Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

PHYSICS LETTERS B

From hybrid to quadratic inflation with high-scale supersymmetry breaking

CrossMark

Constantinos Pallis [a],[*], Qaisar Shafi [b]

[a] Departament de Física Teòrica, IFIC, Universitat de València-CSIC, E-46100 Burjassot, Spain
[b] Bartol Research Institute, Department of Physics and Astronomy, University of Delaware, Newark, DE 19716, USA

ARTICLE INFO

ABSTRACT

Motivated by the reported discovery of inflationary gravity waves by the Bicep2 experiment, we propose an inflationary scenario in supergravity, based on the standard superpotential used in hybrid inflation. The new model yields a tensor-to-scalar ratio $r \simeq 0.14$ and scalar spectral index $n_s \simeq 0.964$, corresponding to quadratic (chaotic) inflation. The important new ingredients are the high-scale, $(1.6\text{–}10) \cdot 10^{13}$ GeV, soft supersymmetry breaking mass for the gauge singlet inflaton field and a shift symmetry imposed on the Kähler potential. The end of inflation is accompanied, as in the earlier hybrid inflation models, by the breaking of a gauge symmetry at $(1.2\text{–}7.1) \cdot 10^{16}$ GeV, comparable to the grand-unification scale.

1. Introduction

The discovery of B-modes in the polarization of the cosmic microwave background radiation at large angular scales by the Bicep2 experiment [1] has created much excitement among inflationary model builders, since this effect can be caused by an early inflationary era with a large tensor-to-scalar ratio $r = 0.16^{+0.06}_{-0.05}$ – after substraction of a dust foreground. Although other interpretations [2,3] of this result are possible, it motivates us to explore how realistic *supersymmetric* (SUSY) inflation models can accommodate such large r values.

The textbook quadratic inflationary model [4] predicting $r = 0.13 - 0.16$, and a (scalar) spectral index $n_s = 0.96 - 0.967$, seems to be in good agreement with Bicep2 (r) and the WMAP [5] and Planck [6] measurements (n_s). Quadratic inflation can be accompanied by a *Grand Unified Theory* (GUT) phase transition in non-supersymmetric inflation models, based either on the Coleman–Weinberg or Higgs [7] potential, which yield predictions for n_s that more or less overlap with the prediction of the quadratic model [8, 9]. However, significant differences appear between the predictions of r in these models which can be settled through precision measurements. The consistent supersymmetrization of these models is a highly non-trivial task due to the trans-Planckian values of the inflaton field which aggravate the well-known η-problem within *supergravity* (SUGRA).

One of the more elegant SUSY models which nicely combines inflation with a GUT phase transition is the model of *F-term hybrid inflation* [10,11] – referred to as FHI. It is based on a unique renormalizable superpotential, dictated by a $U(1)$ R-symmetry, employs sub-Planckian values for the inflaton field and can be naturally followed by the breaking of a GUT gauge symmetry, G, such as $G_{B-L} = G_{SM} \times U(1)_{B-L}$ [12] – where $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ is the gauge group of the *Standard Model* (SM) – $G_{LR} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [13], and flipped $SU(5)$ [14], with gauge symmetry $G_{5_X} = SU(5) \times U(1)_X$. The embedding of the simplest model of FHI within a GUT based on a higher gauge group may suffer from the production of disastrous cosmic defects which can be evaded, though, by using shifted [15] or smooth [16] FHI.

In the simplest realization of FHI the standard [10] superpotential is accompanied by a minimal (or canonical) Kähler potential. The resulting n_s is found to be in good agreement with the WMAP and Planck data after including in the inflationary potential *radiative corrections* (RCs) [10] and the *soft SUSY breaking* (SSB) linear term [12,18] – with a mass parameter in the TeV range – an SSB mass term for the inflaton in the same energy region can be ignored in this analysis. This scenario yields [12] r values which lie many orders of magnitude below the measurement reported [1] by Bicep2. A more elaborate extension of this standard FHI scenario exploits non-minimal, quasi-canonical Kähler potentials [19,21] or SSB mass of magnitude as large as 10^{10} GeV for the inflaton field [20]. Depending on the underlying assumptions, the predictions for r are considerably enhanced compared to the minimal scenario of Refs. [12,18]. Thus, r values as large as 0.01 to 0.03 have been reported [20,21]; this fact certainly puts r in the observable range, but it still remains an order of magnitude below the Bicep2 measurement – however, see Ref. [22] for models of FHI with Kähler potential not-respecting the R-symmetry.

* Corresponding author.
   E-mail addresses: cpallis@ific.uv.es (C. Pallis), shafi@bartol.udel.edu (Q. Shafi).

Motivated by this apparent discrepancy between the large r values reported by Bicep2 and the predictions of FHI models, we present here a modified scenario of F-term inflation in which a quadratic potential dictates the inflationary phase, thus replicating the predictions of quadratic inflation, employing the well-studied standard superpotential of FHI. The two key elements for successfully implementing this scenario include a judicious choice of the Kähler potential and a high-scale SUSY breaking. In particular, following earlier similar attempts [23] a shift symmetry is imposed on the Kähler potential to protect the inflationary potential from SUGRA corrections which are dangerous due to trans-Planckian inflaton field values. Moreover, we assume that SUSY is broken at an intermediate scale, $\widetilde{m} \sim 10^{13}$ GeV, which can be identified with the SSB mass of the inflaton. In the context of high-scale SUSY [24,25], such a large SSB scale can become consistent with the LHC results [26] on the mass, $m_h \simeq 126$ GeV, of the SM Higgs boson, h. The end of inflation can be accompanied by the breaking of some gauge symmetry such as G_{LR} or G_{5_X} with the gauge symmetry breaking scale M assuming values close to the SUSY GUT scale $M_{GUT} \simeq 2.86 \cdot 10^{16}$ GeV.

Below, we describe in Section 2 the basic ingredients of our inflationary scenario. Employing a number of constraints presented in Section 3, we provide restrictions on the model parameters in Section 4. Our conclusions are summarized in Section 5. Henceforth we use units where the reduced Planck scale $m_P = 2.44 \cdot 10^{18}$ GeV is taken equal to unity.

