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Performance evaluation of multi-input–single-output (MISO) production process using transfer function and fuzzy logic: Case study of a brewery

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Brewery;
Modeling

Abstract This work reports an improved and novel new method of evaluating the performance of multi input single output (MISO) processes, as exemplified by a brewery. This new method involves the combination of transfer function modeling and fuzzy logic and was used in evaluating the six years performance of a brewery. Of the six years, the period 2010–2011 with a performance rating λ of 0.810 which corresponds to the linguistic variable 'Good' recorded the best performance while the period 2008–2009 with a performance rating λ of 0.381 which corresponds to the linguistic variable 'Fair' recorded the worst performance. The result of this study is expected to open new ways of improving maintenance effectiveness, utilization of raw materials and efficiency of multi input single output (MISO) production processes.

1. Introduction

In brewing as in many other production processes, there are problems of wastes, work in progress (WIP) and low quality raw materials. If the plant/facility is not functioning very well there could be excessive build up of wastes and work in progress, which causes cost to the organization. Low quality raw materials would result to increase in the quantity of additives added to the drink in order to improve its quality. Ideally, if it is possible to produce the drink without the additives, the operation performance in terms of raw material consumption would be perfect. But this scenario is only theoretical, because most of cereals require additives when being brewed into beer

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Nomenclature, Symbols and Notations

k	lag variable	Ξ	coefficient of output variable of differential equation
β_t	pretreated output series	H	coefficient of input variable of differential equation
α_t	prewhitened input series	χ	covariance function
$v(B)$	transfer function	b	transfer function lag
B	backshift operator	ω	difference equation variable for input
Y_t	process output at time t	δ	difference equation variable for output
X_t	process input at time t	r	order of the output series
y_t	differenced output series	s	order of the input series
x_t	differenced input series	S	sample standard deviation
$\hat{Y}_t$	output forecast	σ	population standard deviation
$\hat{X}_t$	input forecast	ρ	auto correlation function
a_t	error term/white noise	γ	cross correlation function
υ_k	impulse response weight at lag k	μ	mean
h	ACF/PACF lag	ACF	Auto Correlation Function
q	order of moving average operator	PACF	Partial Auto Correlation Function
p	order of autoregressive operator	N_t	noise term
d	number of differencing		
θ	autoregressive operator		

or malt drink. If the plant is not working well, then the actual percentage of cereal consumed daily that is transformed into finished product/drink is low. So the objective of the production manager is to transform as many cereals as possible to finished and bottled drink while minimizing the consumption of additives.

Proper process monitoring and control would be of immense benefit to any organization involved in the production of goods and services. Good process monitoring tools would help reduce downtime, reduce rejects and effectively monitor raw materials usage. An excellent process monitoring and control tool is the transfer function modeling [1–6].

Transfer function modeling is a complex task, and it is especially elaborate when the number of input is more than one. In a typical multi input single output production process, as typified by brewing, there is variability in inputs and the output as depicted in Fig. 1.

Transfer functions, which are superior to regression analysis and its derivatives, are used to determine the causal relationship between the input(s) and output of processes [1–4]. Nwobi-Okoye and Igboanugo [3,4] used transfer function modeling for performance evaluation of power generation systems. They considered only the single input single output (SISO) case, as typified by power generation systems. Evaluating the performance of multi input single output (MISO) processes using transfer function modeling is quite challenging and necessitates the introduction of Fuzzy Logic.

Fuzzy logic which was pioneered by Zadeh [7] has been extensively used by engineers and scientists in modern times as a tool for managing uncertainty. Traditionally engineers and scientists have always sought for precision in measurements and design. Thus, uncertainty has not always been embraced by the scientific community [8,9]. In the mid 20th century, scientists started thinking of other ways of looking at uncertainties and vagueness. This effort paid off first with the introduction of the studies of vagueness by the philosopher Max Black in 1937 [9,10] and by the introduction of fuzzy logic by Lotfi Zadeh in 1965 [7]. The introduction of fuzzy logic has had a profound effect in our understanding and management of uncertainty.

Fuzzy logic has been extensively used in condition monitoring and assessment. For example civil engineers use it to assess the conditions of bridges and other civil engineering structures [9]. Ertuğrul and Karakaşoğlu [11] used fuzzy logic to evaluate the performance of the firms by using financial ratios and at the same time, taking subjective judgments of decision makers into consideration. They used the method to evaluate the performance of the fifteen Turkish cement firms in the Istanbul Stock Exchange by using their financial tables and determined the rankings of the firms according to their results. Seçme et al. [12] used a fuzzy multi-criteria decision model to evaluate the performances of banks. They selected, examined and evaluated the largest five commercial banks of Turkish Banking in terms of several financial and non-financial indicators. Fuzzy Analytic Hierarchy Process (FAHP) and Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) methods were integrated in their model. Their results showed that not only financial performance but also non-financial performance should be taken into account in a competitive

