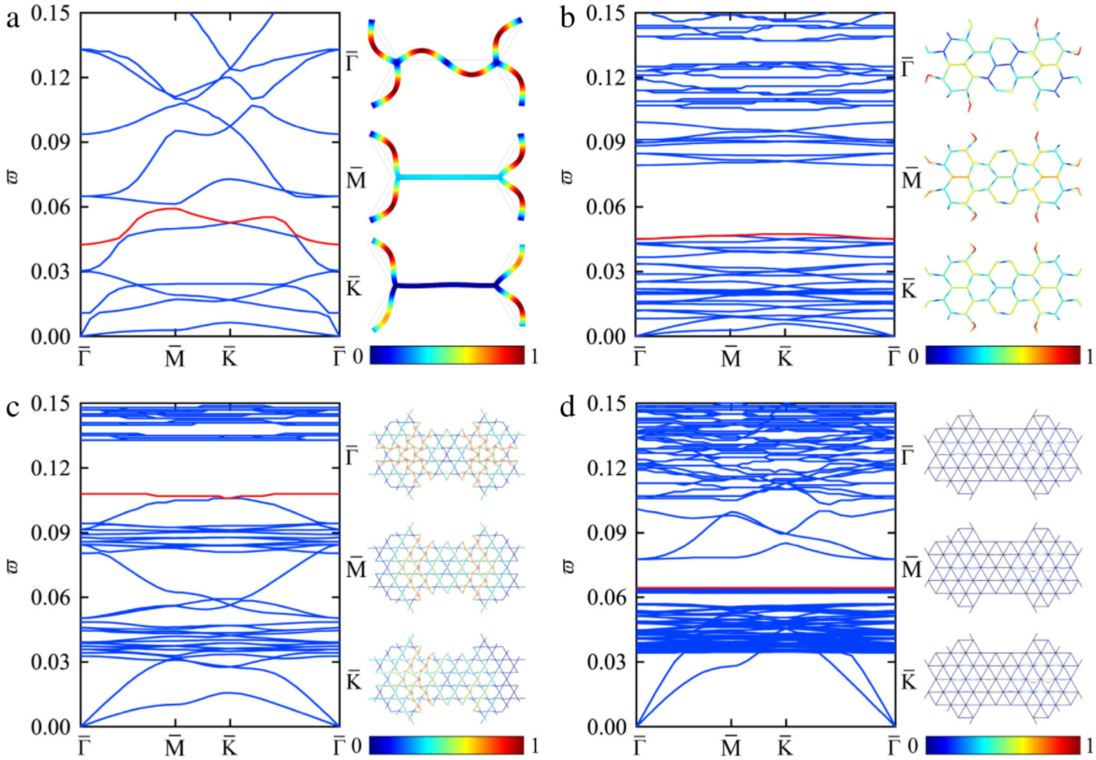

Fig. 2. Phonon dispersion relations of regular and hierarchical honeycombs with $\gamma = 1/5$, $N = 1$, and relative density $\rho/\rho_s = 0.06$. (a) Regular honeycomb; (b)–(d) hexagonal, kagome, and triangular hierarchical honeycombs, respectively. Eigenmodes of the high-symmetry points $\bar{\Gamma}$, $\bar{M}$, and $\bar{K}$ at the lower band edges of the band gaps are also plotted. Here $\bar{\Gamma} = (0, 0)$, $\bar{M} = (2\pi/3l_0, 0)$, and $\bar{K} = (2\pi/3l_0, 2\sqrt{3}\pi/9l_0)$. The normalized frequency is defined as $\varpi = \omega a/2\pi c_t$, where ω is frequency, a is the length of lattice constant, c_t is the transverse wave velocity of the constituent material. The legends indicate the amplitude of normalized displacement.