2. The inflationary scenario

2.1. The GUT symmetry breaking

In the standard FHI we adopt the superpotential

$$W = \kappa S \big(\bar{\Phi} \Phi - M^2 \big), \tag{1}$$

which is the most general renormalizable superpotential consistent with a continuous R-symmetry [10] under which

$$S \to e^{i\varphi} S, \qquad \bar{\Phi} \Phi \to \bar{\Phi} \Phi, \qquad W \to e^{i\varphi} W. \tag{2}$$

Here S is a G-singlet left-handed superfield, and the parameters κ and M are made positive by field redefinitions. In our approach $\bar{\Phi}$, Φ are identified with a pair of left-handed superfields conjugate under G which break G down to G_{SM}. Indeed, along the D-flat direction $|\bar{\Phi}| = |\Phi|$ the SUSY potential, V_{SUSY}, extracted – see e.g. Ref. [28] – from W in Eq. (1), reads

$$V_{SUSY} = \kappa^2 \big(\big(|\Phi|^2 - M^2 \big)^2 + 2|S|^2 |\Phi|^2 \big). \tag{3}$$

From V_{SUSY} in Eq. (3) we find that the SUSY vacuum lies at

$$\big| \langle S \rangle \big| = 0 \quad \text{and} \quad \big| \langle \Phi \rangle \big| = \big| \langle \bar{\Phi} \rangle \big| = M, \tag{4}$$

where the vacuum expectation values of Φ and $\bar{\Phi}$ lie along their SM singlet components. As a consequence, W leads to the spontaneous breaking of G to G_{SM}.

2.2. The inflationary set-up

It is well-known [10] that W also gives rise to FHI since, for values of $|S| \gg M$, there exist a flat direction

$$s \equiv \sqrt{2}\,\text{Im}[S] = 0 \quad \text{and} \quad \bar{\Phi} = \Phi = 0, \tag{5}$$

which provides us with a constant potential energy $\kappa^2 M^4$ suitable for supporting FHI. The inclusion of SUGRA corrections with canonical (minimal) Kähler potential does not affect this result at the

lowest order in the expansion of S – due to a miraculous cancellation occurring. The SUGRA corrections with quasi-canonical Kähler potential [19,21] can be kept under control by mildly tuning the relevant coefficients thanks to sub-Planckian S values required by FHI. The resulting n_s values can be fully compatible with the data [5,6] but the predicted r [20,21] remains well below the purported measurement reported by Bicep2.

In order to safely implement quadratic inflation, favored by Bicep2, within SUGRA and employing W in Eq. (1), we have to tame the η problem which is more challenging due to the trans-Planckian values needed for the inflaton superfield, S. To this end, we exploit a Kähler potential which respects the following symmetries:

$$S \to S + c \quad \text{and} \quad S \to -S, \tag{6}$$

where c is a real number – cf. Ref. [23]. Namely we take

$$\begin{aligned} K = {}&-\frac{1}{2}\big(S - S^* \big)^2 + |\Phi|^2 + |\bar{\Phi}|^2 \\ &+ \frac{(S - S^*)^2}{2\Lambda^2} \big(k_S \big(S - S^* \big)^2 + k_{S\Phi} |\Phi|^2 + k_{S\bar{\Phi}} |\bar{\Phi}|^2 \big) \\ &+ \frac{1}{\Lambda^2} \big(k_\Phi |\Phi|^4 + k_{\bar{\Phi}} |\bar{\Phi}|^4 \big) + \cdots. \end{aligned} \tag{7}$$

Here $k_S, k_\Phi, k_{\bar{\Phi}}, k_{S\Phi}$ and $k_{S\bar{\Phi}}$ are positive or negative constants of order unity – for simplicity we take $k_{S\Phi} = k_{S\bar{\Phi}}$ – and Λ is a cutoff scale determined below. Although K is not invariant under the R symmetry of Eq. (2), the fields $\Phi^\alpha = S, \Phi, \bar{\Phi}$ are canonically normalized, i.e., $K_{\alpha\bar{\beta}} = \delta_{\alpha\bar{\beta}}$ – note that the complex scalar components of the various superfields are denoted by the same symbol.

The F-term (tree level) SUGRA scalar potential, V_{I0}, of our model is obtained from W in Eq. (1) and K in Eq. (7) by applying the standard formula:

$$V_{I0} = e^K \big(K^{\alpha\bar{\beta}} F_\alpha F_{\bar{\beta}} - 3|W|^2 \big), \tag{8}$$

with $K_{\alpha\bar{\beta}} = K_{,\Phi^\alpha \Phi^{*\bar{\beta}}}$, $K^{\bar{\beta}\alpha} K_{\alpha\bar{\gamma}} = \delta^{\bar{\beta}}_{\bar{\gamma}}$ and $F_\alpha = W_{,\Phi^\alpha} + K_{,\Phi^\alpha} W$. We explicitly verify that the SUSY vacuum of Eq. (4) remains intact for the choice of K in Eq. (7). Along the field direction in Eq. (5) the only surviving terms of V_{I0} are

$$V_{I0} = e^K \big(K^{SS^*} |W_{,S}|^2 - 3|W|^2 \big) = \kappa^2 M^4 \Big(1 - \frac{3}{2}\sigma^2 \Big), \tag{9}$$

where the canonically normalized inflaton, σ, is defined by

$$S = (\sigma + is)/\sqrt{2}. \tag{10}$$

As shown from Eq. (9), V_{I0} is not suitable to drive inflation mainly due to the minus sign which renders V_{I0} unbounded from below for large σ's – cf. Ref. [17]. On the other hand, the symmetries in Eq. (6) ensure a complete disappearance of the exponential prefactor in Eq. (9), which could ruin any inflationary solution for large σ's.