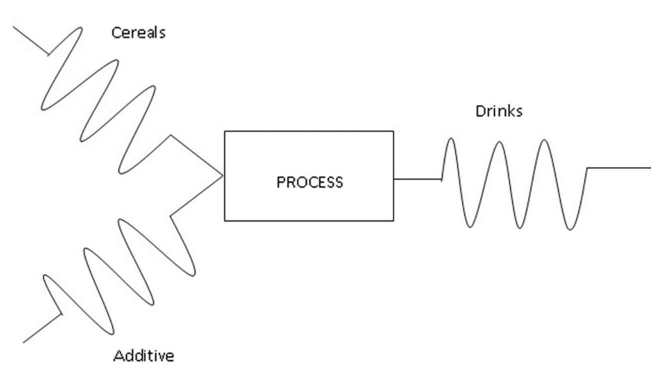

Figure 1 Schematic of the input–output relationship of a brewery.

environment. Chen [13] evaluated the performance of weapon systems using fuzzy arithmetic operations. They discovered that because the proposed methods use simplified fuzzy arithmetic operations of fuzzy numbers rather than the complicated entropy weight calculations used in a previous research, its execution is much faster than the one presented in the previous research. Tseng [14] used a hybrid approach which combines the analytic network process (ANP), used to analyze the dependence aspects; the decision-making trial and evaluation laboratory (DEMATEL), used to deal with the interactive criteria; and the fuzzy set theory, used to evaluate the uncertainty in balanced scorecard (BSC) a multi-criteria evaluation concept that highlights the importance of performance measurement. His results show that student acquisition is the most influential and weighty criterion, and the annual growth in revenue is the most effective criterion. Yang and Chen [15] introduced an evaluation model that integrates triangular fuzzy numbers and the analytical hierarchy process to develop a fuzzy multiple-attribute decision-making (FMADM) model for key quality-performance evaluation. Their results demonstrate that decision-makers can use the flexibility of the proposed model by adjusting the confidence coefficient to express their degree of understanding with respect to the importance of each component and are also a significant contributor to quality improvement. Sadiq et al. [16] used fuzzy logic to evaluate and predict the performance of slow sand filters used for wastewater treatment using the uncertainties in the control parameters and processes. The results were compared with a multiple regression model and performed creditably well. Yeh et al. [17] developed an effective fuzzy multicriteria analysis (MA) approach to performance evaluation for urban public transport systems involving multiple criteria of multilevel hierarchies and subjective assessments of decision alternatives. The approach was found to be computationally efficient, and its underlying concepts are simple and comprehensible. They used a case study on 10 bus companies of an urban public transport system in Taiwan to illustrate the effectiveness of the approach. Wu [18] developed an integrated approach to rate decision alternatives using data envelopment analysis and fuzzy preference relations. Other works which used fuzzy for performance evaluation include: Cheng and Lin [19], Lin et al. [20], etc.

The fact that transfer functions represent performance evaluation, condition and process monitoring tools just like fuzzy logic, as listed in the literature, means that combining the two tools would result to better process monitoring and control. This work attempts to combine the accuracy and precisions of transfer function modeling and the ability of fuzzy logic to deal with vagueness and uncertainty to evaluate and monitor the performance of a multi input single output process as exemplified by the brewing process. The hub of our investigation is a local brewing company known as Consolidated Breweries PLC located at Awo Omamma, Imo State, Nigeria. The company produces malt drinks and beer.

2. Theoretical brief

2.1. Multiple input transfer function models

In terms of the impulse response weights $v(B)$, the transfer function can be represented as [2]:

$$Y_t = v(B)X_{t-b} + N_t \tag{1}$$

Recalling that $(B) = \delta^{-1}(B)\omega(B)$ [2], we obtain:

$$Y_t = \delta^{-1}(B)\omega(B)X_{t-b} + N_t \tag{2}$$

Allowing for several inputs, $X_{1,t}, X_{2,t},\ldots,X_{m,t}$ we have:

$$Y_t = v_1(B)X_{1,t} + \ldots + v_m(B)X_{m,t} + N_t \tag{3}$$

$$Y_t = \delta^{-1}(B)\omega_1(B)X_{1,t-b} + \ldots + \delta^{-1}(B)\omega_m(B)X_{m,t-b} + N_t \tag{4}$$

Here $v_j(B)$ is the generating function of the impulse response weights relating to $X_{j,t}$ to the output. Assuming differencing is applied to the input and output series we obtain:

$$y_t = v_1(B)x_{1,t} + \ldots + v_m(B)x_{m,t} + n_t \tag{5}$$

Multiplying throughout by $X_{1,t-k}, X_{2,t-k}, \ldots, X_{m,t-k}$ in turn and taking expectations and forming the generating functions, we obtain:

$$\begin{aligned}
\gamma^{x_1y}(B) &= v_1(B)\gamma^{x_1x_1}(B) + v_2(B)\gamma^{x_1x_2}(B) + \ldots + v_m(B)\gamma^{x_1x_m}(B)\\
\gamma^{x_2y}(B) &= v_1(B)\gamma^{x_2x_1}(B) + v_2(B)\gamma^{x_2x_2}(B) + \ldots + v_m(B)\gamma^{x_2x_m}(B)\\
\vdots \quad &\vdots\\
\gamma^{x_my}(B) &= v_1(B)\gamma^{x_mx_1}(B) + v_2(B)\gamma^{x_mx_2}(B) + \ldots + v_m(B)\gamma^{x_mx_m}(B)
\end{aligned} \tag{6}$$