Having demonstrated that the broadband and multiple band gaps are dictated by the slenderness ratio and coordination number, we now examine effects of two geometric parameters, γ and N, on the evolution of band gaps. Note that for a given relative density and a type of hierarchical honeycomb, the slenderness ratio of the lattice is uniquely controlled by γ and N. Here we consider the case that the relative density $\rho/\rho_s = 0.16$ for the hierarchical honeycombs to ensure that the cell walls of each lattice have considerable thickness at large N. To quantitatively evaluate the wave attenuation capability of the hierarchical honeycombs, we define two indicators to consider the broadband and multiband features: maximum relative band gaps, $(\Delta\omega/\omega_*)_{\max}$ and total relative band gaps, $\sum(\Delta\omega/\omega_*)$, where $\Delta\omega$ is band gap width and ω_* is the midgap frequency. As shown in Fig. 3, both the maximum band gaps and total band gaps tend to diminish for $N > 2$. On one hand, for a given relative density, larger N indicates more substructures and larger slenderness ratio. While for wave propagation in lattice structures, slenderness ratio is critical to the band gaps formation [26]. On the other hand, with the increase of N, the effect of structural hierarchy becomes weaker. At the maximum N, all hierarchical structures reduce into regular lattice materials, which do not have or only have small band gaps, depending on the slenderness ratio and node connectivity [26,27]. For a given $N \leq 2$, the maximum band gaps and total band gaps tend to decrease when the hierarchical length ratio decreases from 1/2 to 1/11. The maximum band gaps and total band gaps of regular honeycombs are also plotted in Fig. 3 for the purpose of comparison ($\gamma = 1$). We observe that hierarchical honeycombs with $\gamma = 1/2$ exhibit comparable or larger maximum band gaps, whereas the total band gaps strongly depend on the shape of the lattice when compared with that of the regular honeycomb. These quantitative analyses not only support our conclusion concerning the mechanisms underlying the band gap formation, but also provide clues to design phononic crystals with desired wave attenuation capability.

To demonstrate the potential of designing lightweight phononic crystals, a quantitative investigation is carried out to examine the effect of relative density on the band gap evolution (Fig. 4). Here we choose $\gamma = 1/5$ and $N = 1$, and $\rho/\rho_s = 0.06$–0.32. Noticeably, both the maximum band gaps and total band gaps are inversely proportional to the relative density. For the maximum band gaps, hierarchical honeycombs show much smaller exponents than that of regular honeycombs, indicating that hierarchical honeycombs can be potentially designed with light weight. For the total band gaps, we note that hexagonal and triangular hierarchical honeycombs exhibits smaller exponents. Although the exponent of kagome hierarchical honeycomb is larger than that of regular honeycomb, an inverse relation between total band gaps and relative density still can be observed.

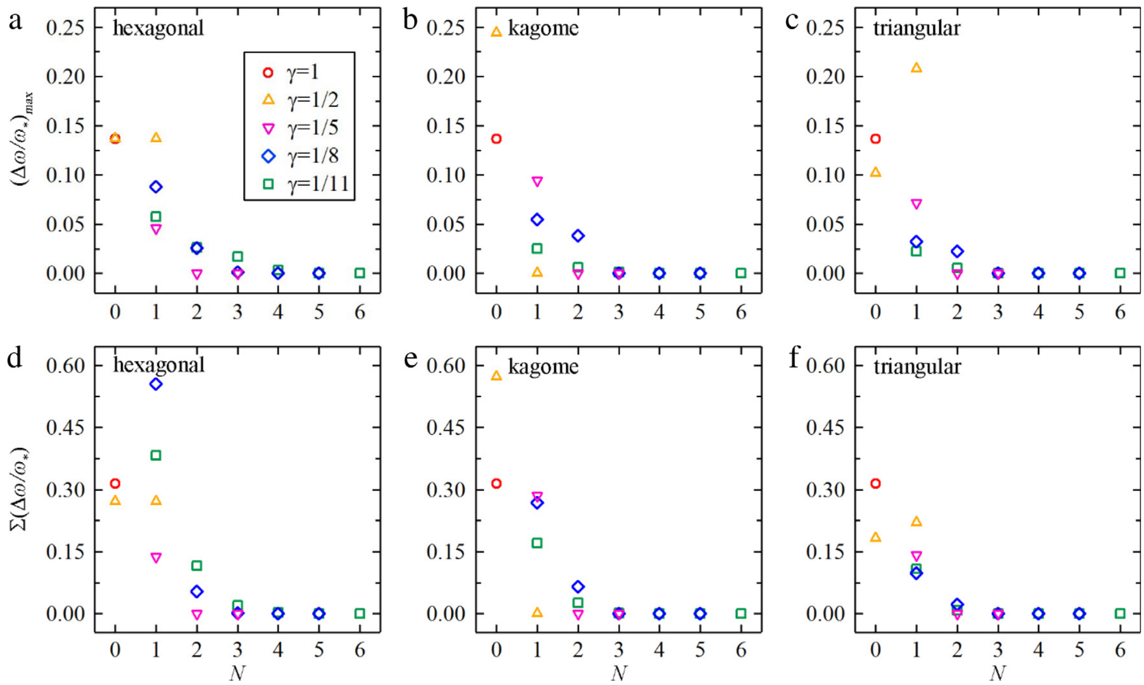

Fig. 3. Effects of hierarchical length ratio and number of lattice on the evolution of maximum band gaps and total band gaps. (a)–(c) Maximum band gaps in hexagonal, kagome, and triangular hierarchical honeycombs, respectively; (d)–(f) total band gaps in hexagonal, kagome, and triangular hierarchical honeycombs, respectively. $\Delta\omega$ is band gap width and ω_* is the midgap frequency. Here the relative density is $\rho/\rho_s = 0.16$.