A satisfactory solution can be achieved, if we consider an intermediate-scale SSB mass parameter $\widetilde{m}$, whose contribution can exceed the negative contribution to V_{I0} for conveniently selected κ and M. Such a heavy mass parameter is normally generated following the usual SUSY breaking procedures – see e.g. Ref. [27] – provided that the gravitino mass is of similar size and the Polonyi field has canonical Kähler potential. The contributions to the inflationary potential from the SSB effects [12,18] can be parameterized as follows:

$$V_{IS} = \widetilde{m}^2 \sum_\alpha \big| \Phi^\alpha \big|^2 - \big(a_S \kappa M^2 S - \kappa A_\kappa S \Phi \bar{\Phi} + \text{c.c.} \big), \tag{11a}$$

Table 1
The mass spectrum of the model along the path in Eq. (5).

Fields	Eigenstates	Masses squared
Bosons		
1 real scalar	σ	$m_\sigma^2 = \widetilde{m}^2 - 3\kappa^2 M^4$
1 real scalar	s	$m_s^2 = \widetilde{m}^2 + \kappa^2 M^4$
		$\qquad \cdot ((3 - \sigma^2) - 24k_S/\Lambda^2)$
2N complex scalars	$\phi_{i\pm} = \frac{\bar{\phi}_i \pm \phi_i}{\sqrt{2}}$	$m_{\phi_\pm}^2 \simeq \frac{k_{S\Phi}\kappa^2 M^4}{\Lambda^2} \mp \frac{\kappa|A_\kappa|\sigma}{\sqrt{2}}$
	$(i = 1, 2)$	$\qquad + \widetilde{m}^2 + \kappa^2 (\frac{(1\pm M^2)\sigma^2}{2} \mp M^2)$
Fermions		
1 Weyl spinor	ψ_S	$m_{\psi_S}^2 = \kappa^2 M^2 \sigma^2/2$
2N Weyl spinors	$\psi_\pm = \frac{\psi_{\bar\Phi} \pm \psi_\Phi}{\sqrt{2}}$	$m_{\psi_\pm}^2 = \kappa^2\sigma^2/2$

where we assume for simplicity that there is a universal SSB mass $\widetilde{m}$ for all the superfields $\Phi^\alpha = S, \Phi, \bar\Phi$ of our model. Also a_S and A_κ are mass parameters comparable to $\widetilde{m}$. Along the field configuration in Eq. (5), V_{IS} reads

$$V_{IS} = \widetilde{m}^2\sigma^2/2 - \sqrt{2}a_S\kappa M^2\sigma. \tag{11b}$$

We note in passing that, due to Eq. (11a), $|\langle S \rangle|$ is shifted [13] from its value in Eq. (4) to

$$\big|\langle S \rangle\big| \simeq \big(|A_\kappa| - |a_S|\big)/2\kappa \big(1 + \widetilde{m}^2/2\kappa^2 M^2\big), \tag{12}$$

where we selected conveniently the phases of A_κ and a_S so that $\langle V_{SUSY} + V_{IS} \rangle$ is minimized.

2.3. Beyond the tree-level potential

Expanding the various fields, besides S – see Eq. (10) – in real and imaginary parts according to the prescription

$$X = (x_1 + ix_2)/\sqrt{2} \tag{13}$$

where $X = \Phi, \bar\Phi$ and $x = \phi, \bar\phi$ respectively, we are able to check the stability of the field directions in Eq. (5). Namely, we check the validity of the conditions

$$\partial V_{tr}/\partial \chi^\alpha = 0 \quad \text{and} \quad m_{\chi^\alpha}^2 > 0, \tag{14a}$$

where $\chi^\alpha = \sigma, s, \phi_i$ and $\bar\phi_i$ with $i = 1, 2$ and V_{tr} stands for the tree-level inflationary potential

$$V_{tr} = V_{I0} + V_{IS} \tag{14b}$$

with V_{I0} and V_{IS} given in Eqs. (9) and (11b). Note that the imposed $\mathbb{Z}_2$ symmetry on K – see Eq. (6) – excludes the terms $(S - S^*)$ or $(S - S^*)^3$ which could violate the first condition in Eq. (14a) for $\chi^\alpha = s$. Moreover, in Eq. (14a), $m_{\chi^\alpha}^2$ are the eigenvalues of the mass squared matrix $M_{\alpha\beta}^2 = \partial^2 V_{tr}/\partial \chi^\alpha \partial \chi^\beta$ which are presented in Table 1. Setting

$$\widetilde{m} \geq \sqrt{3}\kappa M^2, \qquad \Lambda \leq \frac{2\sqrt{3}|k_S|}{\sqrt{2N_\star - 3}} \tag{15a}$$

(where we employ Eq. (21a) and set $a_S \ll 1$ for the derivation of the latter expression above) and, neglecting M^4 terms,

$$\sigma \geq \sigma_c \simeq \frac{\sqrt{2}\sqrt{\kappa^2 M^2 - \widetilde{m}^2}}{\kappa\sqrt{1 + M^2}} \quad \text{with } M > \frac{\widetilde{m}}{\kappa} \tag{15b}$$

assists us to achieve the positivity of m_σ^2, m_s^2 and $m_{\phi_+}^2$, respectively. Note that the two first terms in the expression for $m_{\phi_\pm}^2$ are neglected in the derivation of Eq. (15b), since their contribution is suppressed for $k_{S\Phi} \sim 1$ and $|A_\kappa| \simeq 10^{-6} - 10^{-5}$. In Table 1 we also present the masses squared of the chiral fermions of the model

along the trajectory in Eq. (5). We remark that the fermionic and bosonic degrees of freedom are equal to $2(1 + 2N)$. Inserting these masses into the well-known Coleman–Weinberg formula, we can find the one-loop RCs, ΔV, which can be written as

$$\Delta V = \frac{1}{64\pi^2}\bigg(m_\sigma^4 \ln\frac{m_\sigma^2}{Q^2} + m_s^4 \ln\frac{m_s^2}{Q^2} - 2m_{\psi_S}^4 \ln\frac{m_{\psi_S}^2}{Q^2}$$
$$+ 2N\bigg(\sum_{i=\pm} m_{\phi_i}^4 \ln\frac{m_{\phi_i}^2}{Q^2} - 2m_{\psi_\pm}^4 \ln\frac{m_{\psi_\pm}^2}{Q^2}\bigg)\bigg). \tag{16}$$

Here Q is a renormalization group mass scale and N is the dimensionality of the representations to which $\bar\Phi$ and Φ belong – we have [12,21] $N = 1, 2, 10$ for $G = G_{B-L}, G_{LR}$ and G_{5_X}, correspondingly.