Substituting $B = e^{-i2\pi f}$, the spectral equations are obtained. For the case of $m = 2$, the spectral equations are:

$$p_{x_1y}(f) = H_1(f)p_{x_1x_1}(f) + H_m(f)p_{x_1x_2}(f) \tag{7}$$

$$p_{x_2y}(f) = H_1(f)p_{x_2x_1}(f) + H_m(f)p_{x_2x_2}(f) \tag{8}$$

The frequency response functions $H_1(f) = v_1(e^{-i2\pi f}), H_2(f) = v_2(e^{-i2\pi f})$ can be calculated through methods outlined in the literature on spectral analysis such as Koopmans [21], Jenkins and Watts [22], etc. The impulse response weights can be obtained by the inverse transformation thus:

$$v_k = \int_{-\frac{1}{2}}^{\frac{1}{2}} v(e^{-i2\pi f})e^{i2\pi f}df \tag{9}$$

2.2. Transfer function-fuzzy logic modeling

For the models in Eqs. (2) and (3), the simplest case occurs when r, s and N_t are zero and b is a constant. For other case scenarios, δ, ω, b and N_t are regarded as fuzzy numbers. Thus for the parameter, ω, we can define a fuzzy set such that:

$$\mu_{A_i}(\omega) \in [0, 1] \tag{10}$$

where A_i denotes membership function i of ω.

Since δ, ω, b and N_t are fuzzy variables, they could be used as inputs to a fuzzy inference system using either the MAMDANI, SUGENO or any other suitable fuzzy inference model to generate an output, λ, which measures the efficiency or performance of the process or system.

The parameters δ, ω, b and N_t are regarded as minor coefficients of performance (COP_{minor}), while the parameter, λ, is regarded as the major coefficient of performance (COP_{major}).

3. Methodology

The six year data obtained from the brewery was subjected to exploratory data analysis to detect outliers and patterns in the data. After the exploratory data analysis, the transfer function model according to Eq. (4) was obtained using the input–output data for the periods 2006–2007, 2008–2009 and 2010–2011.

In order to realize the transfer function model based on Eq. (4), a plot of the 3-year input–output data was done using SPSS software. Following the plot, the data were investigated for stationarity, using the plots of the autocorrelation functions (ACF) and Partial autocorrelation functions (PACF). The inputs and output series derived from the plots were investigated for stationarity. Nonstationary series were differenced to achieve stationarity. A univariate model was individually fitted to the input X_{1t} and output Y_t, and input X_{2t} and output Y_t for each of the years in order to respectively estimate prewhitened input series α_{1t} and α_{2t}, and pretreated output series β_{1t} and β_{2t} respectively. Calculation of the cross correlation functions, CCF (k) of $\beta_{1t}\alpha_{1t-k}$ and $\beta_{2t}\alpha_{2t-k}$ was used to identify r, s and b parameters of the transfer function model. Sequel to obtaining the nature of the transfer function models, the impulse response weights v_k, estimated with spectral analysis, were used to estimate the transfer function parameters in Eq. (4). After obtaining the transfer function model, the transfer function parameters were combined with fuzzy logic to evaluate the yearly performance of the plant.

The first step in fuzzy logic analysis was the development of the membership functions for the input variables and the performance ratings of the output (plant performance). The membership function/values were assigned by intuition. After developing the membership functions and fuzzification of the input and output variables, the Mamdani fuzzy logic inference system [9] was used to model the effects of the input variables on performance. The defuzzification was done using the centriod method.

Fig. 2 shows the conceptual model of the hybrid transfer function-fuzzy inference modeling system for determining the minor and major coefficients of performance (COP). In the Figure (Fig. 2) a two input single output system with constant lag (b) and zero noise (N_t) is used for simplicity.

4. Results

4.1. Transfer function modeling

Fig. 3 shows the monthly raw materials consumption and the corresponding output (drink production) in the years 2006–2007 for Consolidated Brewery Nigeria Limited. The raw material X_1 are cereals while the raw material X_2 is the additive.

4.1.1. Analysis of the relationship between Input 1 (X_1) and Output (Y)

After the plots shown in Fig. 3, the data (X_1 series) was investigated for stationarity, using the plots of the autocorrelation functions (ACF) and Partial autocorrelation functions (PACF).