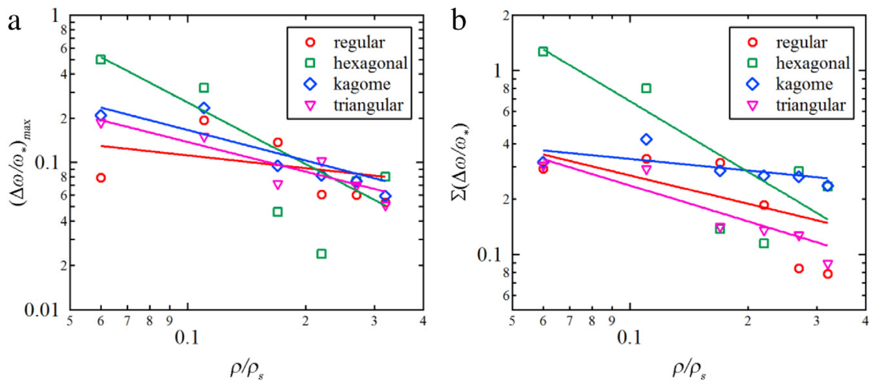

Fig. 4. Effects of relative density on maximum band gaps and total band gaps. (a) Maximum band gaps and (b) total band gaps of regular and hierarchical honeycombs. Here $\gamma = 1/5$ and $N = 1$. The solid lines represent the numerical data fitting using scaling law.

It has been demonstrated that hierarchal honeycombs can have improved mechanical properties. To further explore the possibility to design phononic crystals with relatively high stiffness, we numerically examine the mechanical response of the proposed hierarchical honeycombs under uniaxial compression. A constitutive stress–strain behavior of the glassy polymer SU-8 together with a periodic representative volume element of each regular and hierarchical honeycomb is employed to predict the mechanical response [19,28]. More details concerning the constitutive stress–strain behavior of SU-8 and the implementation of periodic boundary conditions can be found in the Supporting Information (see Appendix A).

Fig. 5(a) reports the mechanical response of the hierarchical honeycombs with $\gamma = 1/5$ and $N = 1$, and $\rho/\rho_s = 0.06$ under uniaxial compression up to 10% macroscopic strain. For the regular honeycomb, we observe a typical stress–strain relation including an initial linear-elastic regime and a following non-linear trend induced by plastic deformation. As compared to the regular honeycomb, hexagonal hierarchical honeycomb exhibits a very similar response, but a slightly lower stress–strain curve, indicating a comparable stiffness. For kagome and triangular hierarchical honeycombs, the stress increases rapidly with the strain followed by higher yield/buckling stress, indicating much higher stiffness. The highly nonlinear behavior of hierarchical honeycombs is dictated by local buckling of the cell walls together with the plastic deformation of SU-8. Importantly, these deformation mechanisms will endow hierarchical honeycombs with enhanced energy ab-