All in all, the full potential of our model is

$$V_I = V_{tr} + \Delta V, \tag{17}$$

with V_{tr} and ΔV given in Eqs. (14b) and (16) respectively.

3. Constraining the model parameters

Based on V_I in Eq. (17) we proceed to explore the allowed parameter space of our model employing the standard slow-roll approximation [28]. The free parameters are

$$\kappa, M, k_S, k_{S\Phi}, \Lambda, \widetilde{m}, a_S, |A_\kappa| \quad \text{and} \quad N.$$

The parameters $k_S, k_{S\Phi}$ and $|A_\kappa|$ exclusively influence the values of m_s^2 and $m_{\phi\pm}^2$ – see Table 1 – and so, we take for them a convenient value, close to unity, which can assist us to achieve the positivity and heaviness – see below – of these masses squared, e.g., $k_S = -k_{S\Phi} = -5$ and $|A_\kappa| = 10^{-6}$. The remaining parameters can be restricted by imposing a number of observational (1, 3) and theoretical (2) restrictions specified below.

3.1. Inflationary observables

The number of e-foldings, $N_\star$, that the pivot scale $k_\star = 0.05/\text{Mpc}$ undergoes during inflation, and the amplitude A_s of the power spectrum of the curvature perturbation can be calculated using the standard formulae

$$N_\star = \int_{\sigma_f}^{\sigma_\star} d\sigma \frac{V_I}{V_I'} \quad \text{and} \quad \sqrt{A_s} = \frac{1}{2\sqrt{3}\pi}\frac{V_I^{3/2}(\sigma_\star)}{|V_I'(\sigma_\star)|} \tag{18}$$

where the prime denotes derivation with respect to σ, $\sigma_\star$ is the value of σ when $k_\star$ crosses outside the horizon of inflation, and σ_f is the value of σ at the end of inflation which coincides with σ_c, Eq. (15b), if $\epsilon(\sigma_c) \leq 1$ and $\eta(\sigma_c) \leq 1$ or is determined by the condition:

$$\max\{\epsilon(\sigma), \eta(\sigma)\} = 1 \quad \text{for } \sigma \geq \sigma_c. \tag{19a}$$

Here ϵ and η are the well-known [28] slow-roll parameters defined as follows:

$$\epsilon = \big(V_I'/\sqrt{2}V_I\big)^2 \quad \text{and} \quad \eta = V_I''/V_I. \tag{19b}$$

Agreement with the observations [5,6] requires

$$N_\star \simeq 55 \quad \text{and} \quad \sqrt{A_s} \simeq 4.686 \cdot 10^{-5}, \tag{20}$$

which allow us to restrict $\sigma_\star$ and $\widetilde{m}$. Neglecting ΔV in Eq. (17) and assuming that a_S is adequately suppressed we approach the quadratic inflationary model with

$$\epsilon = \eta = 2/\sigma^2, \qquad \sigma_{\rm f} \simeq \sqrt{2} \quad \text{and} \quad \sigma_\star \simeq 2\sqrt{N_\star}. \tag{21a}$$

Hence, inflation takes place for $\sigma \gg 1$ with $\sigma_{\rm f} \sim 1$ and $\sigma_{\rm c} \ll 1$ – see Eq. (15b). Employing the last equalities in Eqs. (18) and (21a) we find

$$\tilde{m} \simeq \sqrt{3}\sqrt{\kappa^2 M^4 N_\star^2 + 2A_s\pi^2}/N_\star = (6\text{–}40) \cdot 10^{-6}, \tag{21b}$$

for the values of Eq. (20) and κ and M of order 0.01. Therefore, the range of the $\tilde{m}$ values is somehow extended compared to those obtained in the quadratic model.

We can finally calculate $n_{\rm s}$, its running, $\alpha_{\rm s}$, and r, via the relations:

$$n_{\rm s} = 1 - 6\epsilon_\star + 2\eta_\star \simeq 1 - 2/N_\star = 0.964, \tag{22a}$$

$$\alpha_{\rm s} = \frac{2}{3}\left(4\eta_\star^2 - (n_{\rm s}-1)^2\right) - 2\xi_\star \simeq \frac{-2}{N_\star^2} = -6 \cdot 10^{-4}, \tag{22b}$$

$$r = 16\epsilon_\star \simeq 8/N_\star = 0.14, \tag{22c}$$

where $\xi \simeq m_{\rm P}^4\, V_{\rm I}' V_{\rm I}'''/V_{\rm I}^2$ and all the variables with the subscript $\star$ are evaluated at $\sigma = \sigma_\star$. These results are in agreement with the observational data [1,5,6] derived in the framework of the ΛCDM model.

Since there is no observational hint [6] for large non-Gaussianity in the cosmic microwave background, we should make sure that the masses squared of the scalar excitations in Table 1, besides m_σ^2, are greater than the Hubble parameter squared, $H_{\rm I}^2 = V_{\rm I}/3m_{\rm P}^2$, during the last 50–60 e-foldings of inflation, so that the observed curvature perturbation is generated wholly by σ as assumed in Eq. (20). The lowest $m_{\chi^\alpha}^2$ in Table 1, by far, is the one for $\chi^\alpha = s$ and its ratio to $H_{\rm I}^2$ is estimated to be

$$\frac{m_s^2}{H_{\rm I}^2}(\sigma_\star) \simeq \frac{\kappa^2 M^4 N_\star(\Lambda^2(3-2N_\star)-12k_S)}{2A_s\Lambda^2\pi^2} + \frac{3}{2N_\star}, \tag{23a}$$

employing Eq. (21b) and under the assumptions made above. Given that $m_s^2/H_{\rm I}^2$ increases as σ drops, we end up with the following condition:

$$m_s^2/H_{\rm I}^2(\sigma_\star) \geq 1, \tag{23b}$$

from which we can derive an upper bound, more restrictive than that of Eq. (15a), on Λ