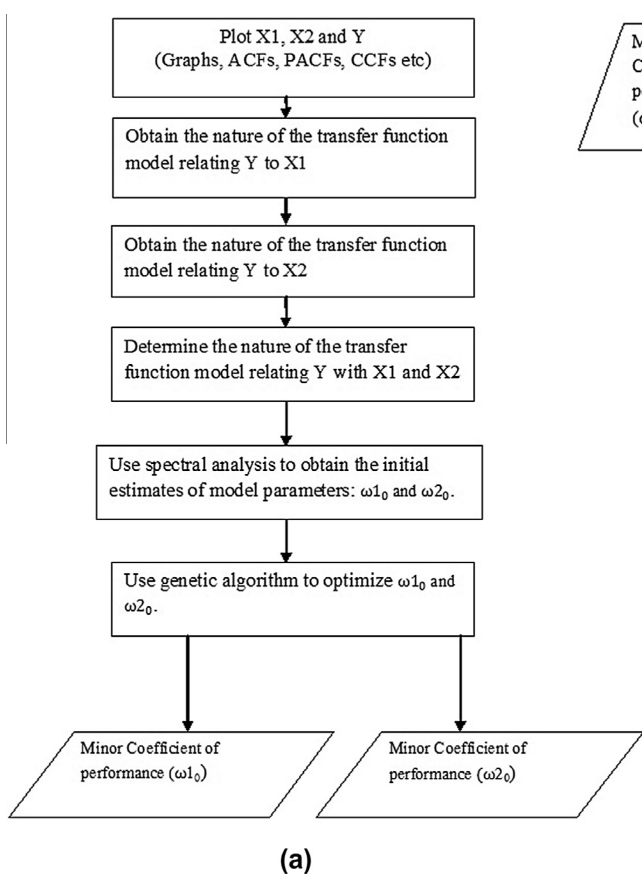

Figure 2 (2a.) Transfer function modeling process for determining the minor COP ($\omega 1_0$ and $\omega 2_0$). (2b.) Fuzzy inference system for determining the major COP (λ). Conceptual model of the transfer function-fuzzy modeling system.

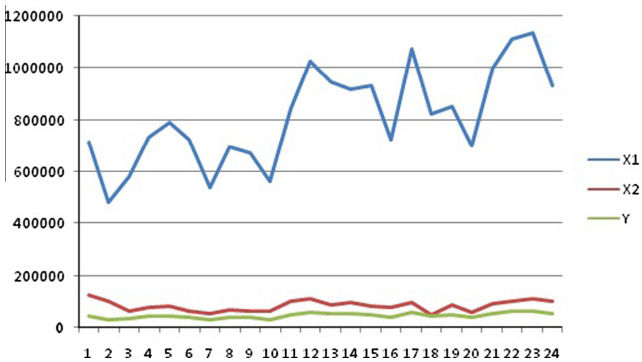

Figure 3 Weekly raw material consumption and output.

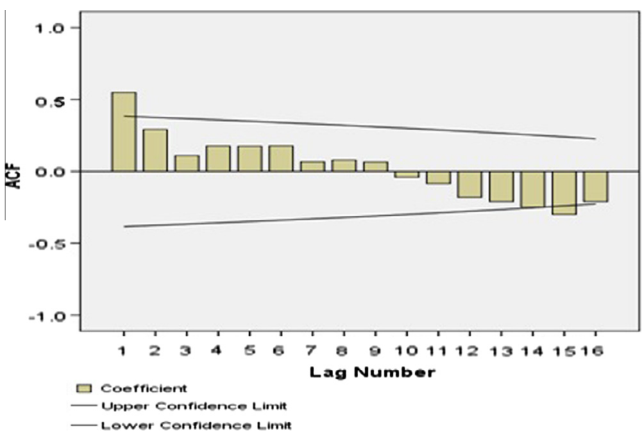

Figure 4 ACF of the input series (X_1).

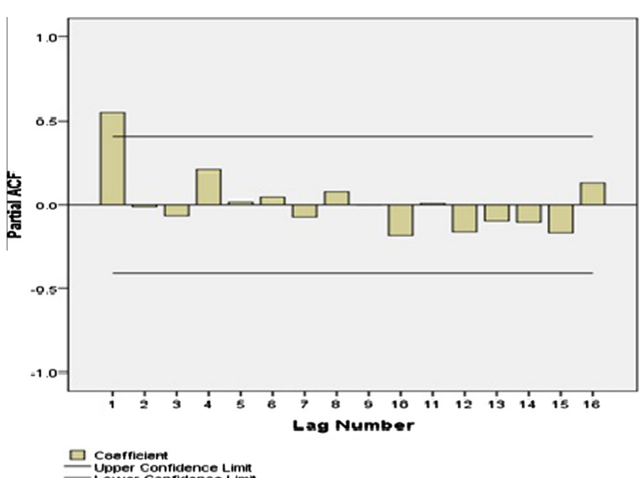

Figure 5 PACF of the input series (X_1).

The X_1 series upon analysis was found to be stationary, hence differencing was not used. Examination of the ACF and PACF in Figs. 4 and 5 is indicative that auto regression one (AR (1)) model is the appropriate model to use.