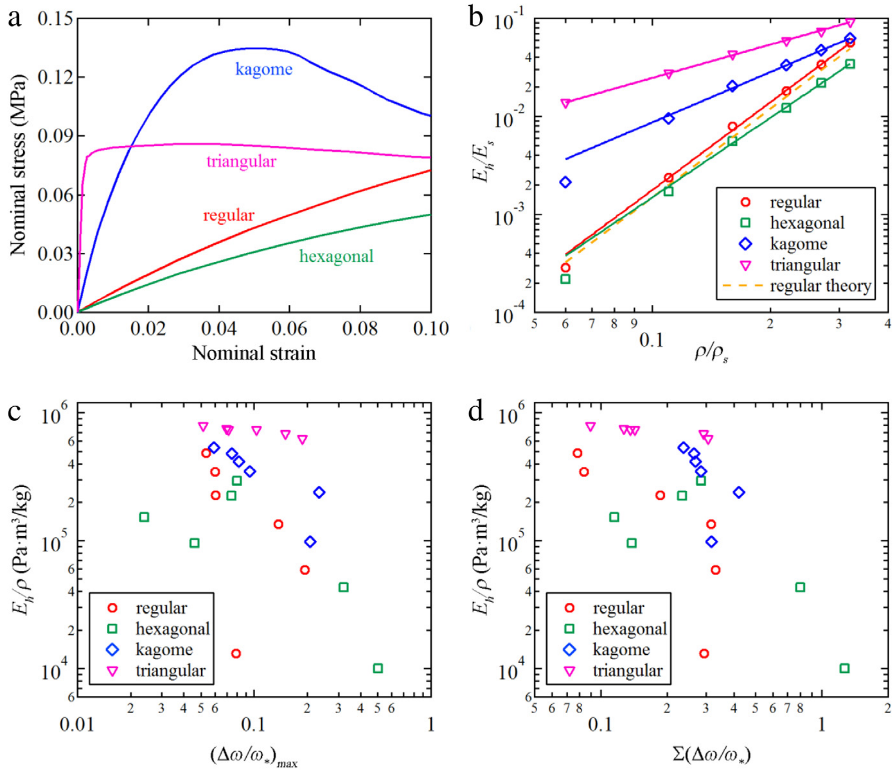

Fig. 5. (a) Stress–strain relations of regular honeycomb and hierarchical honeycombs compressed along y direction; (b) Relations between stiffness and relative density of regular honeycomb and hierarchical honeycombs; (c) and (d): Ashby-type plots of specific modulus and maximum band gaps and total band gaps. E_h and E_s are the Young's modulus of hierarchical honeycombs and constituent material SU-8, respectively. Here $\gamma = 1/5$ and $N = 1$.

sorption capacity. It should be emphasized that hierarchical honeycombs can be constructed by replacing each vertex of regular honeycombs with a smaller self-similar hexagon [29]. In this case, broad and multiple band gaps can be retained, the stiffness, however, will be significantly sacrificed.

Fig. 5(b) shows the stiffness of the hierarchical honeycombs at relative density range $\rho/\rho_s = 0.06$–0.32. For the regular honeycombs, the simulated stiffness agrees well with predictions from linear elastic theory [30], indicating that our numerical framework can accurately predict the mechanical response of the regular honeycombs. Hexagonal hierarchical honeycombs have comparable yet slightly lower stiffness than that of regular honeycomb. Notably, kagome and triangular hierarchical honeycombs show significantly improved stiffness. For example, at low relative density ($\rho/\rho_s = 0.06$), kagome and triangular hierarchical honeycombs exhibit an improved stiffness by nearly one and two orders of magnitude as compared to the regular honeycomb and the hexagonal hierarchical honeycomb, respectively. We fit the stiffness as a function of relative density using a scaling law, $E_h/E_s = C(\rho/\rho_s)^n$, where E_h and E_s are the stiffness of hierarchical honeycombs and solid constituent material SU-8, respectively, C is geometry-dependent proportionality constant, and n is the scaling exponent. As a result, the scaling exponents for regular honeycomb, hexagonal, kagome, and triangular hierarchical honeycombs are 2.97, 2.69, 1.70, and 1.13, respectively, indicating that regular honeycomb and the hexagonal hierarchical honeycomb exhibit a bending-dominated deformation behavior, whereas kagome and triangular hierarchical honeycombs have a stretching-dominated deformation behavior. Intrinsically, the discrepancy between the bending-dominated and stretching-dominated behavior is governed by the geometric features of the lattice, i.e., slenderness ratio and coordinate numbers.

By combining the simulated stiffness and band gaps of the hierarchical honeycombs with different relative densities, we obtain the Ashby-type plots of specific modulus versus maximum band gaps and total band gaps, as shown in Fig. 5(c) and (d), respectively. Compared with the regular honeycombs, hexagonal hierarchical honeycombs retains comparable specific modulus but broader and multiple band gaps. Remarkably, kagome and triangular hierarchical honeycombs can achieve specific stiffness that are 40–60 times higher while having similar maximum band gaps and total band gaps, compared with that of regular honeycombs. From a practical perspective, the proposed hierarchical honeycombs have great potential applications in areas where lightweight, wave attenuation, and load carrying capacity are simultaneously desired.

In summary, the numerical analyses in this work provide insights into the effect of structural hierarchy on the wave attenuation and load-carrying capabilities. We have demonstrated that broad and multiple band gaps can be achieved in the proposed hierarchical honeycombs, providing that the geometric parameters are rationally selected. In addition, kagome and triangular hierarchical honeycombs exhibit improved specific stiffness compared with regular honeycombs. We emphasize that the achieved outstanding wave attenuation capability and enhanced mechanical properties are attributed to the introduction of the structural hierarchy. Thus the proposed hierarchical honeycombs can be termed a new type of metamaterials. The findings reported here will provide new opportunities to design lightweight and stiff phononic crystals for various applications including underwater wave mitigation in submarines and other structural vibration mitigation in defense, aerospace, and automotive industries.

Acknowledgments

The authors gratefully acknowledge the financial support from the National Science Foundation (CMMI-1462270) and the Office of Naval Research (Dr. Yapa Rajapakse). Y. Chen thanks Dr. Pai Wang (Harvard University) and Dr. Yongtao Sun (Tianjin University) for helpful discussion. The authors also thank Yue Wang (Stony Brook University) for helping run part of the simulations.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.eml.2016.05.009.

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