$$\Lambda \lesssim \frac{\sqrt{6|k_S|N_\star}\,\kappa M^2}{\sqrt{\kappa^2 M^4 N_\star^2 + A_s\pi^2}} \tag{23c}$$

ranging from 0.74 to 0.3 as κ and M vary from 0.1 to 0.01 – recall that we use $k_S < 0$, as dictated by Eq. (23a). The most natural scale close to these Λ values is the string scale, i.e., $\Lambda = 0.1 \cdot (5/2.44) \simeq 0.2$; we thus confine ourselves to this choice for Λ onwards and restrict κ or M – with given Λ. E.g., Eq. (23b) implies:

$$M \gtrsim \sqrt{\frac{\Lambda\pi}{\kappa}}\sqrt[4]{\frac{2A_s}{N_\star(\Lambda^2(3-2N_\star)-12k_S)}}, \tag{23d}$$

which turns out to be more restrictive than that of Eq. (15b) if we make use of Eq. (21b).

3.2. The GUT phase transition

One outstanding feature of our proposal is that the inflationary scenario is followed by a GUT phase transition, in sharp contrast to the original quadratic inflation [4]. We should note, however, that $V_{\rm tr}$, Eq. (14b), develops along the track of Eq. (5) an absolute minimum at

$$\sigma_0 = \frac{\sqrt{2}\kappa\,a_S M^2}{\tilde{m}^2 - 3\kappa^2 M^4}, \tag{24}$$

which has the sign of a_S and a possible complication may be that σ gets trapped in this false vacuum and consequently no GUT phase transition takes place if $\sigma_{\rm c} \leq \sigma_0$ for $\sigma_\star > 0$, or $\sigma_{\rm c} \geq \sigma_0$ for $\sigma_\star < 0$. Note that the inflationary observables remain unchanged under the replacements

$$a_S \to -a_S \quad \text{and} \quad \sigma \to -\sigma, \tag{25}$$

since $V_{\rm tr}$ remains invariant. To assure a timely destabilization of $\bar{\Phi}$–Φ system – in the ϕ_{1+} or ϕ_{2-} direction – we impose the condition

$$\sigma_{\rm c} \geq \sigma_0 \quad \text{for } \sigma_\star > 0, \quad \text{or} \quad \sigma_{\rm c} \leq \sigma_0 \quad \text{for } \sigma_\star < 0. \tag{26}$$

The structure of $V_{\rm tr}$ for $\sigma_\star > 0$ [$\sigma_\star < 0$] is visualized in Fig. 1(a) [Fig. 1(b)], where we present $V_{\rm tr}$ – conveniently normalized such that $V_{\rm tr}(\sigma_0) = 0$ – as a function of σ for the same κ and M ($\kappa = 0.01$ and $M = 0.012$) and two different a_S values with constant $|a_S|$ taking into account Eq. (20). Namely, in Fig. 1(a), we take $a_S = -[+]2 \cdot 10^{-5}$ – gray [light gray] line – corresponding to $\sigma_\star = 13.95$ [15.9] and $\sigma_{\rm f} = 0.44$ [2.4]. As anticipated from Eqs. (26) and (15b), $V_{\rm tr}$ develops minima at the points $|\sigma_0| \simeq 0.97$, whereas $\sigma_{\rm c} \simeq 0.017$ is constant in all cases since it is independent of a_S. We observe that for $a_S < 0$, we obtain $\sigma_0 < \sigma_{\rm c}$ and so the GUT phase transition can proceed without doubt, whereas for $a_S > 0$ we have $\sigma_0 > \sigma_{\rm c}$, making the destabilization of the ϕ_+ direction – see Table 1 – rather uncertain. In Fig. 1(b), we present $V_{\rm tr}$ versus σ changing the signs of a_S and $\sigma_\star$ according Eq. (25), i.e., we set $a_S = +[-]2 \cdot 10^{-5}$ with $\sigma_\star = -13.95$ [-15.9] – gray [light gray] line. We remark that the case with $a_S < 0$ remains problematic since σ meets first $\sigma_0 = -0.97 < \sigma_{\rm c} = 0.017$ and its trapping in the minimum is possible, whereas the case with $a_S > 0$ is free from such a problem, since $\sigma_0 = 0.97 > \sigma_{\rm c} = 0.017$. Given this situation we henceforth concentrate on the case with $\sigma_\star > 0$. The results for the case with $\sigma_\star < 0$ are obtained by flipping the sign of a_S as suggested by the symmetry of $V_{\rm tr}$, Eq. (25).

3.3. Compatibility with the formation of cosmic strings

If $G = G_{B-L}$, $B-L$ cosmic strings are produced during the GUT phase transition, at the end of inflation. The tension $\mu_{\rm cs}$ of these defects has to respect the bound [12,29,30]:

$$\mu_{\rm cs} = \frac{9.6\pi M^2}{\ln(2/\beta)} \leq 8 \cdot 10^{-6}$$

$$\Rightarrow \quad M \leq 0.001\left(\frac{\ln(2/\beta)}{1.2\pi}\right)^{1/2}, \tag{27}$$

where $\beta = \kappa^2/8g^2 \leq 10^{-2}$ with $g \simeq 0.7$ being the gauge coupling constant close to $M_{\rm GUT}$. From Eq. (27), for $\kappa = 0.1, 0.01$ and 0.001, we obtain $10^3 M \leq 1.33, 1.7$ and 2, whereas Eq. (23d) entails $10^3 M \geq 2.7, 8.7$ and 27 respectively. As a consequence, our scheme is not compatible with the choice $G = G_{B-L}$. This negative result can be, most probably, avoided if we invoke the superpotential employed in shifted [15] or smooth [16] FHI. In that cases, Φ and $\bar{\Phi}$ are confined to some non-vanishing value during inflation; thus, the $B-L$ strings can be easily inflated away.