The formula for AR (1) models [2,23] is given by Eq. (11):

$$X_{1t} = \theta_0 + \emptyset_1 X_{1t-1} + e_t \tag{11}$$

But for AR (1) models, we have:

$$ACF(1) = \emptyset_1 = 0.549 \tag{12}$$

$$\theta_0 = (1 - \emptyset_1)\mu \tag{13}$$

$$\mu = 811883.96$$

$$\theta_0 = (1 - 0.549)811883.96$$

$$\theta_0 = 366159.66596$$

Fitting the coefficients θ_0 and $\emptyset_1$ into the formula for AR (1) models, Eq. (14) is obtained.

$$X_{1t} = 366159.66596 + 0.549X_{1t-1} + e_t \tag{14}$$

But

$$e_t = \alpha_t \tag{15}$$

In forecasting form Eq. (14) is transformed to Eq. (16):

$$\widehat{X}_{1t} = 20496.74864 + 0.549X_{1t-1} \tag{16}$$

Pre-treating the output in the same way the input was transformed, we obtain:

$$\emptyset_1 = 0.549$$

$$\theta_0 = (1 - \emptyset_1)\mu$$

$$\mu = 45447.33623$$

$$\theta_0 = (1 - 0.549)45447.33623$$

$$\theta_0 = 20496.74864$$

$$Y_t = 20496.74864 + 0.549\,Y_{t-1} + e_t \tag{17}$$

But

$$e_t = \beta_t \tag{18}$$

In forecasting form Eq. (20) is transformed to Eq. (22):

$$\widehat{Y}_t = 20496.74864 + 0.549\,Y_{t-1} \tag{19}$$

The CCF between β_t and α_t is shown in Fig. 6. It has one significant CCF at lag zero (0). Hence, according to Box et al. [2] and DeLurgio [23], the parameters r, s and b of the transfer function that supports such CCF pattern are 0, 0 and 0 respec-

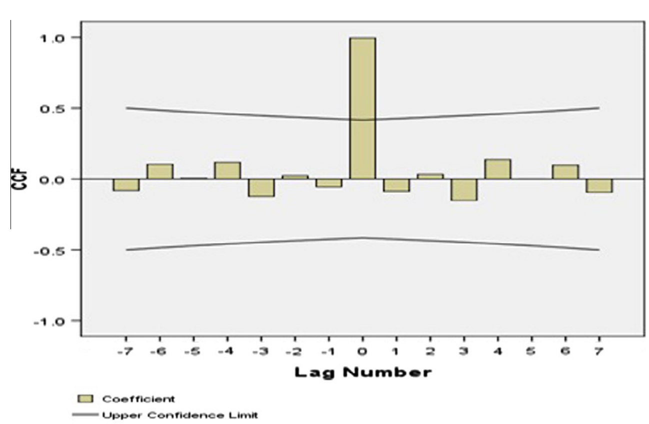

Figure 6 CCF of the pre-whitened series (Y vs X_1).

tively. In view of this fact, the CCF supports the following transfer function model:

$$y_t = \omega 1_0 x_{1t} + N_t \tag{20}$$

Based on Ljung-Box statistics and analysis of the residuals, the transfer function was found to have white noise residuals, hence we disregarded the noise term N_t, to obtain Eq. (21).

$$y_t = \omega 1_0 x_{1t} \tag{21}$$

As shown by Box et al. [2] and DeLurgio [23],

$$v1_0 = \omega 1_0 \tag{22}$$

$$v1_0 = \textit{impulse response for } X_1$$

But

$$X_{1t} - \mu_1 = x_{1t} \tag{23}$$

And

$$Y_t - \mu_y = y_t \tag{24}$$

Substituting Eq. (24) into Eqs. (21) and (25) is obtained.

$$Y_t = \mu_y + \omega 1_0 x_{1t} \tag{25}$$

4.1.2. Analysis of the relationship between Input 2 (X_2) and Output (Y)

After the plots shown in Fig. 3, the data were investigated for stationarity, using the plots of the autocorrelation functions (ACF) and Partial autocorrelation functions (PACF).

The input series derived from the plots were found not to be stationary, hence differencing was used to achieve stationarity. Stochastic regularity was achieved after the second differencing. The plots of the ACF and PACF after differencing are shown in Figs. 7 and 8 respectively.

Examination of the ACF shown in Fig. 7, the ACF at lag 1 is significant. But examination of the PACF shown in Fig. 8, only the ACF at lag 1 is significant, and this is indicative that MA (1) model is the appropriate model to use.

The formula for MA (1) models [2,23] is given by Eq. (26):

$$X_{2t} = \mu + \theta_1 e_{2t-1} + e_{2t} \tag{26}$$

$$x_{2t} = X_{2t} - X_{2t-1} \tag{27}$$

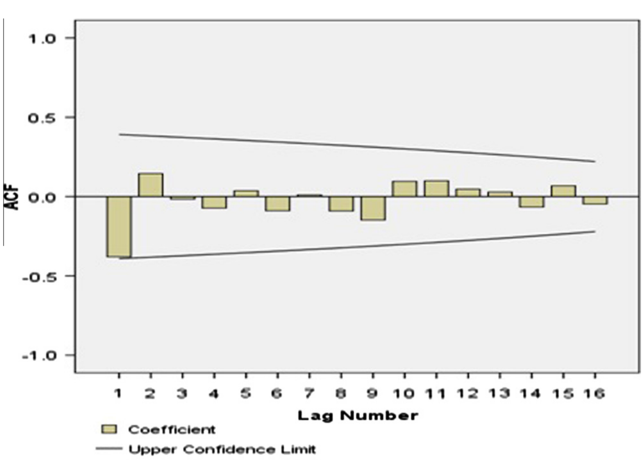

Figure 7 ACF of the input series (X_2).