4. Results

Following our previous discussion we henceforth concentrate our analysis on $G = G_{\rm LR}$ or G_{5X}. For both selected G's, M can be related to the GUT scale since the non-singlet under $G_{\rm SM}$ gauge
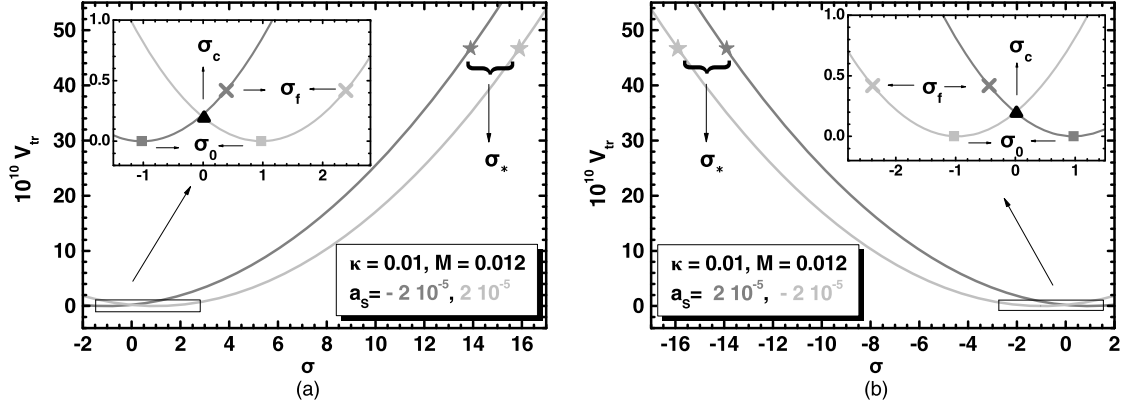
Fig. 1. Tree level inflationary potential V_{tr} as a function of σ for $\sigma_\star > 0$ and $a_S = -[+] \, 2 \cdot 10^{-5}$ (a) for $\sigma_\star < 0$ and $a_S = +[-]2 \cdot 10^{-5}$ (b) – gray [light gray] line. We set $\kappa = 0.01$, $M = 0.012$, $k_S = -5$ and $|A_\kappa| = 10^{-6}$. The values of $\sigma_\star, \sigma_f, \sigma_0$ and σ_c are also depicted.
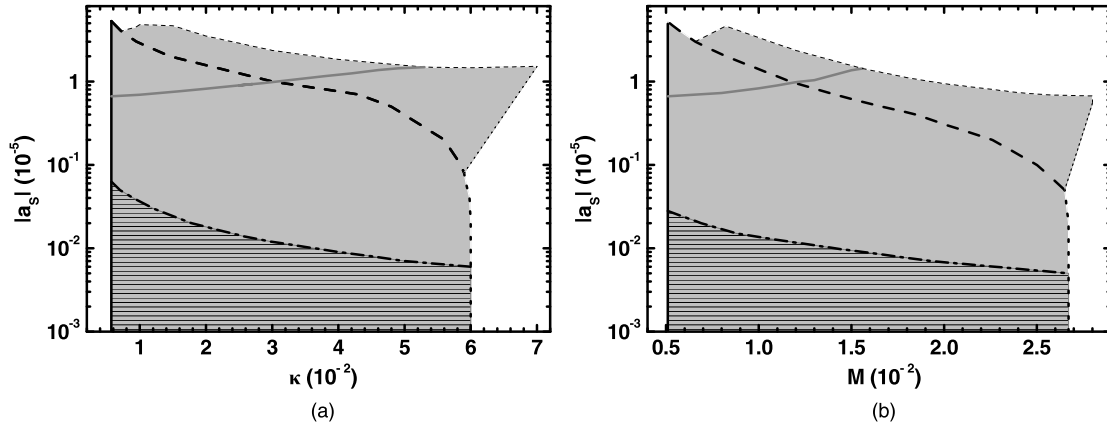

Fig. 2. Allowed (shaded [lined]) regions for $\sigma_\star > 0$ and $a_S < 0$ [$a_S > 0$] in the κ–$|a_S|$ plane with $M = 0.012$ (a) and in the M–$|a_S|$ plane with $\kappa = 0.03$ (b). Along the gray line we set $a_S = -\tilde{m}$. We take $k_S = -k_{S\Phi} = -5$, $|A_\kappa| = 10^{-6}$ and $\Lambda = 0.2$.

bosons acquire mass equal to gM at the SUSY vacuum, Eq. (4) – see Ref. [21]. However, in high-scale SUSY [24,25] the GUT scale is model dependent and so any M value between 0.001 and 0.1 is, in principle, acceptable. For reference we mention that the conventional SUSY GUT scale corresponds to the choice $gM = (2/2.44) \cdot 10^{-2}$, i.e., $M \simeq 0.012$. Recall finally that we set $k_S = -k_{S\Phi} = -5$ $|A_\kappa| = 10^{-6}$ and $\Lambda = 0.2$ throughout.

In our numerical calculations, we use the complete formulae for V_I, $N_\star$, A_S and the slow-roll parameters – see Eqs. (17), (18) and (19b) – and not the approximate relations listed in Section 3 for the sake of presentation. As regards Q in Eq. (16), we determine it by requiring [31] $\Delta V(\sigma_\star) = 0$. Note that Q is not well-defined if we impose the alternative condition [31] $\Delta V(\sigma_f) = 0$ since $m_{\phi+}$ instantaneously vanishes when $\sigma_f = \sigma_c$. To reduce the possible [31, 32] dependence of our results on the choice of Q, we confine ourselves to values of κ, M and a_S which do not enhance ΔV. As a consequence, our findings are highly independent of the specific choice of G. For definiteness we mention that we take $G = G_{5_X}$.

Confronting our model with the imposed constraints, we depict the allowed (lightly gray shaded [lined]) regions for $a_S < 0$ [$a_S > 0$] in the κ–$|a_S|$ plane with $M = 0.012$ and in the M–$|a_S|$ plane with $\kappa = 0.03$ – see Figs. 2(a) and 2(b) respectively. The left bounds in both plots come from the saturation of Eq. (23b). It is straightforward to show that the (simplified) analytical expression in Eq. (23d) is in accordance with the bound, 0.0057 [0.0051] depicted in Fig. 2(a) [Fig. 2(b)]. Had we used $k_S = -1$, this bound in Fig. 2(a) [Fig. 2(b)] would have been moved to 0.014 [0.008] cutting a minor slice of the allowed region. It is clear from Eq. (26)

that the allowed region for $a_S > 0$ is considerably shrunk compared to that for $a_S < 0$, since $a_S < 0$ implies $\sigma_0 < 0$, and so Eq. (26) is automatically fulfilled thanks to the positivity of σ_c – see Eq. (15b). Indeed, the saturation of Eq. (26) gives the upper bound of the allowed (lined) regions for $a_S < 0$. On the other hand, for $a_S < 0$ no solution to Eq. (20) exists beyond the thin dashed line. In the shaded region between the thick and thin dashed lines the end of inflation is found by the condition $\sigma_f = \sigma_c$ and not the one in Eq. (19a) which exclusively gives σ_f for $a_S > 0$, and in the regions below the thick dashed lines for $a_S < 0$. Note that for $a_S < 0$ we have allowed parameters even for $|a_S| = \tilde{m}$ which are depicted by the gray lines. Finally, beyond the (thin and thick) dotted lines, our results become unstable with respect to the variations of Q; the model predictions are, thus, less trustable and we do not pursue it any further.

Summarizing our findings from Fig. 2 the parameters of W in Eq. (1) are bounded as follows:

$$0.57 \lesssim \kappa/10^{-2} \lesssim 7 \quad \text{and} \quad 0.51 \lesssim M/10^{-2} \lesssim 2.9. \tag{28}$$

Moreover, the SSB mass parameters in Eq. (11b) are confined in the following ranges:

$$0.66 \lesssim \tilde{m}/10^{-5} \lesssim 4.4 \quad \text{and} \quad |a_S|/10^{-5} \lesssim 5.3 \, [0.063] \tag{29}$$

for $a_S < 0$ [$a_S > 0$]. The most natural framework of SSB in which our model can be embedded is that of high-scale SUSY since the $\tilde{m}$ values encountered here are roughly consistent with $m_h \simeq 126$ GeV [24]. On the other hand, split SUSY cannot be directly combined with our proposal since requiring $m_h \simeq 126$ GeV implies [25]

$\widetilde{m} \lesssim 10^8$ GeV, which is rather low to drive inflation. However, a possible coupling of S with the electroweak higgses of the minimal SUSY SM can modify this conclusion as outlined in Ref. [25].

It is worth noticing that, contrary to Ref. [20], κ and M are constrained so that the contribution to V_I from Eq. (11b) exceeds that from Eq. (9). As a consequence, our model here shares identical predictions with the original quadratic inflationary model as regards n_s, α_s and r, and so it is consistent with Bicep2 findings [1]. Indeed, for $N_\star = 55$ we find $0.12 \lesssim r \lesssim 0.14$ and

$$0.963 \lesssim n_s \lesssim 0.969, \qquad 4.7 \lesssim -\alpha_s/10^{-4} \lesssim 6.8 \qquad (30)$$

which are consistent with WMAP [5] and Planck [6] results within the ΛCDM model. Contrary to quadratic model, however, our model implies a built-in mechanism for spontaneous breaking of G at the scale M, Eq. (28), compatible with the SUSY GUT scale, $M \simeq 0.012$. The resulting mass of the inflaton at the SUSY vacuum takes values

$$6.5 \lesssim m_\sigma/10^{-6} \lesssim 8.7, \qquad (31)$$

which allow for the decay of the inflaton to right-handed neutrinos, if the relevant couplings exist. Thus, a successful scenario of non-thermal leptogenesis, along the lines of Refs. [12,33], can be easily constructed.

5. Conclusions

We have presented a framework for implementing quadratic (chaotic) inflation in realistic SUSY models which have previously been used for FHI. Namely, we have retained a $U(1)$ R-symmetry from earlier FHI which yields a unique superpotential, W, at renormalizable level, linear with respect the inflaton field. On the other hand, the Kähler potential, K, is judiciously chosen so that no extensive SUGRA corrections arise. Our model is thus protected against contributions from higher order terms in both K and W. We showed that the model displays a wide and natural range of the parameters κ, M and a_S which allows quadratic inflation to be successfully implemented, provided that the SSB mass parameter $\widetilde{m}$ lies at the intermediate energy scale motivated by high-scale (or, under some special circumstances, split) SUSY breaking. As a consequence the inflationary observables are in excellent agreement with the combined analysis of the Planck, WMAP and Bicep2 measurements.