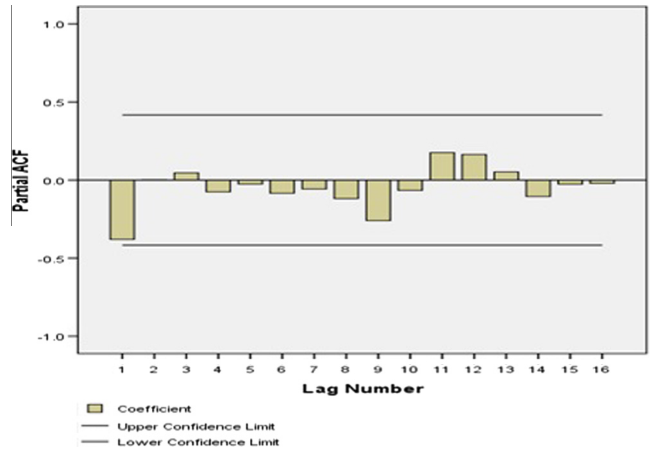

Figure 8 PACF of the input series (X_2).

$$x_{2t} = \theta_1 e_{2t-1} + e_{2t} \tag{28}$$

But for MA (1) models, we have:

$$ACF(1) = \rho_1 = -0.379 \tag{29}$$

But

$$\rho_1 = \frac{-\theta_1}{1 + \theta_1^2} \tag{30}$$

Therefore

$$-0.379 = \frac{-\theta_1}{1 + \theta_1^2} \tag{31}$$

$$0.379\theta_1^2 - \theta_1 + 0.379 = 0 \tag{32}$$

$$\theta_1 = -0.4588$$

Hence, fitting the coefficient θ_1 into the formula for MA (1) models, Eq. (33) is obtained.

$$x_{2t} = -0.4588 e_{2t-1} + e_{2t} \tag{33}$$

Substituting Eq. (33) into Eq. (27) we obtain:

$$X_{2t} - X_{2t-1} = -0.4588 e_{2t-1} + e_{2t} \tag{34}$$

$$X_{2t} = X_{2t-1} - 0.4588 e_{2t-1} + e_{2t} \tag{35}$$

But

$$e_t = \alpha_t \tag{36}$$

In forecasting form Eq. (35) is transformed to Eq. (37):

$$\widehat{X}_{2t} = X_{2t-1} - 0.4588 e_{2t-1} \tag{37}$$

Pre-treating the output in the same way the input was transformed, we obtain:

$$Y_t = Y_{t-1} - 0.4588 e_{t-1} + e_t \tag{38}$$

But

$$e_t = \beta_t \tag{39}$$

In forecasting form Eq. (38) is transformed to Eq. (40):

$$\widehat{Y}_t = Y_{t-1} - 0.4588 e_{t-1} \tag{40}$$

The CCF between β_t and α_t is shown in Fig. 9. It has one significant CCF at lag zero (0). Hence, according to Box et al. [2],

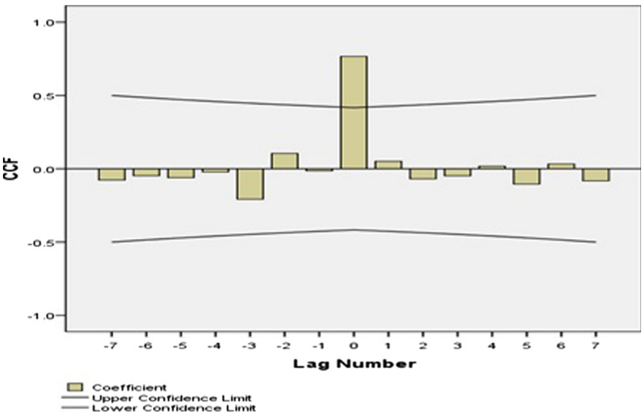

Figure 9 CCF of the pre-whitened series (Y vs X_2).

the parameters r, s and b of the transfer function that supports such CCF pattern are 0, 0 and 0 respectively. In view of this fact, the CCF supports the following transfer function model:

$$y_t = \omega 2_0 x_{2t} + N_t \tag{41}$$

Based on Ljung-Box statistics and analysis of the residuals, the transfer function was found to have white noise residuals, hence we disregarded the noise term N_t, to obtain Eq. (42).

$$y_t = \omega 2_0 x_{2t} \tag{42}$$

As shown by Box et al. [2] and DeLurgio [23],

$$v2_0 = \omega 1_0 \tag{43}$$

$v2_0 = impulse\ response\ for\ X_2$

But

$$X_{2t} - \mu = x_{2t} \tag{44}$$

And

$$Y_t - \mu_y = y_t \tag{45}$$

Substituting Eq. (45) into Eqs. (42) and (46) is obtained.