References

[1] P.A.R. Ade, et al., Bicep2 Collaboration, arXiv:1403.3985.
[2] H. Liu, P. Mertsch, S. Sarkar, arXiv:1404.1899.
[3] M.J. Mortonson, U. Seljak, arXiv:1405.5857;
    R. Flauger, J.C. Hill, D.N. Spergel, arXiv:1405.7351.
[4] A.D. Linde, Phys. Lett. B 129 (1983) 177.
[5] G. Hinshaw, et al., WMAP Collaboration, arXiv:1212.5226.
[6] P.A.R. Ade, et al., Planck Collaboration, arXiv:1303.5082.
[7] M.U. Rehman, Q. Shafi, J.R. Wickman, Phys. Rev. D 78 (2008) 123516, arXiv:0810.3625.
[8] N. Okada, V.N. Şenoğuz, Q. Shafi, arXiv:1403.6403;
    S.M. Boucenna, S. Morisi, Q. Shafi, J.W.F. Valle, arXiv:1404.3198.
[9] T. Kobayashi, O. Seto, Phys. Rev. D 89 (2014) 103524, arXiv:1403.5055.
[10] G.R. Dvali, Q. Shafi, R.K. Schaefer, Phys. Rev. Lett. 73 (1994) 1886, arXiv:hep-ph/9406319.
[11] E.J. Copeland, et al., Phys. Rev. D 49 (1994) 6410, arXiv:astro-ph/9401011.
[12] C. Pallis, Q. Shafi, Phys. Lett. B 725 (2013) 327, arXiv:1304.5202.
[13] G.R. Dvali, G. Lazarides, Q. Shafi, Phys. Lett. B 424 (1998) 259, arXiv:hep-ph/9710314.
[14] B. Kyae, Q. Shafi, Phys. Lett. B 635 (2006) 247, arXiv:hep-ph/0510105.
[15] R. Jeannerot, et al., J. High Energy Phys. 10 (2000) 012, arXiv:hep-ph/0002151;
    S. Khalil, M.U. Rehman, Q. Shafi, E.A. Zaakouk, Phys. Rev. D 83 (2011) 063522, arXiv:1104.4143;
    M. Civiletti, M.U. Rehman, Q. Shafi, J.R. Wickman, Phys. Rev. D 84 (2011) 103505, arXiv:1010.3657;
    T. Kobayashi, O. Seto, arXiv:1404.3102.
[16] G. Lazarides, C. Panagiotakopoulos, Phys. Rev. D 52 (1995) R559, arXiv:hep-ph/9506325;
    G. Lazarides, C. Panagiotakopoulos, N.D. Vlachos, Phys. Rev. D 54 (1996) 1369, arXiv:hep-ph/9606297;
    R. Jeannerot, S. Khalil, G. Lazarides, Phys. Lett. B 506 (2001) 344, arXiv:hep-ph/0103229;
    M.U. Rehman, Q. Shafi, Phys. Rev. D 86 (2012) 027301, arXiv:1202.0011;
    S. Khalil, Q. Shafi, A. Sil, Phys. Rev. D 86 (2012) 073004, arXiv:1208.0731.
[17] A.D. Linde, A. Riotto, Phys. Rev. D 56 (1997) 1841, arXiv:hep-ph/9703209;
    V.N. Şenoğuz, Q. Shafi, Phys. Lett. B 567 (2003) 79, arXiv:hep-ph/0305089.
[18] V.N. Şenoğuz, Q. Shafi, Phys. Rev. D 71 (2005) 043514, arXiv:hep-ph/0412102;
    M.U. Rehman, Q. Shafi, J.R. Wickman, Phys. Lett. B 683 (2010) 191, arXiv:0908.3896.
[19] M. Bastero-Gil, S.F. King, Q. Shafi, Phys. Lett. B 651 (2007) 345, arXiv:hep-ph/0604198;
    B. Garbrecht, C. Pallis, A. Pilaftsis, J. High Energy Phys. 12 (2006) 038, arXiv:hep-ph/0605264;
    M.U. Rehman, V.N. Şenoğuz, Q. Shafi, Phys. Rev. D 75 (2007) 043522, arXiv:hep-ph/0612023.
[20] Q. Shafi, J.R. Wickman, Phys. Lett. B 696 (2011) 438, arXiv:1009.5340.
[21] M.U. Rehman, Q. Shafi, J.R. Wickman, Phys. Rev. D 83 (2011) 067304, arXiv:1012.0309;
    M. Civiletti, C. Pallis, Q. Shafi, Phys. Lett. B 733 (2014) 276, arXiv:1402.6254.
[22] F. Brummer, V. Domcke, V. Sanz, arXiv:1405.4868.
[23] M. Kawasaki, M. Yamaguchi, T. Yanagida, Phys. Rev. Lett. 85 (2000) 3572, arXiv:hep-ph/0004243;
    P. Brax, J. Martin, Phys. Rev. D 72 (2005) 023518, arXiv:hep-th/0504168;
    S. Antusch, K. Dutta, P.M. Kostka, Phys. Lett. B 677 (2009) 221, arXiv:0902.2934;
    R. Kallosh, A. Linde, T. Rube, Phys. Rev. D 83 (2011) 043507, arXiv:1011.5945;
    T. Li, Z. Li, D.V. Nanopoulos, J. Cosmol. Astropart. Phys. 02 (2014) 028, arXiv:1311.6770;
    K. Harigaya, T.T. Yanagida, arXiv:1403.4729;
    J. Ellis, M.A.G. Garcia, D.V. Nanopoulos, K.A. Olive, arXiv:1403.7518;
    R. Kallosh, A. Linde, A. Westphal, arXiv:1405.0270;
    A. Mazumdar, T. Noumi, M. Yamaguchi, arXiv:1405.3959.
[24] L.E. Ibáñez, I. Valenzuela, arXiv:1403.6081;
    L.J. Hall, Y. Nomura, S. Shirai, arXiv:1403.8138;
    S. Zheng, arXiv:1405.2775, and references therein.
[25] G.F. Giudice, A. Strumia, Nucl. Phys. B 858 (2012) 63, arXiv:1108.6077, and references therein.
[26] ATLAS Collaboration, CDF Collaboration, CMS Collaboration, D0 Collaboration, arXiv:1403.4427.
[27] H.P. Nilles, Phys. Rep. 110 (1984) 1;
    N. Arkani-Hamed, S. Dimopoulos, G.F. Giudice, A. Romanino, Nucl. Phys. B 709 (2005) 3, arXiv:hep-ph/0409232.
[28] D.H. Lyth, A. Riotto, Phys. Rep. 314 (1999) 1, arXiv:hep-ph/9807278;
    A. Mazumdar, J. Rocher, Phys. Rep. 497 (2011) 85, arXiv:1001.0993.
[29] P.A.R. Ade, et al., Planck Collaboration, arXiv:1303.5085.
[30] J. Rocher, M. Sakellariadou, J. Cosmol. Astropart. Phys. 03 (2005) 004, arXiv:hep-ph/0406120;
    W. Buchmüller, V. Domcke, K. Schmitz, Nucl. Phys. B 862 (2012) 587, arXiv:1202.6679.
[31] K. Enqvist, M. Karčiauskas, J. Cosmol. Astropart. Phys. 02 (2014) 034, arXiv:1312.5944.
[32] V.N. Şenoğuz, Q. Shafi, Phys. Lett. B 668 (2008) 6, arXiv:0806.2798.
[33] G. Lazarides, Q. Shafi, Phys. Lett. B 258 (1991) 305;
    K. Kumekawa, T. Moroi, T. Yanagida, Prog. Theor. Phys. 92 (1994) 437, arXiv:hep-ph/9405337.