$$Y_t = \mu_y + \omega 2_0 x_{2t} \tag{46}$$

4.1.3. Obtaining the transfer function models

Having related X_1 and Y in Section 4.1.1, as well as X_2 and Y in Section 4.1.2 from the analysis above, it is obvious that the transfer function relating Y with X_1 and X_2 is of the form:

$$y_t = \omega 1_0 x_{1t} + \omega 2_0 x_{2t} \tag{47}$$

Since $y_t = Y_t - \mu_y$, $x_{1t} = X_{1t} - \mu_1$ and $x_{2t} = X_{2t} - \mu_2$

$$Y_t = \mu_y + \omega 1_0 (X_{1t} - \mu_1) + \omega 2_0 (X_{2t} - \mu_2) \tag{48}$$

Since

$v1_0 = \omega 1_0$ and $v2_0 = \omega 2_0$

where

$v1_0 = impulse\ response\ for\ X_1$ and $v2_0 = impulse\ response\ for\ X_2$

$$Y_t = \mu_y + v1_0 (X_{1t} - \mu_1) + v2_0 (X_{2t} - \mu_2) \tag{49}$$

$v1_0$ and $v2_0$ were obtained by spectral analysis. After spectral analysis and parameter optimization using genetic algorithm the values of $v1_0$ and $v2_0$ obtained were:

$v1_0 = 0.048234991$ and $v2_0 = 0.060100632$

Therefore for 2006–2007 operation of the brewery, the transfer function is given by:

$$Y_t = 45447.329 + 0.048234991(X_{1t} - \mu_1) + 0.060100632(X_{2t} - \mu_2) \tag{50}$$

Table 1 shows the transfer function models for the six years operation of the plant.

Table 2 shows the Coefficients of Performance (COP) of the Drink Plant for the years 2011, 2012 and 2013. As shown in Table 2, raw material consumption was least in the year 2011 while the rate of transformation of the cereals to finished drink was highest in the same year. The overall plant performance can only be determined by fuzzy logic or fuzzy inference process.

4.2. Fuzzy logic

A typical fuzzy set A_{rl} for low raw material consumption is given by:

$$A_{rl} = \left\{ \frac{0}{0} + \frac{1}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} \right\} \tag{51}$$

The fuzzy sets were used to develop the membership functions. The membership functions for the input variables, which model the coefficients of performance on Table 2, and the output variable (plant performance) are shown in Figs. 10–13. As

Table 1 Transfer function models of the brewery.

Year	Transfer function model ($v(B)$)
2010–2011	$Y_t = 64785.94 + 0.04898(X_{1t} - \mu_1) + 0.05470(X_{2t} - \mu_2)$
2008–2009	$Y_t = 60639.75 + 0.04513(X_{1t} - \mu_1) + 0.07470(X_{2t} - \mu_2)$
2006–2007	$Y_t = 45447.33 + 0.04823(X_{1t} - \mu_1) + 0.06010(X_{2t} - \mu_2)$

Table 2 Minor coefficients of performance of the brewery.

Year	Coefficient of performance (cereals) $\omega 1_0$	Coefficient of performance (additive) $\omega 2_0$
2010–2011	0.04898	0.05470
2008–2009	0.04513	0.07470
2006–2007	0.04823	0.06010

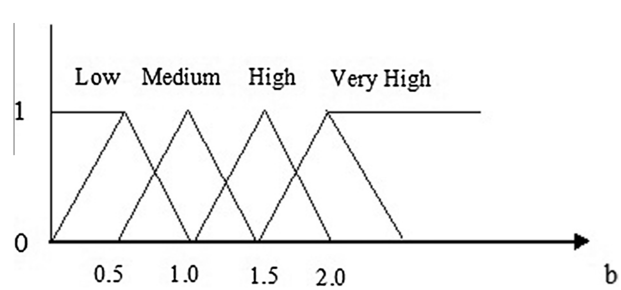

Figure 10 Membership function for lag variable, b.

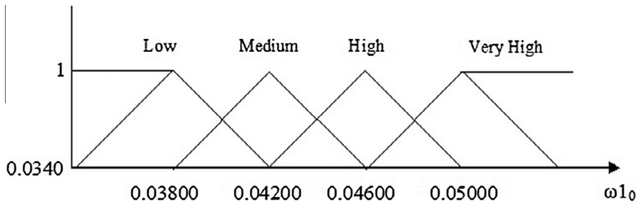

Figure 11 Membership function for coefficient of Input 1 (cereals).

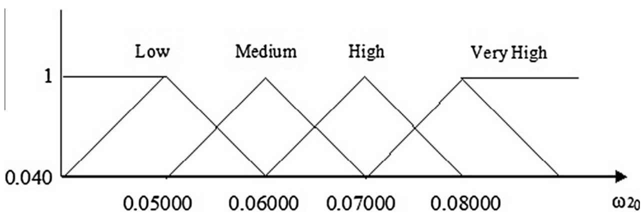

Figure 12 Membership function for coefficient of Input 2 (additive).

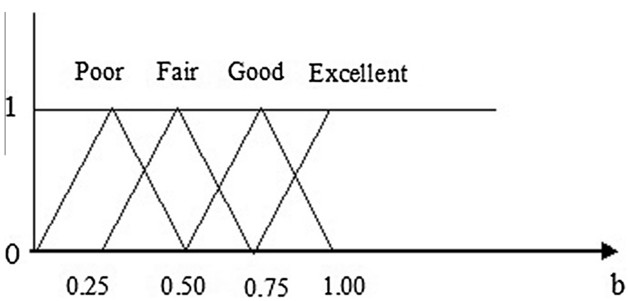

Figure 13 Membership function for performance rating.

shown in the figures, the membership functions are triangular and were developed by intuition.

Since from Table 2, the lag variable is zero in all the transfer function models, we considered only the parameters $\omega 1_0$ and $\omega 2_0$ in our analysis. Hence, we developed a two dimensional performance evaluation matrix shown in Table 3 and linguistic variables for performance rating shown in Table 4.

Table 4 Performance Ratings.

Colour	Performance Rating
	Excellent
	Good
	Fair
	Poor

4.2.1. *Fuzzy rules*

From Tables 3 and 4, the fuzzy logic rules were developed. Some of the rules are:

If Cereal is Low AND additive is Very High THEN Performance Rating is Poor.

If Cereal is Low AND additive is Low THEN Performance Rating is Fair.

Altogether sixteen rules were developed. The rules were aggregated to a single fuzzy output and defuzzified using centroid method to obtain the performance ratings shown in Table 5.

As shown in Table 5, the best performance rating denoted by λ is 0.810 which occurred in the year 2010–2011. The linguistic variable corresponding to 0.810 as shown in the table is 'Good'. Hence, the best performance over the six years is rated 'Good'. Similarly, the worst performance occurred in the year 2008–2009 with λ value equal to 0.381. The linguistic variable corresponding to 0.381 as shown in the table is 'Fair' and the worst performance is rated 'Fair'.

5. Discussion

The graphs of the inputs and output shown in Fig. 4 are stochastic, confirming the fact that in a production system the input and output are stochastic in nature as stated by Nwobi-Okoye and Igboanugo [3,4]. The results here are espe-

Table 3 Fuzzy Performance Evaluation Matrix.

		Input 2 (Additive)			
		Very High	High	Medium	Low
		1	***2***	***3***	***4***
Input 1 (Cereals)					
Low	1				
Medium	2				
High	3				
Very High	4				

Table 5 Major coefficients of performance of the brewery.

Year	Coefficient of performance (cereals) $\omega 1_0$	Coefficient of performance (additive) $\omega 2_0$	Performance rating (overall) λ	Linguistic variable of performance rating
2010–2011	0.04898	0.05470	0.810	Good
2008–2009	0.04513	0.07470	0.381	Fair
2006–2007	0.04823	0.06010	0.786	Good

cially very important because the concept of using Coefficient of Performance (COP), a superior metric for evaluating the performance of processes, introduced by Nwobi-Okoye and Igboanugo [3,4] is simple to use in single input single output (SISO) processes. Extending the concept to multi input single output (MISO) processes is difficult and as the inputs increase the complexity increases. Application of fuzzy logic solves this problem of handling the complexity of evaluating the performance of multi input single output (MISO) processes using the concept of Coefficient of Performance as this study demonstrated.

The plant's coefficients of performance and overall performance rating are affected by the level of maintenance and quality of machines used in the production processes, as well as the quality of raw materials used in the production process. A comparison of the periods 2010–2011 and 2006–2007 performances shows that whereas the rate of transformation of cereals to finished drink is slightly lower in 2006–2007, as indicted by lower value of $\boldsymbol{\omega} 1_0$, the quality of the cereals used in the year was slightly lower, as indicted by higher value of $\boldsymbol{\omega} 2_0$, thus making the overall performance rating $\boldsymbol{\lambda}$ lower than in the period 2006–2007. Higher values of $\omega 1_0$ indicate that the machines functioned better during the period.

We would regard $\omega 1_0$ and $\omega 2_0$, which were obtained from transfer function modeling, as minor performance indicators or coefficients of (COP_{minor}) because they measure efficiency or system performance. Neither $\omega 1_0$ nor $\omega 2_0$ alone can measure the performance of the system under study because it is a multi (two) input system. Since $\omega 1_0$ and $\omega 2_0$ are fuzzy numbers (variables), they were fuzzified and used as inputs to a fuzzy inference system using the MAMDANI fuzzy inference model to generate an output, λ, which measures the overall efficiency or performance of the process or system under study (the brewery). The value, λ, would be regarded as the major coefficient of performance (COP_{major}). Hence, with transfer function modeling alone one can only have a partial view or vague idea of the performance of the system. In order words, the system performance is ambiguous. But combining transfer function and fuzzy logic, which is a powerful tool for handling vagueness, a complete, clear and unambiguous view of system performance is obtained.

6. Conclusion

Modeling multivariate processes is quite challenging. The complexity increases as the number of inputs increases. This work considered only the two input single output process as typified by the brewing process studied herein. The result of this study is very significant because we have been able to model the complex interaction between raw material quality and operations efficiency to determine plant performance rating. The result will even be more significant as the number of interacting inputs increases. The method developed in this paper is a statistically sound and robust method of evaluating the performance of multi input single output (MISO) systems.

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